

# Computer algebra independent integration tests

5-Inverse-trig-functions/5.6-Inverse-cosecant/5.6.1-u-a+b-arccsc-c-x^n

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3.145	$\int \frac{a+b \csc^{-1}(cx)}{x^2\sqrt{d+ex^2}} dx$	627
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3.148	$\int \frac{x^3(a+b \csc^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	641
3.149	$\int \frac{x(a+b \csc^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	646
3.150	$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex^2)^{3/2}} dx$	649
3.151	$\int \frac{a+b \csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}} dx$	651
3.152	$\int \frac{x^4(a+b \csc^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	653
3.153	$\int \frac{x^2(a+b \csc^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	655
3.154	$\int \frac{a+b \csc^{-1}(cx)}{(d+ex^2)^{3/2}} dx$	657
3.155	$\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}} dx$	660
3.156	$\int \frac{x^5(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	664
3.157	$\int \frac{x^3(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	669

3.158	$\int \frac{x(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	674
3.159	$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex^2)^{5/2}} dx$	678
3.160	$\int \frac{a+b \csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}} dx$	680
3.161	$\int \frac{x^6(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	682
3.162	$\int \frac{x^4(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	684
3.163	$\int \frac{x^2(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	686
3.164	$\int \frac{a+b \csc^{-1}(cx)}{(d+ex^2)^{5/2}} dx$	690
3.165	$\int (fx)^m (d+ex^2)^3 (a+b \csc^{-1}(cx)) dx$	694
3.166	$\int (fx)^m (d+ex^2)^2 (a+b \csc^{-1}(cx)) dx$	698
3.167	$\int (fx)^m (d+ex^2) (a+b \csc^{-1}(cx)) dx$	702
3.168	$\int \frac{(fx)^m (a+b \csc^{-1}(cx))}{d+ex^2} dx$	706
3.169	$\int \frac{(fx)^m (a+b \csc^{-1}(cx))}{(d+ex^2)^2} dx$	708
3.170	$\int (fx)^m (d+ex^2)^{3/2} (a+b \csc^{-1}(cx)) dx$	710
3.171	$\int (fx)^m \sqrt{d+ex^2} (a+b \csc^{-1}(cx)) dx$	712
3.172	$\int \frac{(fx)^m (a+b \csc^{-1}(cx))}{\sqrt{d+ex^2}} dx$	714
3.173	$\int \frac{(fx)^m (a+b \csc^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	716
3.174	$\int \frac{x^{11}(a+b \csc^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$	718
3.175	$\int \frac{x^7(a+b \csc^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$	723
3.176	$\int \frac{x^3(a+b \csc^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$	728
3.177	$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{1-c^4x^4}} dx$	732
3.178	$\int \frac{a+b \csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}} dx$	734

#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 178 ]. This is test number [ 158 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 178 )	% 0. ( 0 )
Mathematica	% 98.88 ( 176 )	% 1.12 ( 2 )
Maple	% 82.58 ( 147 )	% 17.42 ( 31 )
Maxima	% 29.78 ( 53 )	% 70.22 ( 125 )
Fricas	% 61.8 ( 110 )	% 38.2 ( 68 )
Sympy	% 7.87 ( 14 )	% 92.13 ( 164 )
Giac	% 26.97 ( 48 )	% 73.03 ( 130 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

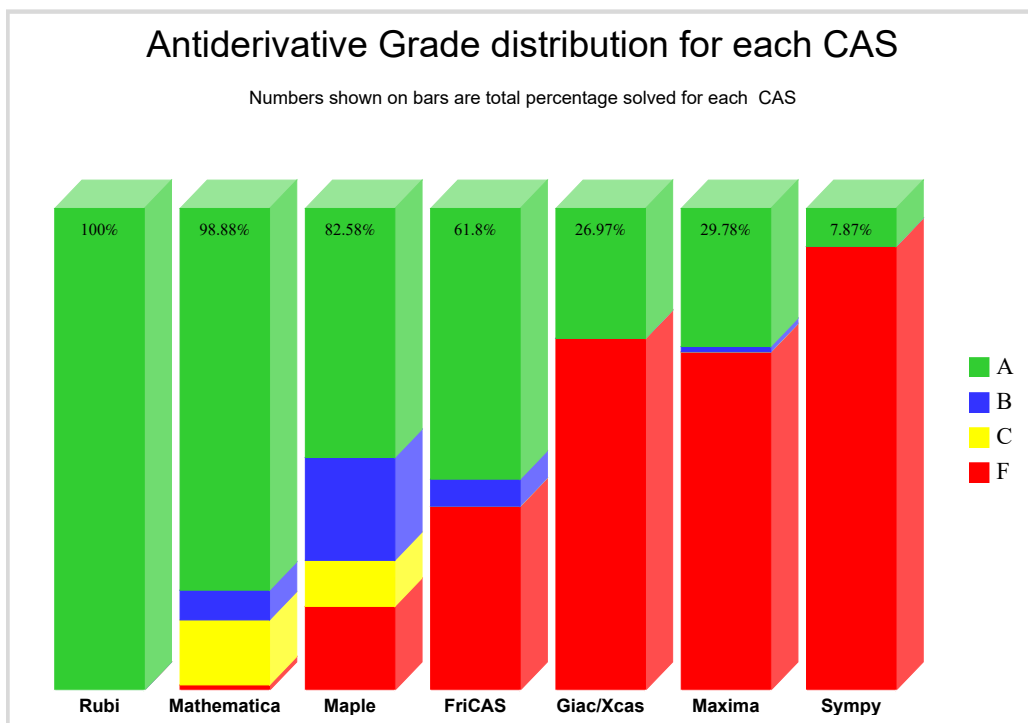
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

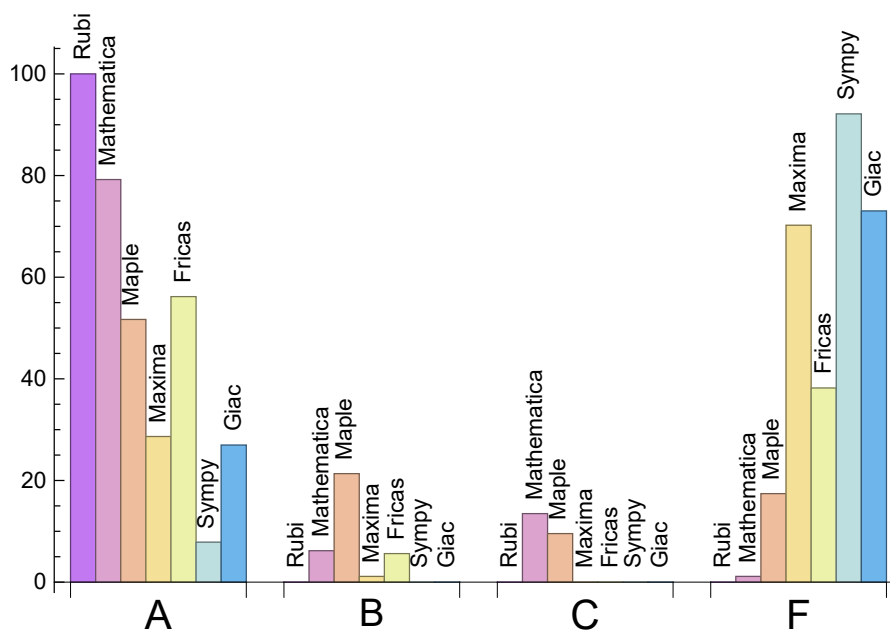
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	79.21	6.18	13.48	1.12
Maple	51.69	21.35	9.55	17.42
Maxima	28.65	1.12	0.	70.22
Fricas	56.18	5.62	0.	38.2
Sympy	7.87	0.	0.	92.13
Giac	26.97	0.	0.	73.03



The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.5	206.92	0.75	134.5	1.
Mathematica	3.68	283.32	0.93	134.5	0.9
Maple	1.48	437.35	1.5	174.	1.2
Maxima	0.82	152.57	1.31	127.	1.35
Fricas	3.06	522.86	2.94	146.5	1.61
Sympy	0.19	2.64	0.08	0.	0.
Giac	0.05	2.62	0.08	0.	0.

## 1.4 list of integrals that has no closed form antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {8, 16, 18, 19, 24, 25, 26, 27, 28, 48, 51, 53, 57, 60, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 115, 116, 117, 118, 128}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())
```

```
try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

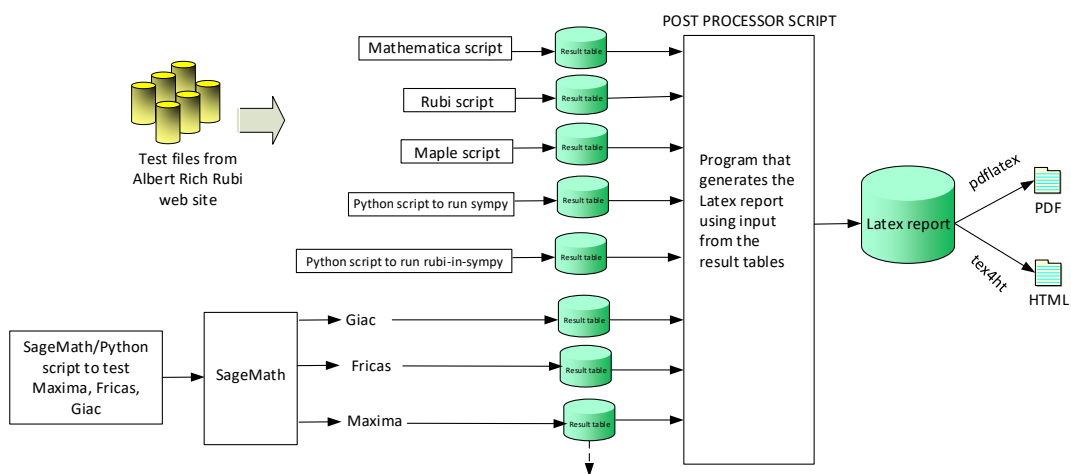
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)**

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

Nasser M. Abbasi  
June 22, 2018



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 60, 61, 62, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 101, 108, 109, 110, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { 25, 53, 72, 98, 99, 100, 102, 103, 104, 107, 111 }

C grade: { 51, 52, 56, 57, 58, 59, 63, 64, 65, 69, 70, 71, 105, 112, 113, 118, 126, 127, 128, 136, 137, 146, 155, 164 }

F grade: { 106, 114 }

#### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 18, 22, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 5, 10, 12, 15, 16, 17, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 44, 45, 48, 49, 50, 51, 52, 56, 70, 71, 72, 75, 77, 78, 88, 89, 105, 112, 113 }

C grade: { 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117 }

F grade: { 41, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 17, 20, 29, 33, 34, 35, 42, 43, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 15, 22 }

C grade: { }

F grade: { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 39, 40, 41, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 105, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 7, 17, 47, 49, 50, 112, 113, 156, 157, 158 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 126, 127, 136, 137, 145, 146, 154, 155, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.1.6 Sympy

A grade: { 9, 33, 34, 35, 40, 42, 43, 61, 121, 124, 125, 141, 144, 177 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178 }

## 2.1.7 Giac

A grade: { 7, 33, 34, 35, 39, 40, 42, 43, 47, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { }

C grade: { }



F grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	107	177	217	275	0	0
normalized size	1	1.	0.94	1.55	1.9	2.41	0.	0.
time (sec)	N/A	0.061	0.143	0.178	0.952	2.707	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	72	83	108	143	0	0
normalized size	1	1.	0.81	0.93	1.21	1.61	0.	0.
time (sec)	N/A	0.042	0.132	0.174	0.957	2.464	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	97	150	178	248	0	0
normalized size	1	1.	1.09	1.69	2.	2.79	0.	0.
time (sec)	N/A	0.048	0.075	0.177	0.975	2.567	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	74	80	119	0	0
normalized size	1	1.	0.97	1.16	1.25	1.86	0.	0.
time (sec)	N/A	0.027	0.101	0.177	0.956	2.386	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	85	123	131	219	0	0
normalized size	1	1.	1.33	1.92	2.05	3.42	0.	0.
time (sec)	N/A	0.036	0.054	0.178	0.971	2.475	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	65	49	90	0	0
normalized size	1	1.	1.28	1.67	1.26	2.31	0.	0.
time (sec)	N/A	0.012	0.024	0.171	0.985	2.329	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	72	154	0	63
normalized size	1	1.	1.87	1.19	2.32	4.97	0.	2.03
time (sec)	N/A	0.021	0.066	0.169	0.957	2.462	0.	1.095

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	53	140	0	0	0	0
normalized size	1	1.	0.83	2.19	0.	0.	0.	0.
time (sec)	N/A	0.085	0.03	0.257	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	63	45	62	37	0
normalized size	1	1.	1.28	1.97	1.41	1.94	1.16	0.
time (sec)	N/A	0.02	0.029	0.171	0.982	2.2	2.634	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	118	112	93	0	0
normalized size	1	1.	1.29	2.31	2.2	1.82	0.	0.
time (sec)	N/A	0.032	0.035	0.174	1.488	2.177	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	75	78	97	0	0
normalized size	1	1.	0.98	1.25	1.3	1.62	0.	0.
time (sec)	N/A	0.038	0.065	0.176	0.977	2.238	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	78	147	169	122	0	0
normalized size	1	1.	1.03	1.93	2.22	1.61	0.	0.
time (sec)	N/A	0.045	0.061	0.174	1.479	2.292	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	83	103	123	0	0
normalized size	1	1.	0.84	1.01	1.26	1.5	0.	0.
time (sec)	N/A	0.049	0.085	0.173	0.978	2.269	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	174	223	150	0	0
normalized size	1	1.	0.87	1.72	2.21	1.49	0.	0.
time (sec)	N/A	0.062	0.102	0.177	1.473	2.246	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	124	208	266	339	0	0
normalized size	1	1.	1.16	1.94	2.49	3.17	0.	0.
time (sec)	N/A	0.106	0.223	0.225	2.107	2.966	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	210	327	0	0	0	0
normalized size	1	1.	1.51	2.35	0.	0.	0.	0.
time (sec)	N/A	0.119	1.339	0.352	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	89	133	113	266	0	0
normalized size	1	1.	1.62	2.42	2.05	4.84	0.	0.
time (sec)	N/A	0.07	0.146	0.222	1.002	2.219	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	147	196	0	0	0	0
normalized size	1	1.	1.75	2.33	0.	0.	0.	0.
time (sec)	N/A	0.068	0.202	0.266	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	137	361	0	0	0	0
normalized size	1	1.	1.51	3.97	0.	0.	0.	0.
time (sec)	N/A	0.123	0.142	0.295	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	118	107	140	0	0
normalized size	1	1.	1.42	2.36	2.14	2.8	0.	0.
time (sec)	N/A	0.061	0.147	0.219	1.002	1.874	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	102	191	0	196	0	0
normalized size	1	1.	1.16	2.17	0.	2.23	0.	0.
time (sec)	N/A	0.078	0.116	0.214	0.	2.031	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	108	154	266	224	0	0
normalized size	1	1.	1.06	1.51	2.61	2.2	0.	0.
time (sec)	N/A	0.096	0.225	0.219	2.11	2.014	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	148	265	0	275	0	0
normalized size	1	1.	1.1	1.98	0.	2.05	0.	0.
time (sec)	N/A	0.112	0.175	0.228	0.	1.883	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	285	510	0	0	0	0
normalized size	1	1.	1.38	2.46	0.	0.	0.	0.
time (sec)	N/A	0.217	0.86	0.408	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	580	648	0	0	0	0
normalized size	1	1.	2.64	2.95	0.	0.	0.	0.
time (sec)	N/A	0.188	7.751	0.446	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	182	349	0	0	0	0
normalized size	1	1.	1.44	2.77	0.	0.	0.	0.
time (sec)	N/A	0.147	0.467	0.33	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	265	437	0	0	0	0
normalized size	1	1.	1.84	3.03	0.	0.	0.	0.
time (sec)	N/A	0.112	0.294	0.319	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	242	666	0	0	0	0
normalized size	1	1.	1.95	5.37	0.	0.	0.	0.
time (sec)	N/A	0.143	0.213	0.333	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	135	199	198	238	0	0
normalized size	1	1.	1.69	2.49	2.48	2.98	0.	0.
time (sec)	N/A	0.086	0.201	0.25	1.027	2.378	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	186	315	0	342	0	0
normalized size	1	1.	1.49	2.52	0.	2.74	0.	0.
time (sec)	N/A	0.105	0.211	0.26	0.	2.259	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	204	299	0	401	0	0
normalized size	1	1.	1.2	1.76	0.	2.36	0.	0.
time (sec)	N/A	0.15	0.3	0.261	0.	2.352	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	283	472	0	512	0	0
normalized size	1	1.	1.36	2.27	0.	2.46	0.	0.
time (sec)	N/A	0.176	0.34	0.268	0.	2.385	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	2.896	1.02	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	3.366	0.371	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.285	0.729	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	48	0	0	0	0
normalized size	1	1.	0.91	1.02	0.	0.	0.	0.
time (sec)	N/A	0.106	0.081	0.214	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	0
normalized size	1	1.	0.89	0.92	0.	0.	0.	0.
time (sec)	N/A	0.134	0.077	0.214	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	102	0	0	0	0
normalized size	1	1.	0.78	0.87	0.	0.	0.	0.
time (sec)	N/A	0.229	0.172	0.218	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	5.503	2.495	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.582	2.259	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	83	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.173	2.089	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.391	1.878	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.778	1.842	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	165	485	363	633	0	0
normalized size	1	1.	0.99	2.9	2.17	3.79	0.	0.
time (sec)	N/A	0.402	0.305	0.138	1.046	3.767	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	122	361	267	462	0	0
normalized size	1	1.	0.99	2.93	2.17	3.76	0.	0.
time (sec)	N/A	0.277	0.185	0.141	1.003	2.872	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	113	140	124	302	0	0
normalized size	1	1.	1.36	1.69	1.49	3.64	0.	0.
time (sec)	N/A	0.167	0.196	0.18	1.001	2.197	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	72	154	0	63
normalized size	1	1.	1.87	1.19	2.32	4.97	0.	2.03
time (sec)	N/A	0.023	0.053	0.173	0.977	2.073	0.	1.13

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	411	881	0	0	0	0
normalized size	1	1.	1.6	3.43	0.	0.	0.	0.
time (sec)	N/A	0.406	0.658	0.417	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	141	214	0	960	0	0
normalized size	1	1.	1.38	2.1	0.	9.41	0.	0.
time (sec)	N/A	0.155	0.221	0.263	0.	2.369	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	250	1007	0	2223	0	0
normalized size	1	1.	1.45	5.85	0.	12.92	0.	0.
time (sec)	N/A	0.292	0.515	0.174	0.	6.219	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	693	870	1222	0	0	0	0
normalized size	1	1.4	1.75	2.46	0.	0.	0.	0.
time (sec)	N/A	2.827	13.909	0.285	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	502	368	836	0	0	0	0
normalized size	1	1.24	0.91	2.07	0.	0.	0.	0.
time (sec)	N/A	2.195	1.614	0.281	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	657	388	0	0	0	0
normalized size	1	1.	2.09	1.23	0.	0.	0.	0.
time (sec)	N/A	0.485	13.262	0.273	0.	0.	0.	0.



Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	17.512	5.796	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	7.997	4.339	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	333	810	0	0	0	0
normalized size	1	1.	0.9	2.18	0.	0.	0.	0.
time (sec)	N/A	0.755	1.423	0.278	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	714	714	873	1248	0	0	0	0
normalized size	1	1.	1.22	1.75	0.	0.	0.	0.
time (sec)	N/A	2.589	13.747	0.286	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	530	784	862	0	0	0	0
normalized size	1	1.	1.48	1.63	0.	0.	0.	0.
time (sec)	N/A	1.907	13.573	0.276	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	289	412	0	0	0	0
normalized size	1	1.	0.84	1.2	0.	0.	0.	0.
time (sec)	N/A	1.6	1.429	0.278	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	243	254	0	0	0	0
normalized size	1	1.	1.15	1.2	0.	0.	0.	0.
time (sec)	N/A	0.319	3.5	0.272	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	6.289	3.53	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	8.757	4.408	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	814	890	0	0	0	0
normalized size	1	1.	1.48	1.62	0.	0.	0.	0.
time (sec)	N/A	2.474	13.83	0.281	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	750	441	0	0	0	0
normalized size	1	1.	2.03	1.2	0.	0.	0.	0.
time (sec)	N/A	1.814	13.676	0.278	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	226	277	0	0	0	0
normalized size	1	1.	0.95	1.16	0.	0.	0.	0.
time (sec)	N/A	1.575	1.74	0.276	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	124	217	0	0	0	0
normalized size	1	1.	1.04	1.82	0.	0.	0.	0.
time (sec)	N/A	0.225	0.203	0.27	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	11.07	4.336	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	13.905	3.789	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	602	602	887	1077	0	0	0	0
normalized size	1	1.	1.47	1.79	0.	0.	0.	0.
time (sec)	N/A	2.896	13.935	0.288	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	856	1038	0	0	0	0
normalized size	1	1.	1.95	2.36	0.	0.	0.	0.
time (sec)	N/A	2.185	13.789	0.289	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	345	913	0	0	0	0
normalized size	1	1.	1.1	2.91	0.	0.	0.	0.
time (sec)	N/A	2.001	1.649	0.286	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	725	886	0	0	0	0
normalized size	1	1.	2.43	2.97	0.	0.	0.	0.
time (sec)	N/A	0.409	13.513	0.279	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	25.151	3.867	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	25.371	4.704	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	540	637	1002	1640	0	0	0	0
normalized size	1	1.18	1.86	3.04	0.	0.	0.	0.
time (sec)	N/A	0.752	14.053	0.296	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	140	338	400	459	0	0
normalized size	1	1.	0.68	1.64	1.94	2.23	0.	0.
time (sec)	N/A	0.125	0.231	0.182	0.987	4.597	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	121	282	313	398	0	0
normalized size	1	1.	0.75	1.75	1.94	2.47	0.	0.
time (sec)	N/A	0.108	0.166	0.184	0.987	3.564	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	149	194	207	327	0	0
normalized size	1	1.	1.37	1.78	1.9	3.	0.	0.
time (sec)	N/A	0.051	0.261	0.184	1.017	3.453	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	104	136	120	288	0	0
normalized size	1	1.	1.2	1.56	1.38	3.31	0.	0.
time (sec)	N/A	0.065	0.119	0.186	0.989	2.908	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	69	121	127	157	0	0
normalized size	1	1.	0.66	1.15	1.21	1.5	0.	0.
time (sec)	N/A	0.078	0.104	0.187	0.983	2.603	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	94	140	185	217	0	0
normalized size	1	1.	0.62	0.92	1.22	1.43	0.	0.
time (sec)	N/A	0.092	0.16	0.185	0.973	2.626	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	110	158	232	273	0	0
normalized size	1	1.	0.56	0.8	1.18	1.39	0.	0.
time (sec)	N/A	0.118	0.165	0.19	0.979	2.523	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	115	152	247	304	0	0
normalized size	1	1.	0.59	0.78	1.26	1.55	0.	0.
time (sec)	N/A	0.15	0.227	0.182	0.98	3.008	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	97	134	192	247	0	0
normalized size	1	1.	0.63	0.88	1.25	1.61	0.	0.
time (sec)	N/A	0.122	0.206	0.183	0.992	2.916	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	78	115	132	192	0	0
normalized size	1	1.	0.57	0.83	0.96	1.39	0.	0.
time (sec)	N/A	0.096	0.095	0.178	1.003	2.783	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	103	198	0	0	0	0
normalized size	1	1.	0.83	1.6	0.	0.	0.	0.
time (sec)	N/A	0.286	0.128	0.554	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	137	201	0	0	0	0
normalized size	1	1.	1.	1.47	0.	0.	0.	0.
time (sec)	N/A	0.302	0.414	0.384	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	184	494	545	641	0	0
normalized size	1	1.	0.73	1.96	2.16	2.54	0.	0.
time (sec)	N/A	0.237	0.35	0.184	1.037	6.069	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	151	371	400	547	0	0
normalized size	1	1.	0.79	1.94	2.09	2.86	0.	0.
time (sec)	N/A	0.114	0.236	0.183	0.969	4.948	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	134	286	266	502	0	0
normalized size	1	1.	0.82	1.75	1.63	3.08	0.	0.
time (sec)	N/A	0.13	0.194	0.187	0.977	3.892	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	125	254	213	501	0	0
normalized size	1	1.	0.8	1.62	1.36	3.19	0.	0.
time (sec)	N/A	0.132	0.268	0.189	0.995	3.296	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	127	191	244	302	0	0
normalized size	1	1.	0.69	1.04	1.33	1.65	0.	0.
time (sec)	N/A	0.16	0.262	0.194	1.027	2.694	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	153	223	325	401	0	0
normalized size	1	1.	0.63	0.93	1.35	1.66	0.	0.
time (sec)	N/A	0.197	0.261	0.191	1.017	2.569	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	159	214	342	431	0	0
normalized size	1	1.	0.66	0.88	1.41	1.78	0.	0.
time (sec)	N/A	0.223	0.319	0.186	1.014	3.361	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	124	182	255	336	0	0
normalized size	1	1.	0.64	0.93	1.31	1.72	0.	0.
time (sec)	N/A	0.144	0.281	0.188	1.002	3.043	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	157	301	0	0	0	0
normalized size	1	1.	0.84	1.62	0.	0.	0.	0.
time (sec)	N/A	0.422	0.364	1.038	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	187	276	0	0	0	0
normalized size	1	1.	0.99	1.46	0.	0.	0.	0.
time (sec)	N/A	0.43	0.64	0.7	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	565	565	1260	407	0	0	0	0
normalized size	1	1.	2.23	0.72	0.	0.	0.	0.
time (sec)	N/A	1.257	1.696	1.534	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	507	507	1123	401	0	0	0	0
normalized size	1	1.	2.21	0.79	0.	0.	0.	0.
time (sec)	N/A	1.139	0.367	0.422	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	529	529	1068	272	0	0	0	0
normalized size	1	1.	2.02	0.51	0.	0.	0.	0.
time (sec)	N/A	0.87	0.381	0.94	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	479	407	2875	0	0	0	0
normalized size	1	1.	0.85	6.	0.	0.	0.	0.
time (sec)	N/A	0.9	0.857	0.57	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	572	572	1241	332	0	0	0	0
normalized size	1	1.	2.17	0.58	0.	0.	0.	0.
time (sec)	N/A	1.1	1.62	1.537	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	628	628	1480	729	0	0	0	0
normalized size	1	1.	2.36	1.16	0.	0.	0.	0.
time (sec)	N/A	1.321	5.614	0.734	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	590	590	1442	541	0	0	0	0
normalized size	1	1.	2.44	0.92	0.	0.	0.	0.
time (sec)	N/A	1.238	2.045	0.536	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	286	354	0	833	0	0
normalized size	1	1.	2.18	2.7	0.	6.36	0.	0.
time (sec)	N/A	0.121	0.726	0.276	0.	3.463	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	566	566	0	3036	0	0	0	0
normalized size	1	1.	0.	5.36	0.	0.	0.	0.
time (sec)	N/A	1.161	41.937	0.839	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	803	803	1634	1888	0	0	0	0
normalized size	1	1.	2.03	2.35	0.	0.	0.	0.
time (sec)	N/A	2.4	6.153	10.047	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	765	765	1482	1722	0	0	0	0
normalized size	1	1.	1.94	2.25	0.	0.	0.	0.
time (sec)	N/A	1.255	2.338	1.572	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	759	759	1477	1716	0	0	0	0
normalized size	1	1.	1.95	2.26	0.	0.	0.	0.
time (sec)	N/A	2.288	2.559	1.395	0.	0.	0.	0.



Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	806	806	1525	1784	0	0	0	0
normalized size	1	1.	1.89	2.21	0.	0.	0.	0.
time (sec)	N/A	2.364	2.468	10.007	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	727	727	2053	1498	0	0	0	0
normalized size	1	1.	2.82	2.06	0.	0.	0.	0.
time (sec)	N/A	1.442	7.565	0.842	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	390	1870	0	2093	0	0
normalized size	1	1.	2.48	11.91	0.	13.33	0.	0.
time (sec)	N/A	0.182	1.058	0.29	0.	6.485	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	385	1840	0	1839	0	0
normalized size	1	1.	1.99	9.53	0.	9.53	0.	0.
time (sec)	N/A	0.191	0.878	0.283	0.	6.341	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	704	704	0	5294	0	0	0	0
normalized size	1	1.	0.	7.52	0.	0.	0.	0.
time (sec)	N/A	1.318	60.225	1.979	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1144	1144	2067	3175	0	0	0	0
normalized size	1	1.	1.81	2.78	0.	0.	0.	0.
time (sec)	N/A	1.6	6.19	2.188	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1144	1144	2075	2329	0	0	0	0
normalized size	1	1.	1.81	2.04	0.	0.	0.	0.
time (sec)	N/A	3.069	6.128	2.568	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1134	1134	2060	3166	0	0	0	0
normalized size	1	1.	1.82	2.79	0.	0.	0.	0.
time (sec)	N/A	4.049	6.061	2.95	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	365	0	0	3779	0	0
normalized size	1	1.	0.91	0.	0.	9.38	0.	0.
time (sec)	N/A	1.312	0.648	6.16	0.	41.437	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	326	0	0	3036	0	0
normalized size	1	1.	1.11	0.	0.	10.33	0.	0.
time (sec)	N/A	0.409	0.721	5.756	0.	16.403	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	265	0	0	2427	0	0
normalized size	1	1.	1.36	0.	0.	12.45	0.	0.
time (sec)	N/A	0.212	0.463	3.025	0.	6.778	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	5.346	6.236	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	5.688	1.717	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	9.034	5.807	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	6.109	5.31	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	1.682	4.628	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	247	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.432	0.627	9.214	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	325	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.62	0.698	7.089	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	339	0	0	3768	0	0
normalized size	1	1.	0.91	0.	0.	10.07	0.	0.
time (sec)	N/A	0.538	0.601	3.198	0.	38.959	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	304	0	0	3006	0	0
normalized size	1	1.	1.16	0.	0.	11.47	0.	0.
time (sec)	N/A	0.288	0.715	1.773	0.	16.302	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	5.962	1.941	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	4.794	1.263	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	9.565	3.258	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	6.676	2.795	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	12.612	3.097	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	6.427	6.571	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	303	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.573	0.658	5.984	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	383	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.891	0.852	5.132	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	328	0	0	3063	0	0
normalized size	1	1.	1.02	0.	0.	9.54	0.	0.
time (sec)	N/A	1.048	0.745	6.129	0.	20.082	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	271	0	0	2449	0	0
normalized size	1	1.	1.2	0.	0.	10.88	0.	0.
time (sec)	N/A	0.327	0.515	6.874	0.	8.089	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	210	0	0	1928	0	0
normalized size	1	1.	1.59	0.	0.	14.61	0.	0.
time (sec)	N/A	0.151	0.223	4.49	0.	4.3	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	1.67	5.735	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	13.024	1.263	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	12.98	4.946	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.986	4.268	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	140	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	0.187	4.654	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	249	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.569	0.658	8.776	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	302	0	0	3167	0	0
normalized size	1	1.	1.2	0.	0.	12.57	0.	0.
time (sec)	N/A	1.074	0.608	3.643	0.	7.196	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	220	0	0	2364	0	0
normalized size	1	1.	1.41	0.	0.	15.15	0.	0.
time (sec)	N/A	0.321	0.349	3.068	0.	4.971	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	96	0	0	625	0	0
normalized size	1	1.	1.22	0.	0.	7.91	0.	0.
time (sec)	N/A	0.117	0.145	1.764	0.	3.675	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	18.216	3.144	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	23.796	1.072	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	9.31	2.306	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	4.774	2.06	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	112	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.192	1.883	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	213	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.318	0.477	3.06	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	311	0	0	4487	0	0
normalized size	1	1.	1.28	0.	0.	18.47	0.	0.
time (sec)	N/A	1.057	0.552	3.697	0.	7.852	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	173	0	0	1378	0	0
normalized size	1	1.	1.06	0.	0.	8.45	0.	0.
time (sec)	N/A	0.246	0.265	3.102	0.	5.39	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	157	0	0	1188	0	0
normalized size	1	1.	1.14	0.	0.	8.61	0.	0.
time (sec)	N/A	0.135	0.206	1.777	0.	5.2	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	34.161	1.883	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	43.148	1.093	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	13.406	3.532	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	11.675	2.412	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	185	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.283	0.278	2.079	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	249	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.554	1.879	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	566	402	0	0	0	0	0
normalized size	1	0.97	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	2.405	1.672	8.362	0.	0.	0.	0.



Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	352	293	0	0	0	0	0
normalized size	1	0.95	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.432	0.845	4.826	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	202	171	0	0	0	0	0
normalized size	1	0.94	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.511	3.074	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	1.923	2.106	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	2.643	2.455	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	1.084	3.12	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.108	5.697	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	1.407	7.631	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	1.637	4.121	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	194	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	2.527	0.291	22.796	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	159	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	2.027	0.401	7.663	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	135	138	0	0	0	0	0
normalized size	1	1.07	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.176	3.027	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.398	6.214	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	3.436	1.84	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder

the integral was to solve. In this test, problem number [69] had the largest ratio of [ 0.8571 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.	12	0.417
2	A	4	3	1.	12	0.25
3	A	6	5	1.	12	0.417
4	A	3	3	1.	12	0.25
5	A	5	5	1.	12	0.417
6	A	2	2	1.	10	0.2
7	A	5	4	1.	8	0.5
8	A	6	6	1.	12	0.5
9	A	2	2	1.	12	0.167
10	A	4	4	1.	12	0.333
11	A	4	3	1.	12	0.25
12	A	5	4	1.	12	0.333
13	A	4	3	1.	12	0.25
14	A	6	4	1.	12	0.333
15	A	5	5	1.	14	0.357
16	A	8	6	1.	14	0.429
17	A	4	4	1.	12	0.333
18	A	7	5	1.	10	0.5
19	A	6	6	1.	14	0.429
20	A	4	3	1.	14	0.214
21	A	4	3	1.	14	0.214
22	A	5	5	1.	14	0.357
23	A	5	3	1.	14	0.214
24	A	10	10	1.	14	0.714
25	A	11	8	1.	14	0.571
26	A	7	7	1.	12	0.583
27	A	9	6	1.	10	0.6
28	A	7	7	1.	14	0.5
29	A	5	3	1.	14	0.214
30	A	6	6	1.	14	0.429
31	A	8	6	1.	14	0.429
32	A	10	6	1.	14	0.429
33	A	0	0	0.	0	0.
34	A	0	0	0.	0	0.
35	A	0	0	0.	0	0.
36	A	4	4	1.	14	0.286
37	A	6	6	1.	14	0.429
38	A	9	5	1.	14	0.357
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	3	3	1.	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.
44	A	11	9	1.	16	0.562
45	A	10	9	1.	16	0.562
46	A	9	9	1.	14	0.643
47	A	5	4	1.	8	0.5
48	A	4	2	1.	16	0.125
49	A	7	7	1.	16	0.438
50	A	8	8	1.	16	0.5
51	A	31	16	1.4	21	0.762
52	A	24	15	1.24	19	0.79
53	A	15	11	1.	18	0.611
54	A	0	0	0.	0	0.
55	A	0	0	0.	0	0.
56	A	22	13	1.	18	0.722
57	A	27	17	1.	21	0.81
58	A	20	15	1.	21	0.714
59	A	14	13	1.	19	0.684
60	A	9	9	1.	18	0.5
61	A	0	0	0.	0	0.
62	A	0	0	0.	0	0.
63	A	23	15	1.	21	0.714
64	A	16	13	1.	21	0.619
65	A	11	11	1.	19	0.579
66	A	6	6	1.	18	0.333
67	A	0	0	0.	0	0.
68	A	0	0	0.	0	0.
69	A	31	18	1.	21	0.857
70	A	25	17	1.	21	0.81
71	A	19	14	1.	19	0.737
72	A	12	11	1.	18	0.611
73	A	0	0	0.	0	0.
74	A	0	0	0.	0	0.
75	A	19	14	1.18	18	0.778
76	A	7	7	1.	19	0.368
77	A	6	7	1.	19	0.368
78	A	5	5	1.	16	0.312
79	A	4	5	1.	19	0.263
80	A	4	5	1.	19	0.263
81	A	5	6	1.	19	0.316
82	A	6	6	1.	19	0.316
83	A	5	5	1.	19	0.263
84	A	5	5	1.	19	0.263
85	A	6	5	1.	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	11	1.	19	0.579
87	A	13	13	1.	19	0.684
88	A	7	8	1.	21	0.381
89	A	6	7	1.	18	0.389
90	A	6	7	1.	21	0.333
91	A	6	7	1.	21	0.333
92	A	5	6	1.	21	0.286
93	A	6	7	1.	21	0.333
94	A	5	6	1.	21	0.286
95	A	6	5	1.	19	0.263
96	A	12	13	1.	21	0.619
97	A	14	15	1.	21	0.714
98	A	25	12	1.	21	0.571
99	A	26	9	1.	19	0.474
100	A	19	7	1.	18	0.389
101	A	19	7	1.	21	0.333
102	A	24	10	1.	21	0.476
103	A	31	14	1.	21	0.667
104	A	29	12	1.	21	0.571
105	A	7	5	1.	19	0.263
106	A	24	10	1.	21	0.476
107	A	51	15	1.	21	0.714
108	A	27	10	1.	21	0.476
109	A	47	11	1.	18	0.611
110	A	50	13	1.	21	0.619
111	A	33	13	1.	21	0.619
112	A	6	7	1.	21	0.333
113	A	8	6	1.	19	0.316
114	A	28	11	1.	21	0.524
115	A	35	11	1.	21	0.524
116	A	63	12	1.	21	0.571
117	A	81	12	1.	18	0.667
118	A	12	12	1.	23	0.522
119	A	11	12	1.	23	0.522
120	A	9	9	1.	21	0.429
121	A	0	0	0.	0	0.
122	A	0	0	0.	0	0.
123	A	0	0	0.	0	0.
124	A	0	0	0.	0	0.
125	A	0	0	0.	0	0.
126	A	11	11	1.	23	0.478
127	A	12	12	1.	23	0.522
128	A	12	12	1.	23	0.522
129	A	10	10	1.	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	0	0	0.	0	0.
131	A	0	0	0.	0	0.
132	A	0	0	0.	0	0.
133	A	0	0	0.	0	0.
134	A	0	0	0.	0	0.
135	A	0	0	0.	0	0.
136	A	12	12	1.	23	0.522
137	A	13	12	1.	23	0.522
138	A	11	12	1.	23	0.522
139	A	10	12	1.	23	0.522
140	A	8	8	1.	21	0.381
141	A	0	0	0.	0	0.
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	0	0	0.	0	0.
145	A	11	11	1.	23	0.478
146	A	11	12	1.	23	0.522
147	A	10	11	1.	23	0.478
148	A	9	11	1.	23	0.478
149	A	4	4	1.	21	0.19
150	A	0	0	0.	0	0.
151	A	0	0	0.	0	0.
152	A	0	0	0.	0	0.
153	A	0	0	0.	0	0.
154	A	5	5	1.	20	0.25
155	A	10	11	1.	23	0.478
156	A	10	11	1.	23	0.478
157	A	7	8	1.	23	0.348
158	A	5	5	1.	21	0.238
159	A	0	0	0.	0	0.
160	A	0	0	0.	0	0.
161	A	0	0	0.	0	0.
162	A	0	0	0.	0	0.
163	A	10	10	1.	23	0.435
164	A	10	11	1.	20	0.55
165	A	6	7	0.97	23	0.304
166	A	6	7	0.95	23	0.304
167	A	5	6	0.94	21	0.286
168	A	0	0	0.	0	0.
169	A	0	0	0.	0	0.
170	A	0	0	0.	0	0.
171	A	0	0	0.	0	0.
172	A	0	0	0.	0	0.
173	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
174	A	16	11	1.	26	0.423
175	A	13	11	1.	26	0.423
176	A	8	9	1.07	26	0.346
177	A	0	0	0.	0	0.
178	A	0	0	0.	0	0.





# Chapter 3

## Listing of integrals

### 3.1 $\int x^6 \left( a + b \operatorname{csc}^{-1}(cx) \right) dx$

**Optimal.** Leaf size=114

$$\frac{1}{7}x^7 \left( a + b \operatorname{csc}^{-1}(cx) \right) + \frac{bx^6 \sqrt{1 - \frac{1}{c^2x^2}}}{42c} + \frac{5bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{168c^3} + \frac{5bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{112c^5} + \frac{5b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{112c^7}$$

[Out] (5\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(112\*c^5) + (5\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4)/(168\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^6)/(42\*c) + (x^7\*(a + b\*ArcCsc[c\*x]))/7 + (5\*b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(112\*c^7)

---

**Rubi [A]** time = 0.0614145, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5221, 266, 51, 63, 208}

$$\frac{1}{7}x^7 \left( a + b \operatorname{csc}^{-1}(cx) \right) + \frac{bx^6 \sqrt{1 - \frac{1}{c^2x^2}}}{42c} + \frac{5bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{168c^3} + \frac{5bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{112c^5} + \frac{5b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{112c^7}$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*ArcCsc[c\*x]), x]

[Out] (5\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(112\*c^5) + (5\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4)/(168\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^6)/(42\*c) + (x^7\*(a + b\*ArcCsc[c\*x]))/7 + (5\*b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(112\*c^7)

#### Rule 5221

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rubi steps

$$\begin{aligned}
\int x^6 (a + b \csc^{-1}(cx)) dx &= \frac{1}{7} x^7 (a + b \csc^{-1}(cx)) + \frac{b \int \frac{x^5}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{7c} \\
&= \frac{1}{7} x^7 (a + b \csc^{-1}(cx)) - \frac{b \operatorname{Subst}\left(\int \frac{1}{x^4 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{14c} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \csc^{-1}(cx)) - \frac{(5b) \operatorname{Subst}\left(\int \frac{1}{x^3 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{84c^3} \\
&= \frac{5b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \csc^{-1}(cx)) - \frac{(5b) \operatorname{Subst}\left(\int \frac{1}{x^2 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{112c^5} \\
&= \frac{5b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{112c^5} + \frac{5b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \csc^{-1}(cx)) - \frac{(5b) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, \frac{1}{x^2}\right)}{224c^7} \\
&= \frac{5b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{112c^5} + \frac{5b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \csc^{-1}(cx)) + \frac{(5b) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, \frac{1}{x^2}\right)}{224c^7} \\
&= \frac{5b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{112c^5} + \frac{5b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \csc^{-1}(cx)) + \frac{5b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{112c^7}
\end{aligned}$$

**Mathematica [A]** time = 0.142633, size = 107, normalized size = 0.94

$$\frac{ax^7}{7} + b \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} \left( \frac{5x^4}{168c^3} + \frac{5x^2}{112c^5} + \frac{x^6}{42c} \right) + \frac{5b \log\left(x \left(\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + 1\right)\right)}{112c^7} + \frac{1}{7} b x^7 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*ArcCsc[c*x]),x]
```

[Out]  $(a*x^7)/7 + b*\text{Sqrt}[(-1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) + (5*x^4)/(168*c^3) + x^6/(42*c)) + (b*x^7*\text{ArcCsc}[c*x])/7 + (5*b*\text{Log}[x*(1 + \text{Sqrt}[(-1 + c^2*x^2)/(c^2*x^2)])])/(112*c^7)$

**Maple [A]** time = 0.178, size = 177, normalized size = 1.6

$$\frac{x^7 a}{7} + \frac{b x^7 \operatorname{arccsc}(c x)}{7} + \frac{b x^6}{42 c} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{b x^4}{168 c^3} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{5 b x^2}{336 c^5} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{5 b}{112 c^7} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{5 b}{112 c^8 x} \sqrt{c^2 x^2 - 1} \ln\left(\frac{c x + \sqrt{c^2 x^2 - 1}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(a+b*arccsc(c*x)),x)`

[Out]  $1/7*x^7*a+1/7*b*x^7*\operatorname{arccsc}(c*x)+1/42/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^6+1/168/c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^4+5/336/c^5*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^2-5/112/c^7*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}+5/112/c^8*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*\ln(c*x+(c^2*x^2-1)^{(1/2)})$

**Maxima [A]** time = 0.952431, size = 217, normalized size = 1.9

$$\frac{1}{7} a x^7 + \frac{1}{672} \left( 96 x^7 \operatorname{arccsc}(c x) + \frac{2 \left( 15 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} - 40 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 33 \sqrt{-\frac{1}{c^2 x^2} + 1} \right)}{c^6 \left( \frac{1}{c^2 x^2} - 1 \right)^3 + 3 c^6 \left( \frac{1}{c^2 x^2} - 1 \right)^2 + 3 c^6 \left( \frac{1}{c^2 x^2} - 1 \right) + c^6} + \frac{15 \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} + 1\right)}{c^6} - \frac{15 \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1} - 1\right)}{c^6} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out]  $1/7*a*x^7 + 1/672*(96*x^7*\operatorname{arccsc}(c*x) + (2*(15*(-1/(c^2*x^2) + 1))^{(5/2)} - 40*(-1/(c^2*x^2) + 1)^{(3/2)} + 33*\sqrt{-1/(c^2*x^2) + 1})/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*\log(\sqrt{-1/(c^2*x^2) + 1} + 1)/c^6 - 15*\log(\sqrt{-1/(c^2*x^2) + 1} - 1)/c^6)/c)*b$

**Fricas [A]** time = 2.70702, size = 275, normalized size = 2.41

$$\frac{48 a c^7 x^7 - 96 b c^7 \arctan\left(-c x + \sqrt{c^2 x^2 - 1}\right) + 48 \left(b c^7 x^7 - b c^7\right) \operatorname{arccsc}(c x) - 15 b \log\left(-c x + \sqrt{c^2 x^2 - 1}\right) + \left(8 b c^5 x^5 + 10 b c^3 x^3 + 15 b c x\right) \sqrt{c^2 x^2 - 1}}{336 c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out]  $1/336*(48*a*c^7*x^7 - 96*b*c^7*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) + 48*(b*c^7*x^7 - b*c^7)*\operatorname{arccsc}(c*x) - 15*b*\log(-c*x + \sqrt{c^2*x^2 - 1}) + (8*b*c^5*x^5 + 10*b*c^3*x^3 + 15*b*c*x)*\sqrt{c^2*x^2 - 1})/c^7$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^6 (a + b \operatorname{acsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(a+b*acsc(c*x)),x)`

[Out] `Integral(x**6*(a + b*acsc(c*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^6, x)`

### 3.2 $\int x^5 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=89

$$\frac{1}{6}x^6 (a + b \csc^{-1}(cx)) + \frac{bx^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{30c} + \frac{2bx^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^3} + \frac{4bx \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^5}$$

[Out]  $(4*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(45*c^5) + (2*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3)/(45*c^3) + (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*\text{ArcCsc}[c*x]))/6$

**Rubi [A]** time = 0.0418436, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5221, 271, 191}

$$\frac{1}{6}x^6 (a + b \csc^{-1}(cx)) + \frac{bx^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{30c} + \frac{2bx^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^3} + \frac{4bx \sqrt{1 - \frac{1}{c^2 x^2}}}{45c^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5*(a + b*\text{ArcCsc}[c*x]), x]$

[Out]  $(4*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(45*c^5) + (2*b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^3)/(45*c^3) + (b*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*\text{ArcCsc}[c*x]))/6$

#### Rule 5221

$\text{Int}[(a + \text{ArcCsc}[c*x])*(d*x)^m, x] \text{Symbol} \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcCsc}[c*x])/(d*(m+1)), x] + \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{m-1}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 271

$\text{Int}[x^m*(a + b*x^n)^p, x] \text{Symbol} \rightarrow \text{Simp}[x^{m+1}*(a + b*x^n)^{p+1}/(a*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1)+1))/(a*(m+1)), \text{Int}[x^{m+n}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, m, n, p\}, x \ \&\& \ \text{IntQ}[\text{Simplify}[(m+1)/n + p + 1], 0] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 191

$\text{Int}[(a + b*x^n)^p, x] \text{Symbol} \rightarrow \text{Simp}[x*(a + b*x^n)^{p+1}/a, x] /; \text{FreeQ}\{a, b, n, p\}, x \ \&\& \ \text{EqQ}[1/n + p + 1, 0]$

#### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \csc^{-1}(cx)) dx &= \frac{1}{6} x^6 (a + b \csc^{-1}(cx)) + \frac{b \int \frac{x^4}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{6c} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^5}{30c} + \frac{1}{6} x^6 (a + b \csc^{-1}(cx)) + \frac{(2b) \int \frac{x^2}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{15c^3} \\
&= \frac{2b \sqrt{1 - \frac{1}{c^2 x^2}} x^3}{45c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^5}{30c} + \frac{1}{6} x^6 (a + b \csc^{-1}(cx)) + \frac{(4b) \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{45c^5} \\
&= \frac{4b \sqrt{1 - \frac{1}{c^2 x^2}} x}{45c^5} + \frac{2b \sqrt{1 - \frac{1}{c^2 x^2}} x^3}{45c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^5}{30c} + \frac{1}{6} x^6 (a + b \csc^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.131926, size = 72, normalized size = 0.81

$$\frac{ax^6}{6} + b \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} \left( \frac{2x^3}{45c^3} + \frac{4x}{45c^5} + \frac{x^5}{30c} \right) + \frac{1}{6} bx^6 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcCsc[c\*x]),x]

[Out] (a\*x^6)/6 + b\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)]\*((4\*x)/(45\*c^5) + (2\*x^3)/(45\*c^3) + x^5/(30\*c)) + (b\*x^6\*ArcCsc[c\*x])/6

**Maple [A]** time = 0.174, size = 83, normalized size = 0.9

$$\frac{1}{c^6} \left( \frac{c^6 x^6 a}{6} + b \left( \frac{c^6 x^6 \operatorname{arccsc}(cx)}{6} + \frac{(c^2 x^2 - 1)(3c^4 x^4 + 4c^2 x^2 + 8)}{90cx} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsc(c\*x)),x)

[Out] 1/c^6\*(1/6\*c^6\*x^6\*a+b\*(1/6\*c^6\*x^6\*arccsc(c\*x)+1/90\*(c^2\*x^2-1)\*(3\*c^4\*x^4+4\*c^2\*x^2+8)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c/x))

**Maxima [A]** time = 0.957131, size = 108, normalized size = 1.21

$$\frac{1}{6} ax^6 + \frac{1}{90} \left( 15x^6 \operatorname{arccsc}(cx) + \frac{3c^4 x^5 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} + 10c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 15x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^5} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out]  $1/6*a*x^6 + 1/90*(15*x^6*\arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1))^{(5/2)} + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^{(3/2)} + 15*x*\sqrt{-1/(c^2*x^2) + 1})/c^5)*b$

**Fricas [A]** time = 2.4637, size = 143, normalized size = 1.61

$$\frac{15bc^6x^6 \arccsc(cx) + 15ac^6x^6 + (3bc^4x^4 + 4bc^2x^2 + 8b)\sqrt{c^2x^2 - 1}}{90c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out]  $1/90*(15*b*c^6*x^6*\arccsc(c*x) + 15*a*c^6*x^6 + (3*b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*\sqrt{c^2*x^2 - 1})/c^6$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{acsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*acsc(c*x)),x)`

[Out] `Integral(x**5*(a + b*acsc(c*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^5, x)`

### 3.3 $\int x^4 \left( a + b \csc^{-1}(cx) \right) dx$

**Optimal.** Leaf size=89

$$\frac{1}{5}x^5 \left( a + b \csc^{-1}(cx) \right) + \frac{bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{20c} + \frac{3bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{40c^3} + \frac{3b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{40c^5}$$

[Out] (3\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(40\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4)/(20\*c) + (x^5\*(a + b\*ArcCsc[c\*x]))/5 + (3\*b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(40\*c^5)

**Rubi [A]** time = 0.0475787, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5221, 266, 51, 63, 208}

$$\frac{1}{5}x^5 \left( a + b \csc^{-1}(cx) \right) + \frac{bx^4 \sqrt{1 - \frac{1}{c^2x^2}}}{20c} + \frac{3bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{40c^3} + \frac{3b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{40c^5}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcCsc[c\*x]),x]

[Out] (3\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(40\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4)/(20\*c) + (x^5\*(a + b\*ArcCsc[c\*x]))/5 + (3\*b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(40\*c^5)

#### Rule 5221

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]



Rule 208

$\text{Int}[(a + b \cdot (x^2)^{-1}), x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int x^4 (a + b \csc^{-1}(cx)) dx &= \frac{1}{5} x^5 (a + b \csc^{-1}(cx)) + \frac{b \int \frac{x^3}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{5c} \\ &= \frac{1}{5} x^5 (a + b \csc^{-1}(cx)) - \frac{b \text{Subst}\left(\int \frac{1}{x^3 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{10c} \\ &= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \csc^{-1}(cx)) - \frac{(3b) \text{Subst}\left(\int \frac{1}{x^2 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{40c^3} \\ &= \frac{3b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{40c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \csc^{-1}(cx)) - \frac{(3b) \text{Subst}\left(\int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{80c^5} \\ &= \frac{3b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{40c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \csc^{-1}(cx)) + \frac{(3b) \text{Subst}\left(\int \frac{1}{c^2 - c^2 x^2} dx, x, \sqrt{1 - \frac{1}{c^2 x^2}}\right)}{40c^3} \\ &= \frac{3b \sqrt{1 - \frac{1}{c^2 x^2}} x^2}{40c^3} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \csc^{-1}(cx)) + \frac{3b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{40c^5} \end{aligned}$$

**Mathematica [A]** time = 0.0750523, size = 97, normalized size = 1.09

$$\frac{ax^5}{5} + b \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} \left( \frac{3x^2}{40c^3} + \frac{x^4}{20c} \right) + \frac{3b \log\left(x \left( \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + 1 \right)\right)}{40c^5} + \frac{1}{5} b x^5 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcCsc[c\*x]), x]

[Out] (a\*x^5)/5 + b\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)]\*((3\*x^2)/(40\*c^3) + x^4/(20\*c)) + (b\*x^5\*ArcCsc[c\*x])/5 + (3\*b\*Log[x\*(1 + Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])])/(40\*c^5)

**Maple [A]** time = 0.177, size = 150, normalized size = 1.7

$$\frac{ax^5}{5} + \frac{x^5 b \operatorname{arccsc}(cx)}{5} + \frac{bx^4}{20c} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{bx^2}{40c^3} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{3b}{40c^5} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{3b}{40c^6 x} \sqrt{c^2 x^2 - 1} \ln\left(cx + \sqrt{c^2 x^2 - 1}\right) \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsc(c\*x)), x)

[Out] 1/5\*a\*x^5+1/5\*x^5\*b\*arccsc(c\*x)+1/20/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4+1/40/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2-3/40/c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)+3/40/c^6\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*ln(c\*x+(c^2

$*x^2-1)^{(1/2)}$

---

**Maxima [A]** time = 0.97463, size = 178, normalized size = 2.

$$\frac{1}{5}ax^5 + \frac{1}{80} \left( 16x^5 \operatorname{arccsc}(cx) - \frac{2 \left( 3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 5 \sqrt{-\frac{1}{c^2x^2} + 1} \right) - 3 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} + 1 \right) + 3 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} - 1 \right)}{c^4 \left( \frac{1}{c^2x^2} - 1 \right)^2 + 2c^4 \left( \frac{1}{c^2x^2} - 1 \right) + c^4} + \frac{3 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} + 1 \right) - 3 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} - 1 \right)}{c^4} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] 1/5\*a\*x^5 + 1/80\*(16\*x^5\*arccsc(c\*x) - (2\*(3\*(-1/(c^2\*x^2) + 1)^(3/2) - 5\*sqrt(-1/(c^2\*x^2) + 1))/(c^4\*(1/(c^2\*x^2) - 1)^2 + 2\*c^4\*(1/(c^2\*x^2) - 1) + c^4) - 3\*log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^4 + 3\*log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^4)/c)\*b

---

**Fricas [A]** time = 2.56734, size = 248, normalized size = 2.79

$$\frac{8ac^5x^5 - 16bc^5 \arctan(-cx + \sqrt{c^2x^2 - 1}) + 8(bc^5x^5 - bc^5) \operatorname{arccsc}(cx) - 3b \log(-cx + \sqrt{c^2x^2 - 1}) + (2bc^3x^3 + 3bcx)\sqrt{c^2x^2 - 1}}{40c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] 1/40\*(8\*a\*c^5\*x^5 - 16\*b\*c^5\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + 8\*(b\*c^5\*x^5 - b\*c^5)\*arccsc(c\*x) - 3\*b\*log(-c\*x + sqrt(c^2\*x^2 - 1)) + (2\*b\*c^3\*x^3 + 3\*b\*c\*x)\*sqrt(c^2\*x^2 - 1))/c^5

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*arccsc(c\*x)),x)

[Out] Integral(x\*\*4\*(a + b\*arccsc(c\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^4, x)
```

### 3.4 $\int x^3 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=64

$$\frac{1}{4}x^4(a + b \csc^{-1}(cx)) + \frac{bx^3\sqrt{1 - \frac{1}{c^2x^2}}}{12c} + \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}}{6c^3}$$

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(6\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3)/(12\*c) + (x^4\*(a + b\*ArcCsc[c\*x]))/4

**Rubi [A]** time = 0.0265763, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5221, 271, 191}

$$\frac{1}{4}x^4(a + b \csc^{-1}(cx)) + \frac{bx^3\sqrt{1 - \frac{1}{c^2x^2}}}{12c} + \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}}{6c^3}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcCsc[c\*x]), x]

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(6\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3)/(12\*c) + (x^4\*(a + b\*ArcCsc[c\*x]))/4

#### Rule 5221

Int[((a\_) + ArcCsc[(c\_)\*(x\_)])\*(b\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

#### Rule 191

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rubi steps

$$\begin{aligned} \int x^3 (a + b \csc^{-1}(cx)) dx &= \frac{1}{4}x^4(a + b \csc^{-1}(cx)) + \frac{b \int \frac{x^2}{\sqrt{1 - \frac{1}{c^2x^2}}} dx}{4c} \\ &= \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{4}x^4(a + b \csc^{-1}(cx)) + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2x^2}}} dx}{6c^3} \\ &= \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x}{6c^3} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{4}x^4(a + b \csc^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.101153, size = 62, normalized size = 0.97

$$\frac{ax^4}{4} + b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\left(\frac{x}{6c^3} + \frac{x^3}{12c}\right) + \frac{1}{4}bx^4 \operatorname{csc}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcCsc[c\*x]), x]

[Out] (a\*x^4)/4 + b\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)]\*(x/(6\*c^3) + x^3/(12\*c)) + (b\*x^4\*ArcCsc[c\*x])/4

**Maple [A]** time = 0.177, size = 74, normalized size = 1.2

$$\frac{1}{c^4} \left( \frac{c^4 x^4 a}{4} + b \left( \frac{c^4 x^4 \operatorname{arccsc}(cx)}{4} + \frac{(c^2 x^2 - 1)(c^2 x^2 + 2)}{12 cx} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsc(c\*x)), x)

[Out] 1/c^4\*(1/4\*c^4\*x^4\*a+b\*(1/4\*c^4\*x^4\*arccsc(c\*x)+1/12\*(c^2\*x^2-1)\*(c^2\*x^2+2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c/x))

**Maxima [A]** time = 0.956258, size = 80, normalized size = 1.25

$$\frac{1}{4}ax^4 + \frac{1}{12} \left( 3x^4 \operatorname{arccsc}(cx) + \frac{c^2x^3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 3x\sqrt{-\frac{1}{c^2x^2} + 1}}{c^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x)), x, algorithm="maxima")

[Out] 1/4\*a\*x^4 + 1/12\*(3\*x^4\*arccsc(c\*x) + (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b

**Fricas [A]** time = 2.3862, size = 119, normalized size = 1.86

$$\frac{3bc^4x^4 \operatorname{arccsc}(cx) + 3ac^4x^4 + (bc^2x^2 + 2b)\sqrt{c^2x^2 - 1}}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x)), x, algorithm="fricas")

[Out] 1/12\*(3\*b\*c^4\*x^4\*arccsc(c\*x) + 3\*a\*c^4\*x^4 + (b\*c^2\*x^2 + 2\*b)\*sqrt(c^2\*x^2 - 1))/c^4

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{acsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*acsc(c*x)),x)`

[Out] `Integral(x**3*(a + b*acsc(c*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^3, x)`

### 3.5 $\int x^2 \left( a + b \csc^{-1}(cx) \right) dx$

**Optimal.** Leaf size=64

$$\frac{1}{3}x^3 \left( a + b \csc^{-1}(cx) \right) + \frac{bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{6c} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{6c^3}$$

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(6\*c) + (x^3\*(a + b\*ArcCsc[c\*x]))/3 + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(6\*c^3)

**Rubi [A]** time = 0.035954, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5221, 266, 51, 63, 208}

$$\frac{1}{3}x^3 \left( a + b \csc^{-1}(cx) \right) + \frac{bx^2 \sqrt{1 - \frac{1}{c^2x^2}}}{6c} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2x^2}} \right)}{6c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcCsc[c\*x]), x]

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(6\*c) + (x^3\*(a + b\*ArcCsc[c\*x]))/3 + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(6\*c^3)

#### Rule 5221

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(((d\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] := Simp[(((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int x^2 (a + b \csc^{-1}(cx)) dx &= \frac{1}{3} x^3 (a + b \csc^{-1}(cx)) + \frac{b \int \frac{x}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{3c} \\
 &= \frac{1}{3} x^3 (a + b \csc^{-1}(cx)) - \frac{b \operatorname{Subst}\left(\int \frac{1}{x^2 \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{6c} \\
 &= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}}}{6c} + \frac{1}{3} x^3 (a + b \csc^{-1}(cx)) - \frac{b \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{12c^3} \\
 &= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}}}{6c} + \frac{1}{3} x^3 (a + b \csc^{-1}(cx)) + \frac{b \operatorname{Subst}\left(\int \frac{1}{c^2 - c^2 x^2} dx, x, \sqrt{1 - \frac{1}{c^2 x^2}}\right)}{6c} \\
 &= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}}}{6c} + \frac{1}{3} x^3 (a + b \csc^{-1}(cx)) + \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{6c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.0536283, size = 85, normalized size = 1.33

$$\frac{ax^3}{3} + \frac{bx^2 \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{6c} + \frac{b \log\left(x \left(\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + 1\right)\right)}{6c^3} + \frac{1}{3} bx^3 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcCsc[c\*x]),x]

[Out] (a\*x^3)/3 + (b\*x^2\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(6\*c) + (b\*x^3\*ArcCsc[c\*x])/3 + (b\*Log[x\*(1 + Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])])/(6\*c^3)

**Maple [B]** time = 0.178, size = 123, normalized size = 1.9

$$\frac{x^3 a}{3} + \frac{x^3 b \operatorname{arccsc}(cx)}{3} + \frac{bx^2}{6c} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b}{6c^3} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{b}{6c^4 x} \sqrt{c^2 x^2 - 1} \ln\left(cx + \sqrt{c^2 x^2 - 1}\right) \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x)),x)

[Out] 1/3\*x^3\*a+1/3\*x^3\*b\*arccsc(c\*x)+1/6/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2-1/6/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)+1/6/c^4\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*ln(c\*x+(c^2\*x^2-1)^(1/2))



**Maxima [A]** time = 0.971402, size = 131, normalized size = 2.05

$$\frac{1}{3}ax^3 + \frac{1}{12} \left( 4x^3 \operatorname{arccsc}(cx) + \frac{2\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{\frac{1}{c^2x^2}-1+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 1/12\*(4\*x^3\*arccsc(c\*x) + (2\*sqrt(-1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) - 1) + c^2) + log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*b

**Fricas [A]** time = 2.47486, size = 219, normalized size = 3.42

$$\frac{2ac^3x^3 - 4bc^3 \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + \sqrt{c^2x^2 - 1}bcx + 2(bc^3x^3 - bc^3) \operatorname{arccsc}(cx) - b \log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] 1/6\*(2\*a\*c^3\*x^3 - 4\*b\*c^3\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + sqrt(c^2\*x^2 - 1)\*b\*c\*x + 2\*(b\*c^3\*x^3 - b\*c^3)\*arccsc(c\*x) - b\*log(-c\*x + sqrt(c^2\*x^2 - 1)))/c^3

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*arccsc(c\*x)),x)

[Out] Integral(x\*\*2\*(a + b\*arccsc(c\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^2, x)

### 3.6 $\int x (a + b \operatorname{csc}^{-1}(cx)) dx$

**Optimal.** Leaf size=39

$$\frac{1}{2}x^2 (a + b \operatorname{csc}^{-1}(cx)) + \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}}{2c}$$

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(2\*c) + (x^2\*(a + b\*ArcCsc[c\*x]))/2

**Rubi [A]** time = 0.0119133, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {5221, 191}

$$\frac{1}{2}x^2 (a + b \operatorname{csc}^{-1}(cx)) + \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}}{2c}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(2\*c) + (x^2\*(a + b\*ArcCsc[c\*x]))/2

#### Rule 5221

Int[((a\_) + ArcCsc[(c\_)\*(x\_)])\*(b\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 191

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rubi steps

$$\begin{aligned} \int x (a + b \operatorname{csc}^{-1}(cx)) dx &= \frac{1}{2}x^2 (a + b \operatorname{csc}^{-1}(cx)) + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2x^2}}} dx}{2c} \\ &= \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}x^2 (a + b \operatorname{csc}^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.024409, size = 50, normalized size = 1.28

$$\frac{ax^2}{2} + \frac{bx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c} + \frac{1}{2}bx^2 \operatorname{csc}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcCsc[c\*x]),x]

[Out]  $(a*x^2)/2 + (b*x*\text{Sqrt}[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*\text{ArcCsc}[c*x])/2$

**Maple [A]** time = 0.171, size = 65, normalized size = 1.7

$$\frac{1}{c^2} \left( \frac{c^2 x^2 a}{2} + b \left( \frac{c^2 x^2 \text{arccsc}(cx)}{2} + \frac{c^2 x^2 - 1}{2 cx} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arccsc(c*x)),x)`

[Out]  $1/c^2*(1/2*c^2*x^2*a+b*(1/2*c^2*x^2*\text{arccsc}(c*x)+1/2/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1)))$

**Maxima [A]** time = 0.984624, size = 49, normalized size = 1.26

$$\frac{1}{2} ax^2 + \frac{1}{2} \left( x^2 \text{arccsc}(cx) + \frac{x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out]  $1/2*a*x^2 + 1/2*(x^2*\text{arccsc}(c*x) + x*\text{sqrt}(-1/(c^2*x^2) + 1)/c)*b$

**Fricas [A]** time = 2.32893, size = 90, normalized size = 2.31

$$\frac{bc^2x^2 \text{arccsc}(cx) + ac^2x^2 + \sqrt{c^2x^2 - 1}b}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out]  $1/2*(b*c^2*x^2*\text{arccsc}(c*x) + a*c^2*x^2 + \text{sqrt}(c^2*x^2 - 1)*b)/c^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x(a + b \text{acsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*acsc(c*x)),x)`

[Out] `Integral(x*(a + b*acsc(c*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x)),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x, x)`

### 3.7 $\int (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=31

$$ax + \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \csc^{-1}(cx)$$

[Out] a\*x + b\*x\*ArcCsc[c\*x] + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

**Rubi [A]** time = 0.0205758, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5215, 266, 63, 208}

$$ax + \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCsc[c\*x], x]

[Out] a\*x + b\*x\*ArcCsc[c\*x] + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

#### Rule 5215

Int[ArcCsc[(c\_.)\*(x\_)], x\_Symbol] := Simp[x\*ArcCsc[c\*x], x] + Dist[1/c, Int[1/(x\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /; FreeQ[c, x]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps

$$\begin{aligned}
\int (a + b \csc^{-1}(cx)) dx &= ax + b \int \csc^{-1}(cx) dx \\
&= ax + bx \csc^{-1}(cx) + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{c} \\
&= ax + bx \csc^{-1}(cx) - \frac{b \operatorname{Subst} \left( \int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{2c} \\
&= ax + bx \csc^{-1}(cx) + (bc) \operatorname{Subst} \left( \int \frac{1}{c^2 - c^2 x^2} dx, x, \sqrt{1 - \frac{1}{c^2 x^2}} \right) \\
&= ax + bx \csc^{-1}(cx) + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.0657152, size = 58, normalized size = 1.87

$$ax + \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}} \tanh^{-1} \left( \frac{cx}{\sqrt{c^2 x^2 - 1}} \right)}{\sqrt{c^2 x^2 - 1}} + bx \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCsc[c\*x], x]

[Out] a\*x + b\*x\*ArcCsc[c\*x] + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/Sqrt[-1 + c^2\*x^2]

**Maple [A]** time = 0.169, size = 37, normalized size = 1.2

$$ax + bx \operatorname{arccsc}(cx) + \frac{b}{c} \ln \left( cx + cx \sqrt{1 - \frac{1}{c^2 x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arccsc(c\*x), x)

[Out] a\*x+b\*x\*arccsc(c\*x)+b/c\*ln(c\*x+c\*x\*(1-1/c^2/x^2)^(1/2))

**Maxima [A]** time = 0.956712, size = 72, normalized size = 2.32

$$ax + \frac{\left( 2cx \operatorname{arccsc}(cx) + \log \left( \sqrt{-\frac{1}{c^2 x^2} + 1} + 1 \right) - \log \left( -\sqrt{-\frac{1}{c^2 x^2} + 1} + 1 \right) \right) b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsc(c\*x), x, algorithm="maxima")

[Out] a\*x + 1/2\*(2\*c\*x\*arccsc(c\*x) + log(sqrt(-1/(c^2\*x^2) + 1) + 1) - log(-sqrt(-1/(c^2\*x^2) + 1) + 1))\*b/c

---

**Fricas [B]** time = 2.46162, size = 154, normalized size = 4.97

$$\frac{acx - 2bc \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bcx - bc) \operatorname{arccsc}(cx) - b \log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsc(c\*x),x, algorithm="fricas")

[Out] (a\*c\*x - 2\*b\*c\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + (b\*c\*x - b\*c)\*arccsc(c\*x) - b\*log(-c\*x + sqrt(c^2\*x^2 - 1)))/c

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*acsc(c\*x),x)

[Out] Integral(a + b\*acsc(c\*x), x)

---

**Giac [A]** time = 1.09465, size = 63, normalized size = 2.03

$$\left( x \arcsin\left(\frac{1}{cx}\right) - \frac{c \log\left(\left| -x|c| + \sqrt{c^2x^2 - 1} \right| \right)}{|c|^2 \operatorname{sgn}(x)} \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsc(c\*x),x, algorithm="giac")

[Out] (x\*arcsin(1/(c\*x)) - c\*log(abs(-x\*abs(c) + sqrt(c^2\*x^2 - 1)))/(abs(c)^2\*sgn(x)))\*b + a\*x

### 3.8 $\int \frac{a+b \csc^{-1}(cx)}{x} dx$

**Optimal.** Leaf size=64

$$\frac{1}{2}ib\text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) + \frac{i(a+b \csc^{-1}(cx))^2}{2b} - \log\left(1 - e^{2i \csc^{-1}(cx)}\right)(a+b \csc^{-1}(cx))$$

[Out] ((I/2)\*(a + b\*ArcCsc[c\*x])^2)/b - (a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + (I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]

**Rubi [A]** time = 0.0847952, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5219, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2}ib\text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) + \frac{i(a+b \csc^{-1}(cx))^2}{2b} - \log\left(1 - e^{2i \csc^{-1}(cx)}\right)(a+b \csc^{-1}(cx))$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/x, x]

[Out] ((I/2)\*(a + b\*ArcCsc[c\*x])^2)/b - (a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + (I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]

#### Rule 5219

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] := -Subst[Int[(a + b\*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]

#### Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^n/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 3717

Int[((c\_.) + (d\_.)\*(x\_)^m)\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m \* E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^n)\*((c\_.) + (d\_.)\*(x\_)^m)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^n), x\_Symbol] := Simp[((c + d\*x)^m \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1) \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^n], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]



**Rule 2391**

`Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

**Rubi steps**

$$\begin{aligned}
 \int \frac{a + b \csc^{-1}(cx)}{x} dx &= -\text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x} \right) \\
 &= -\text{Subst} \left( \int (a + bx) \cot(x) dx, x, \csc^{-1}(cx) \right) \\
 &= \frac{i(a + b \csc^{-1}(cx))^2}{2b} + 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)}{1 - e^{2ix}} dx, x, \csc^{-1}(cx) \right) \\
 &= \frac{i(a + b \csc^{-1}(cx))^2}{2b} - (a + b \csc^{-1}(cx)) \log(1 - e^{2i \csc^{-1}(cx)}) + b \text{Subst} \left( \int \log(1 - e^{2ix}) dx, x, \csc^{-1}(cx) \right) \\
 &= \frac{i(a + b \csc^{-1}(cx))^2}{2b} - (a + b \csc^{-1}(cx)) \log(1 - e^{2i \csc^{-1}(cx)}) - \frac{1}{2}(ib) \text{Subst} \left( \int \frac{\log(1 - x)}{x} dx, x, \csc^{-1}(cx) \right) \\
 &= \frac{i(a + b \csc^{-1}(cx))^2}{2b} - (a + b \csc^{-1}(cx)) \log(1 - e^{2i \csc^{-1}(cx)}) + \frac{1}{2}ib \text{Li}_2(e^{2i \csc^{-1}(cx)})
 \end{aligned}$$

**Mathematica [A]** time = 0.0299711, size = 53, normalized size = 0.83

$$\frac{1}{2}ib \left( \csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) \right) + a \log(x) - b \csc^{-1}(cx) \log(1 - e^{2i \csc^{-1}(cx)})$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/x, x]

[Out] -(b\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])]) + a\*Log[x] + (I/2)\*b\*(ArcCsc[c\*x]^2 + PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])

**Maple [A]** time = 0.257, size = 140, normalized size = 2.2

$$a \ln(cx) + \frac{i}{2}b (\text{arccsc}(cx))^2 - b \text{arccsc}(cx) \ln \left( 1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2 x^2}} \right) - b \text{arccsc}(cx) \ln \left( 1 + \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}} \right) + ib \text{polylog}\left(2, -\frac{i}{cx} - \sqrt{1 - \frac{1}{c^2 x^2}}\right) + ib \text{polylog}\left(2, \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x, x)

[Out] a\*ln(c\*x)+1/2\*I\*b\*arccsc(c\*x)^2-b\*arccsc(c\*x)\*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))-b\*arccsc(c\*x)\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I\*b\*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+I\*b\*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( c^2 \int \frac{\sqrt{cx+1}\sqrt{cx-1} \log(x)}{c^4 x^3 - c^2 x} dx + \arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) \log(x) \right) b + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x,x, algorithm="maxima")

[Out] (c^2\*integrate(sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*log(x)/(c^4\*x^3 - c^2\*x), x) + a  
rctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))\*log(x))\*b + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x,x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x,x)

[Out] Integral((a + b\*acsc(c\*x))/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/x, x)

### 3.9 $\int \frac{a+b \csc^{-1}(cx)}{x^2} dx$

**Optimal.** Leaf size=32

$$-\frac{a+b \csc^{-1}(cx)}{x} - bc\sqrt{1-\frac{1}{c^2x^2}}$$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]) - (a + b*\text{ArcCsc}[c*x])/x$

**Rubi [A]** time = 0.0197825, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {5221, 261}

$$-\frac{a+b \csc^{-1}(cx)}{x} - bc\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCsc}[c*x])/x^2, x]$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]) - (a + b*\text{ArcCsc}[c*x])/x$

#### Rule 5221

$\text{Int}[(a + \text{ArcCsc}[c*x])*(d*x)^m, x\_Symbol] :> \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcCsc}[c*x])/(d*(m+1)), x] + \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{m-1}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}[m, -1]$

#### Rule 261

$\text{Int}[(x)^m*(a + (b*x)^n)^p, x\_Symbol] :> \text{Simp}[(a + b*x^n)^{p+1}/(b*n*(p+1)), x] /;$   $\text{FreeQ}\{a, b, m, n, p\}, x \&\& \text{EqQ}[m, n-1] \&\& \text{NeQ}[p, -1]$

#### Rubi steps

$$\begin{aligned} \int \frac{a+b \csc^{-1}(cx)}{x^2} dx &= -\frac{a+b \csc^{-1}(cx)}{x} - \frac{b \int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{c} \\ &= -bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a+b \csc^{-1}(cx)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0287449, size = 41, normalized size = 1.28

$$-\frac{a}{x} - bc\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b \csc^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*\text{ArcCsc}[c*x])/x^2, x]$

[Out]  $-(a/x) - b*c*\text{Sqrt}[(-1 + c^2*x^2)/(c^2*x^2)] - (b*\text{ArcCsc}[c*x])/x$

**Maple [A]** time = 0.171, size = 63, normalized size = 2.

$$c \left( -\frac{a}{cx} + b \left( -\frac{\text{arccsc}(cx)}{cx} - \frac{c^2x^2 - 1}{c^2x^2} \frac{1}{\sqrt{\frac{c^2x^2 - 1}{c^2x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsc(c*x))/x^2,x)`

[Out]  $c*(-1/c/x*a+b*(-1/c/x*\text{arccsc}(c*x)-1/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/c^2/x^2*(c^2*x^2-1)))$

**Maxima [A]** time = 0.982462, size = 45, normalized size = 1.41

$$-\left( c\sqrt{-\frac{1}{c^2x^2} + 1} + \frac{\text{arccsc}(cx)}{x} \right) b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^2,x, algorithm="maxima")`

[Out]  $-(c*\text{sqrt}(-1/(c^2*x^2) + 1) + \text{arccsc}(c*x)/x)*b - a/x$

**Fricas [A]** time = 2.20002, size = 62, normalized size = 1.94

$$\frac{b \text{arccsc}(cx) + \sqrt{c^2x^2 - 1}b + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^2,x, algorithm="fricas")`

[Out]  $-(b*\text{arccsc}(c*x) + \text{sqrt}(c^2*x^2 - 1)*b + a)/x$

**Sympy [A]** time = 2.6338, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{a}{x} - bc\sqrt{1 - \frac{1}{c^2x^2}} - \frac{b \text{acsc}(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a+\infty b}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/x**2,x)`

[Out]  $\text{Piecewise}((-a/x - b*c*\text{sqrt}(1 - 1/(c**2*x**2)) - b*\text{acsc}(c*x)/x, \text{Ne}(c, 0)), (-a + \text{zoo}*b)/x, \text{True})$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/x^2, x)
```

### 3.10 $\int \frac{a+b \csc^{-1}(cx)}{x^3} dx$

**Optimal.** Leaf size=51

$$-\frac{a+b \csc^{-1}(cx)}{2x^2} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2 \csc^{-1}(cx)$$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) + (b*c^2*\text{ArcCsc}[c*x])/4 - (a + b*\text{ArcCsc}[c*x])/(2*x^2)$

**Rubi [A]** time = 0.0315293, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5221, 335, 321, 216}

$$-\frac{a+b \csc^{-1}(cx)}{2x^2} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCsc}[c*x])/x^3, x]$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) + (b*c^2*\text{ArcCsc}[c*x])/4 - (a + b*\text{ArcCsc}[c*x])/(2*x^2)$

#### Rule 5221

$\text{Int}[(a + \text{ArcCsc}[c*x])*(b*x)^m, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcCsc}[c*x])/(d*(m+1)), x] + \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{m-1}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /;$  FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 335

$\text{Int}[(x)^m*((a) + (b)*(x)^n)^p, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{m+2}, x], x, 1/x] /;$  FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

#### Rule 321

$\text{Int}[(c*x)^m*((a) + (b)*(x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c^{n-1}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{m-n}*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 216

$\text{Int}[1/\text{Sqrt}[(a) + (b)*(x)^2], x\_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$  FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^3} dx &= -\frac{a + b \csc^{-1}(cx)}{2x^2} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^4} dx}{2c} \\
&= -\frac{a + b \csc^{-1}(cx)}{2x^2} + \frac{b \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2c} \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{4x} - \frac{a + b \csc^{-1}(cx)}{2x^2} + \frac{1}{4}(bc) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{4x} + \frac{1}{4} bc^2 \csc^{-1}(cx) - \frac{a + b \csc^{-1}(cx)}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0346294, size = 66, normalized size = 1.29

$$-\frac{a}{2x^2} - \frac{bc \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}{4x} + \frac{1}{4} bc^2 \sin^{-1} \left( \frac{1}{cx} \right) - \frac{b \csc^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/x^3,x]

[Out] -a/(2\*x^2) - (b\*c\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(4\*x) - (b\*ArcCsc[c\*x])/(2\*x^2) + (b\*c^2\*ArcSin[1/(c\*x)])/4

**Maple [B]** time = 0.174, size = 118, normalized size = 2.3

$$-\frac{a}{2x^2} - \frac{\operatorname{arccsc}(cx)}{2x^2} + \frac{cb}{4x} \sqrt{c^2 x^2 - 1} \arctan \left( \frac{1}{\sqrt{c^2 x^2 - 1}} \right) \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{cb}{4x} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{b}{4cx^3} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^3,x)

[Out] -1/2\*a/x^2-1/2\*b/x^2\*arccsc(c\*x)+1/4\*c\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*arctan(1/(c^2\*x^2-1)^(1/2))-1/4\*c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x+1/4/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^3

**Maxima [A]** time = 1.48814, size = 112, normalized size = 2.2

$$\frac{1}{4} b \left( \frac{c^4 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^2 x^2 \left( \frac{1}{c^2 x^2} - 1 \right)^{-1}} - c^3 \arctan \left( cx \sqrt{-\frac{1}{c^2 x^2} + 1} \right) \right) - \frac{2 \operatorname{arccsc}(cx)}{x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}b\left(\frac{c^4x\sqrt{-1/(c^2x^2)+1}}{c^2x^2(1/(c^2x^2)-1)-1}-c^3\arctan\left(\frac{c^2x\sqrt{-1/(c^2x^2)+1}}{c}\right)-2\arccsc(cx)/x^2\right)-\frac{1}{2}a/x^2$

**Fricas [A]** time = 2.17711, size = 93, normalized size = 1.82

$$\frac{(bc^2x^2 - 2b)\arccsc(cx) - \sqrt{c^2x^2 - 1}b - 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^3,x, algorithm="fricas")

[Out]  $\frac{1}{4}((b*c^2*x^2 - 2*b)*\arccsc(c*x) - \sqrt{c^2*x^2 - 1}*b - 2*a)/x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*3,x)

[Out] Integral((a + b\*acsc(c\*x))/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/x^3, x)



$$3.11 \quad \int \frac{a+b \csc^{-1}(cx)}{x^4} dx$$

**Optimal.** Leaf size=60

$$-\frac{a+b \csc^{-1}(cx)}{3x^3} + \frac{1}{9}bc^3 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{3}bc^3 \sqrt{1 - \frac{1}{c^2x^2}}$$

[Out]  $-(b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/3 + (b*c^3*(1 - 1/(c^2*x^2))^{(3/2)})/9 - (a + b*\text{ArcCsc}[c*x])/(3*x^3)$

**Rubi [A]** time = 0.0379044, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5221, 266, 43}

$$-\frac{a+b \csc^{-1}(cx)}{3x^3} + \frac{1}{9}bc^3 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{3}bc^3 \sqrt{1 - \frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/x^4,x]

[Out]  $-(b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/3 + (b*c^3*(1 - 1/(c^2*x^2))^{(3/2)})/9 - (a + b*\text{ArcCsc}[c*x])/(3*x^3)$

#### Rule 5221

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(d\*x)^(m+1)\*(a + b\*ArcCsc[c\*x])/(d\*(m+1)), x] + Dist[(b\*d)/(c\*(m+1)), Int[(d\*x)^(m-1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^4} dx &= -\frac{a + b \csc^{-1}(cx)}{3x^3} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{3c} \\
&= -\frac{a + b \csc^{-1}(cx)}{3x^3} + \frac{b \operatorname{Subst}\left(\int \frac{x}{\sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{6c} \\
&= -\frac{a + b \csc^{-1}(cx)}{3x^3} + \frac{b \operatorname{Subst}\left(\int \left(\frac{c^2}{\sqrt{1 - \frac{x}{c^2}}} - c^2 \sqrt{1 - \frac{x}{c^2}}\right) dx, x, \frac{1}{x^2}\right)}{6c} \\
&= -\frac{1}{3}bc^3 \sqrt{1 - \frac{1}{c^2 x^2}} + \frac{1}{9}bc^3 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} - \frac{a + b \csc^{-1}(cx)}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0650652, size = 59, normalized size = 0.98

$$-\frac{a}{3x^3} + b\left(-\frac{2c^3}{9} - \frac{c}{9x^2}\right)\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b \csc^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/x^4, x]

[Out] -a/(3\*x^3) + b\*((-2\*c^3)/9 - c/(9\*x^2))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsc[c\*x])/(3\*x^3)

**Maple [A]** time = 0.176, size = 75, normalized size = 1.3

$$c^3 \left( -\frac{a}{3c^3x^3} + b \left( -\frac{\operatorname{arccsc}(cx)}{3c^3x^3} - \frac{(c^2x^2-1)(2c^2x^2+1)}{9c^4x^4} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^4, x)

[Out] c^3\*(-1/3\*a/c^3/x^3+b\*(-1/3/c^3/x^3\*arccsc(c\*x)-1/9\*(c^2\*x^2-1)\*(2\*c^2\*x^2+1)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4))

**Maxima [A]** time = 0.977066, size = 78, normalized size = 1.3

$$\frac{1}{9}b \left( \frac{c^4 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} - \frac{3 \operatorname{arccsc}(cx)}{x^3} \right) - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^4, x, algorithm="maxima")

[Out]  $1/9*b*((c^4*(-1/(c^2*x^2) + 1))^{3/2} - 3*c^4*\sqrt{-1/(c^2*x^2) + 1})/c - 3*\arccsc(c*x)/x^3) - 1/3*a/x^3$

**Fricas [A]** time = 2.23779, size = 97, normalized size = 1.62

$$\frac{3b \arccsc(cx) + (2bc^2x^2 + b)\sqrt{c^2x^2 - 1} + 3a}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^4,x, algorithm="fricas")`

[Out]  $-1/9*(3*b*\arccsc(c*x) + (2*b*c^2*x^2 + b)*\sqrt{c^2*x^2 - 1} + 3*a)/x^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \arccsc(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x**4,x)`

[Out] `Integral((a + b*arccsc(c*x))/x**4, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arccsc(cx) + a}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^4,x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/x^4, x)`

### 3.12 $\int \frac{a+b \csc^{-1}(cx)}{x^5} dx$

**Optimal.** Leaf size=76

$$-\frac{a+b \csc^{-1}(cx)}{4x^4} - \frac{3bc^3 \sqrt{1-\frac{1}{c^2x^2}}}{32x} - \frac{bc \sqrt{1-\frac{1}{c^2x^2}}}{16x^3} + \frac{3}{32}bc^4 \csc^{-1}(cx)$$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(32*x) + (3*b*c^4*\text{ArcCsc}[c*x])/32 - (a + b*\text{ArcCsc}[c*x])/(4*x^4)$

**Rubi [A]** time = 0.0449829, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5221, 335, 321, 216}

$$-\frac{a+b \csc^{-1}(cx)}{4x^4} - \frac{3bc^3 \sqrt{1-\frac{1}{c^2x^2}}}{32x} - \frac{bc \sqrt{1-\frac{1}{c^2x^2}}}{16x^3} + \frac{3}{32}bc^4 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCsc}[c*x])/x^5, x]$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(32*x) + (3*b*c^4*\text{ArcCsc}[c*x])/32 - (a + b*\text{ArcCsc}[c*x])/(4*x^4)$

#### Rule 5221

$\text{Int}[(a + \text{ArcCsc}[c*x])*(b*x)^m, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcCsc}[c*x])/(d*(m+1)), x] + \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{m-1}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /;$  FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 335

$\text{Int}[(x)^m*((a) + (b)*(x)^n)^p, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{m+2}, x], x, 1/x] /;$  FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

#### Rule 321

$\text{Int}[(c*x)^m*((a) + (b)*(x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c^{n-1}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{n*(m-n+1)})/(b*(m+n*p+1)), \text{Int}[(c*x)^{m-n}*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 216

$\text{Int}[1/\text{Sqrt}[(a) + (b)*(x)^2], x\_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$  FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^5} dx &= \frac{a + b \csc^{-1}(cx)}{4x^4} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}^3} dx}{4c} \\
&= \frac{a + b \csc^{-1}(cx)}{4x^4} + \frac{b \operatorname{Subst} \left( \int \frac{x^4}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4c} \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{16x^3} - \frac{a + b \csc^{-1}(cx)}{4x^4} + \frac{1}{16} (3bc) \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{16x^3} - \frac{3bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{32x} - \frac{a + b \csc^{-1}(cx)}{4x^4} + \frac{1}{32} (3bc^3) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{16x^3} - \frac{3bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{32x} + \frac{3}{32} bc^4 \csc^{-1}(cx) - \frac{a + b \csc^{-1}(cx)}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0606642, size = 78, normalized size = 1.03

$$-\frac{a}{4x^4} + b \left( -\frac{3c^3}{32x} - \frac{c}{16x^3} \right) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + \frac{3}{32} bc^4 \sin^{-1} \left( \frac{1}{cx} \right) - \frac{b \csc^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/x^5, x]

[Out] -a/(4\*x^4) + b\*(-c/(16\*x^3) - (3\*c^3)/(32\*x))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsc[c\*x])/(4\*x^4) + (3\*b\*c^4\*ArcSin[1/(c\*x)])/32

**Maple [B]** time = 0.174, size = 147, normalized size = 1.9

$$-\frac{a}{4x^4} - \frac{b \operatorname{arccsc}(cx)}{4x^4} + \frac{3c^3 b}{32x} \sqrt{c^2 x^2 - 1} \arctan \left( \frac{1}{\sqrt{c^2 x^2 - 1}} \right) \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{3c^3 b}{32x} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{cb}{32x^3} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{b}{16cx^5} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^5, x)

[Out] -1/4\*a/x^4 - 1/4\*b/x^4\*arccsc(c\*x) + 3/32\*c^3\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*arctan(1/(c^2\*x^2-1)^(1/2)) - 3/32\*c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x + 1/32\*c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^3 + 1/16/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^5

**Maxima [A]** time = 1.47882, size = 169, normalized size = 2.22

$$-\frac{1}{32} b \left( \frac{3c^5 \arctan \left( cx \sqrt{-\frac{1}{c^2 x^2} + 1} \right) + \frac{3c^8 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 5c^6 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^4 x^4 \left( \frac{1}{c^2 x^2} - 1 \right)^2 - 2c^2 x^2 \left( \frac{1}{c^2 x^2} - 1 \right) + 1}}{c} + \frac{8 \operatorname{arccsc}(cx)}{x^4} \right) - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^5,x, algorithm="maxima")

[Out]  $-1/32*b*((3*c^5*\arctan(c*x*\sqrt{-1/(c^2*x^2)+1})) + (3*c^8*x^3*(-1/(c^2*x^2)+1)^{(3/2)} + 5*c^6*x*\sqrt{-1/(c^2*x^2)+1}))/c^4*x^4*(1/(c^2*x^2)-1)^2 - 2*c^2*x^2*(1/(c^2*x^2)-1)+1)/c + 8*\arccsc(c*x)/x^4 - 1/4*a/x^4$

**Fricas [A]** time = 2.29235, size = 122, normalized size = 1.61

$$\frac{(3bc^4x^4 - 8b)\operatorname{arccsc}(cx) - (3bc^2x^2 + 2b)\sqrt{c^2x^2 - 1} - 8a}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^5,x, algorithm="fricas")

[Out]  $1/32*((3*b*c^4*x^4 - 8*b)*\operatorname{arccsc}(c*x) - (3*b*c^2*x^2 + 2*b)*\sqrt{c^2*x^2 - 1} - 8*a)/x^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*5,x)

[Out] Integral((a + b\*acsc(c\*x))/x\*\*5, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^5,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/x^5, x)

### 3.13 $\int \frac{a+b \csc^{-1}(cx)}{x^6} dx$

**Optimal.** Leaf size=82

$$-\frac{a+b \csc^{-1}(cx)}{5x^5} - \frac{1}{25}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{5/2} + \frac{2}{15}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{5}bc^5 \sqrt{1 - \frac{1}{c^2x^2}}$$

[Out]  $-(b*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)])/5 + (2*b*c^5*(1 - 1/(c^2*x^2))^{(3/2)})/15 - (b*c^5*(1 - 1/(c^2*x^2))^{(5/2)})/25 - (a + b*\text{ArcCsc}[c*x])/(5*x^5)$

**Rubi [A]** time = 0.0491578, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5221, 266, 43}

$$-\frac{a+b \csc^{-1}(cx)}{5x^5} - \frac{1}{25}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{5/2} + \frac{2}{15}bc^5 \left(1 - \frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{5}bc^5 \sqrt{1 - \frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCsc}[c*x])/x^6, x]$

[Out]  $-(b*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)])/5 + (2*b*c^5*(1 - 1/(c^2*x^2))^{(3/2)})/15 - (b*c^5*(1 - 1/(c^2*x^2))^{(5/2)})/25 - (a + b*\text{ArcCsc}[c*x])/(5*x^5)$

#### Rule 5221

$\text{Int}[(a + \text{ArcCsc}[c*x])*(b*x)^m, x] := \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcCsc}[c*x])/(d*(m+1)), x] + \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{m-1}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 266

$\text{Int}[x^m*(a + b*x)^n, x] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(Simplify[(m+1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

#### Rule 43

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\text{!IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n+1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csc}^{-1}(cx)}{x^6} dx &= -\frac{a + b \operatorname{csc}^{-1}(cx)}{5x^5} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{5c} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{5x^5} + \frac{b \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{10c} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{5x^5} + \frac{b \operatorname{Subst}\left(\int \left(\frac{c^4}{\sqrt{1 - \frac{x}{c^2}}} - 2c^4 \sqrt{1 - \frac{x}{c^2}} + c^4 \left(1 - \frac{x}{c^2}\right)^{3/2}\right) dx, x, \frac{1}{x^2}\right)}{10c} \\
&= -\frac{1}{5}bc^5 \sqrt{1 - \frac{1}{c^2 x^2}} + \frac{2}{15}bc^5 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} - \frac{1}{25}bc^5 \left(1 - \frac{1}{c^2 x^2}\right)^{5/2} - \frac{a + b \operatorname{csc}^{-1}(cx)}{5x^5}
\end{aligned}$$

**Mathematica [A]** time = 0.0850999, size = 69, normalized size = 0.84

$$-\frac{a}{5x^5} + b \left( -\frac{4c^3}{75x^2} - \frac{8c^5}{75} - \frac{c}{25x^4} \right) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} - \frac{b \operatorname{csc}^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/x^6, x]

[Out] -a/(5\*x^5) + b\*((-8\*c^5)/75 - c/(25\*x^4) - (4\*c^3)/(75\*x^2))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsc[c\*x])/(5\*x^5)

**Maple [A]** time = 0.173, size = 83, normalized size = 1.

$$c^5 \left( -\frac{a}{5c^5 x^5} + b \left( -\frac{\operatorname{arccsc}(cx)}{5c^5 x^5} - \frac{(c^2 x^2 - 1)(8c^4 x^4 + 4c^2 x^2 + 3)}{75c^6 x^6} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^6, x)

[Out] c^5\*(-1/5\*a/c^5/x^5+b\*(-1/5/c^5/x^5\*arccsc(c\*x)-1/75\*(c^2\*x^2-1)\*(8\*c^4\*x^4+4\*c^2\*x^2+3)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))

**Maxima [A]** time = 0.978024, size = 103, normalized size = 1.26

$$-\frac{1}{75} b \left( \frac{3c^6 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{5}{2}} - 10c^6 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{3}{2}} + 15c^6 \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} + \frac{15 \operatorname{arccsc}(cx)}{x^5} \right) - \frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^6, x, algorithm="maxima")



[Out]  $-1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^{(5/2)} - 10*c^6*(-1/(c^2*x^2) + 1)^{(3/2)} + 15*c^6*\sqrt{-1/(c^2*x^2) + 1}))/c + 15*\operatorname{arccsc}(c*x)/x^5 - 1/5*a/x^5$

**Fricas [A]** time = 2.26885, size = 123, normalized size = 1.5

$$\frac{15 b \operatorname{arccsc}(cx) + (8bc^4x^4 + 4bc^2x^2 + 3b)\sqrt{c^2x^2 - 1} + 15a}{75x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^6,x, algorithm="fricas")`

[Out]  $-1/75*(15*b*\operatorname{arccsc}(c*x) + (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*\sqrt{c^2*x^2 - 1} + 15*a)/x^5$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x**6,x)`

[Out] `Integral((a + b*arccsc(c*x))/x**6, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^6,x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/x^6, x)`

### 3.14 $\int \frac{a+b \csc^{-1}(cx)}{x^7} dx$

**Optimal.** Leaf size=101

$$-\frac{a+b \csc^{-1}(cx)}{6x^6} - \frac{5bc^5 \sqrt{1-\frac{1}{c^2x^2}}}{96x} - \frac{5bc^3 \sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{bc \sqrt{1-\frac{1}{c^2x^2}}}{36x^5} + \frac{5}{96} bc^6 \csc^{-1}(cx)$$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(144*x^3) - (5*b*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)])/(96*x) + (5*b*c^6*\text{ArcCsc}[c*x])/96 - (a + b*\text{ArcCsc}[c*x])/(6*x^6)$

**Rubi [A]** time = 0.0624712, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5221, 335, 321, 216}

$$-\frac{a+b \csc^{-1}(cx)}{6x^6} - \frac{5bc^5 \sqrt{1-\frac{1}{c^2x^2}}}{96x} - \frac{5bc^3 \sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{bc \sqrt{1-\frac{1}{c^2x^2}}}{36x^5} + \frac{5}{96} bc^6 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCsc}[c*x])/x^7, x]$

[Out]  $-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*\text{Sqrt}[1 - 1/(c^2*x^2)])/(144*x^3) - (5*b*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)])/(96*x) + (5*b*c^6*\text{ArcCsc}[c*x])/96 - (a + b*\text{ArcCsc}[c*x])/(6*x^6)$

#### Rule 5221

$\text{Int}[(a + \text{ArcCsc}[c*x])*(d*x)^m, x] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcCsc}[c*x])/(d*(m+1)), x] + \text{Dist}[(b*d)/(c*(m+1)), \text{Int}[(d*x)^{m-1}/\text{Sqrt}[1 - 1/(c^2*x^2)], x], x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x \} \&\& \text{NeQ}[m, -1]$

#### Rule 335

$\text{Int}[x^m*(a + b*x^n)^p, x] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{m+2}, x], x, 1/x] /;$   $\text{FreeQ}\{a, b, p\}, x \} \&\& \text{ILtQ}[n, 0] \&\& \text{IntegerQ}[m]$

#### Rule 321

$\text{Int}[(c*x)^m*(a + b*x^n)^p, x] \rightarrow \text{Simp}[(c^{n-1}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{m-n}*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, p\}, x \} \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n-1] \&\& \text{NeQ}[m+n*p+1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 216

$\text{Int}[1/\text{Sqrt}[a + b*x^2], x] \rightarrow \text{Simp}[\text{ArcSin}[Rt[-b, 2]*x]/\text{Sqrt}[a]/Rt[-b, 2], x] /;$   $\text{FreeQ}\{a, b\}, x \} \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^7} dx &= \frac{a + b \csc^{-1}(cx)}{6x^6} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} dx}}{6c} \\
&= \frac{a + b \csc^{-1}(cx)}{6x^6} + \frac{b \operatorname{Subst} \left( \int \frac{x^6}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{6c} \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} - \frac{a + b \csc^{-1}(cx)}{6x^6} + \frac{1}{36} (5bc) \operatorname{Subst} \left( \int \frac{x^4}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} - \frac{5bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{144x^3} - \frac{a + b \csc^{-1}(cx)}{6x^6} + \frac{1}{48} (5bc^3) \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} - \frac{5bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{144x^3} - \frac{5bc^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{96x} - \frac{a + b \csc^{-1}(cx)}{6x^6} + \frac{1}{96} (5bc^5) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= -\frac{bc \sqrt{1 - \frac{1}{c^2 x^2}}}{36x^5} - \frac{5bc^3 \sqrt{1 - \frac{1}{c^2 x^2}}}{144x^3} - \frac{5bc^5 \sqrt{1 - \frac{1}{c^2 x^2}}}{96x} + \frac{5}{96} bc^6 \csc^{-1}(cx) - \frac{a + b \csc^{-1}(cx)}{6x^6}
\end{aligned}$$

**Mathematica [A]** time = 0.10211, size = 88, normalized size = 0.87

$$-\frac{a}{6x^6} + b \left( -\frac{5c^3}{144x^3} - \frac{5c^5}{96x} - \frac{c}{36x^5} \right) \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + \frac{5}{96} bc^6 \sin^{-1} \left( \frac{1}{cx} \right) - \frac{b \csc^{-1}(cx)}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/x^7, x]

[Out] -a/(6\*x^6) + b\*(-c/(36\*x^5) - (5\*c^3)/(144\*x^3) - (5\*c^5)/(96\*x))\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsc[c\*x])/(6\*x^6) + (5\*b\*c^6\*ArcSin[1/(c\*x)])/96

**Maple [A]** time = 0.177, size = 174, normalized size = 1.7

$$-\frac{a}{6x^6} - \frac{b \operatorname{arccsc}(cx)}{6x^6} + \frac{5c^5 b}{96x} \sqrt{c^2 x^2 - 1} \arctan \left( \frac{1}{\sqrt{c^2 x^2 - 1}} \right) \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{5c^5 b}{96x} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{5c^3 b}{288x^3} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{cb}{144x^5} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^7, x)

[Out] -1/6\*a/x^6-1/6\*b/x^6\*arccsc(c\*x)+5/96\*c^5\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*arctan(1/(c^2\*x^2-1)^(1/2))-5/96\*c^5\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x+5/288\*c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^3+1/144\*c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^5+1/36\*c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^7

**Maxima [A]** time = 1.47346, size = 223, normalized size = 2.21

$$-\frac{1}{288}b \left( \frac{15c^7 \arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right) - \frac{15c^{12}x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 40c^{10}x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 33c^8x\sqrt{-\frac{1}{c^2x^2}+1}}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3 - 3c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 + 3c^2x^2\left(\frac{1}{c^2x^2}-1\right) - 1}}{c} + \frac{48 \operatorname{arccsc}(cx)}{x^6} \right) - \frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^7,x, algorithm="maxima")

[Out] -1/288\*b\*((15\*c^7\*arctan(c\*x\*sqrt(-1/(c^2\*x^2) + 1)) - (15\*c^12\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 40\*c^10\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 33\*c^8\*x\*sqrt(-1/(c^2\*x^2) + 1))/(c^6\*x^6\*(1/(c^2\*x^2) - 1)^3 - 3\*c^4\*x^4\*(1/(c^2\*x^2) - 1)^2 + 3\*c^2\*x^2\*(1/(c^2\*x^2) - 1) - 1))/c + 48\*arccsc(c\*x)/x^6) - 1/6\*a/x^6

**Fricas [A]** time = 2.24553, size = 150, normalized size = 1.49

$$\frac{3(5bc^6x^6 - 16b) \operatorname{arccsc}(cx) - (15bc^4x^4 + 10bc^2x^2 + 8b)\sqrt{c^2x^2 - 1} - 48a}{288x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^7,x, algorithm="fricas")

[Out] 1/288\*(3\*(5\*b\*c^6\*x^6 - 16\*b)\*arccsc(c\*x) - (15\*b\*c^4\*x^4 + 10\*b\*c^2\*x^2 + 8\*b)\*sqrt(c^2\*x^2 - 1) - 48\*a)/x^6

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*7,x)

[Out] Integral((a + b\*acsc(c\*x))/x\*\*7, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^7,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/x^7, x)

### 3.15 $\int x^3 \left( a + b \operatorname{csc}^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=107

$$\frac{bx^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \operatorname{csc}^{-1}(cx))}{6c} + \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \operatorname{csc}^{-1}(cx))}{3c^3} + \frac{1}{4} x^4 (a + b \operatorname{csc}^{-1}(cx))^2 + \frac{b^2 x^2}{12c^2} + \frac{b^2 \log(x)}{3c^4}$$

[Out] (b^2\*x^2)/(12\*c^2) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(a + b\*ArcCsc[c\*x]))/(3\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3\*(a + b\*ArcCsc[c\*x]))/(6\*c) + (x^4\*(a + b\*ArcCsc[c\*x])^2)/4 + (b^2\*Log[x])/(3\*c^4)

**Rubi [A]** time = 0.106237, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5223, 4410, 4185, 4184, 3475}

$$\frac{bx^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \operatorname{csc}^{-1}(cx))}{6c} + \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \operatorname{csc}^{-1}(cx))}{3c^3} + \frac{1}{4} x^4 (a + b \operatorname{csc}^{-1}(cx))^2 + \frac{b^2 x^2}{12c^2} + \frac{b^2 \log(x)}{3c^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcCsc[c\*x])^2,x]

[Out] (b^2\*x^2)/(12\*c^2) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(a + b\*ArcCsc[c\*x]))/(3\*c^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3\*(a + b\*ArcCsc[c\*x]))/(6\*c) + (x^4\*(a + b\*ArcCsc[c\*x])^2)/4 + (b^2\*Log[x])/(3\*c^4)

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m, x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4410

Int[Cot[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csc[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Csc[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csc[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 4185

Int[(csc[(e\_.) + (f\_.)\*(x\_)])\*(b\_.)^n\*((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := -Simp[(b^2\*(c + d\*x)\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d *x], x]]/d, x] /; FreeQ[{c, d}, x]`

### Rubi steps

$$\begin{aligned} \int x^3 (a + b \csc^{-1}(cx))^2 dx &= -\frac{\text{Subst}\left(\int (a + bx)^2 \cot(x) \csc^4(x) dx, x, \csc^{-1}(cx)\right)}{c^4} \\ &= \frac{1}{4}x^4 (a + b \csc^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \csc^4(x) dx, x, \csc^{-1}(cx)\right)}{2c^4} \\ &= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3 (a + b \csc^{-1}(cx))}{6c} + \frac{1}{4}x^4 (a + b \csc^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \csc^2(x) dx, x, \csc^{-1}(cx)\right)}{3c^4} \\ &= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \csc^{-1}(cx))}{3c^3} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3 (a + b \csc^{-1}(cx))}{6c} + \frac{1}{4}x^4 (a + b \csc^{-1}(cx))^2 \\ &= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \csc^{-1}(cx))}{3c^3} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^3 (a + b \csc^{-1}(cx))}{6c} + \frac{1}{4}x^4 (a + b \csc^{-1}(cx))^2 \end{aligned}$$

**Mathematica [A]** time = 0.222909, size = 124, normalized size = 1.16

$$\frac{cx \left(3a^2c^3x^3 + 2ab\sqrt{1 - \frac{1}{c^2x^2}}(c^2x^2 + 2) + b^2cx\right) + 2bcx \csc^{-1}(cx) \left(3ac^3x^3 + b\sqrt{1 - \frac{1}{c^2x^2}}(c^2x^2 + 2)\right) + 3b^2c^4x^4 \csc^{-1}(cx)^2}{12c^4}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*(a + b*ArcCsc[c*x])^2,x]`

`[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) + 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcCsc[c*x] + 3*b^2*c^4*x^4*ArcCsc[c*x]^2 + 4*b^2*Log[x])/(12*c^4)`

**Maple [B]** time = 0.225, size = 208, normalized size = 1.9

$$\frac{a^2x^4}{4} + \frac{b^2(\operatorname{arccsc}(cx))^2x^4}{4} + \frac{b^2\operatorname{arccsc}(cx)x^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2\operatorname{arccsc}(cx)x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^3} - \frac{b^2}{3c^4}\ln\left(\frac{1}{cx}\right) + \frac{abx^4\operatorname{arccsc}(cx)}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*(a+b*arccsc(c*x))^2,x)`

`[Out] 1/4*a^2*x^4+1/4*b^2*arccsc(c*x)^2*x^4+1/6/c*b^2*arccsc(c*x)*x^3*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/12*b^2*x^2/c^2+1/3/c^3*b^2*arccsc(c*x)*x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/3/c^4*b^2*ln(1/c/x)+1/2*a*b*x^4*arccsc(c*x)+1/6/c*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^3+1/6/c^3*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x-1/3/c^5*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x`

**Maxima [B]** time = 2.10654, size = 266, normalized size = 2.49

$$\frac{1}{4}b^2x^4 \operatorname{arccsc}(cx)^2 + \frac{1}{4}a^2x^4 + \frac{1}{6}\left(3x^4 \operatorname{arccsc}(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 3x\sqrt{-\frac{1}{c^2x^2} + 1}}{c^3}\right)ab + \frac{(2c^4x^4 \arctan(1, \sqrt{cx+1}))}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}b^2x^4\operatorname{arccsc}(cx)^2 + \frac{1}{4}a^2x^4 + \frac{1}{6}(3x^4\operatorname{arccsc}(cx) + (c^2x^3 - 1/\sqrt{c^2x^2 - 1})^{3/2} + 3x\sqrt{-1/\sqrt{c^2x^2 - 1}})/c^3)ab + \frac{1}{12}(2c^4x^4\arctan(1, \sqrt{cx+1})\sqrt{cx-1}) + 2c^2x^2\arctan(1, \sqrt{cx+1})\sqrt{cx-1}) + (c^2x^2 + 2\log(x^2))\sqrt{cx+1}\sqrt{cx-1} - 4\arctan(1, \sqrt{cx+1})\sqrt{cx-1})b^2/(\sqrt{cx+1})\sqrt{cx-1})c^4$

**Fricas [A]** time = 2.96616, size = 339, normalized size = 3.17

$$\frac{3b^2c^4x^4\operatorname{arccsc}(cx)^2 + 3a^2c^4x^4 - 12abc^4\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + b^2c^2x^2 + 4b^2\log(x) + 6(abc^4x^4 - abc^4)\operatorname{arccsc}(cx)}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))^2,x, algorithm="fricas")

[Out]  $\frac{1}{12}(3b^2c^4x^4\operatorname{arccsc}(cx)^2 + 3a^2c^4x^4 - 12a*b*c^4*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) + b^2*c^2*x^2 + 4*b^2*\log(x) + 6*(a*b*c^4*x^4 - a*b*c^4)*\operatorname{arccsc}(c*x) + 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*\operatorname{arccsc}(c*x))*\sqrt{c^2*x^2 - 1})/c^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arccsc}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))\*\*2,x)

[Out] Integral(x\*\*3\*(a + b\*acsc(c\*x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2\*x^3, x)

### 3.16 $\int x^2 \left( a + b \csc^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=139

$$\frac{ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right)}{3c^3} + \frac{ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right)}{3c^3} + \frac{bx^2 \sqrt{1 - \frac{1}{c^2 x^2}} \left( a + b \csc^{-1}(cx) \right)}{3c} + \frac{2b \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right) \left( a - \frac{1}{c} \right)}{3c^3}$$

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCsc[c*x]))/(3*c) +
(x^3*(a + b*ArcCsc[c*x])^2)/3 + (2*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcCsc[c*x])])/(3*c^3) - ((I/3)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + ((I/3)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3
```

**Rubi [A]** time = 0.119006, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5223, 4410, 4185, 4183, 2279, 2391}

$$\frac{ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right)}{3c^3} + \frac{ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right)}{3c^3} + \frac{bx^2 \sqrt{1 - \frac{1}{c^2 x^2}} \left( a + b \csc^{-1}(cx) \right)}{3c} + \frac{2b \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right) \left( a - \frac{1}{c} \right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsc[c*x])^2, x]
```

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCsc[c*x]))/(3*c) +
(x^3*(a + b*ArcCsc[c*x])^2)/3 + (2*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcCsc[c*x])])/(3*c^3) - ((I/3)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + ((I/3)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3
```

#### Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)^ (n_.)*(x_)^ (m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCsc
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
0] || LtQ[m, -1])
```

#### Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

#### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^ (n_.)*((c_.) + (d_.)*(x_)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

#### Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x]
```



$(m - 1) \cdot \text{Log}[1 + E^{(I*(e + f*x))}], x], x] /;$  FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_))], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int x^2 (a + b \csc^{-1}(cx))^2 dx &= -\frac{\text{Subst}\left(\int (a + bx)^2 \cot(x) \csc^3(x) dx, x, \csc^{-1}(cx)\right)}{c^3} \\ &= \frac{1}{3} x^3 (a + b \csc^{-1}(cx))^2 - \frac{(2b) \text{Subst}\left(\int (a + bx) \csc^3(x) dx, x, \csc^{-1}(cx)\right)}{3c^3} \\ &= \frac{b^2 x}{3c^2} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^2 (a + b \csc^{-1}(cx))}{3c} + \frac{1}{3} x^3 (a + b \csc^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \csc^3(x) dx, x, \csc^{-1}(cx)\right)}{3c^3} \\ &= \frac{b^2 x}{3c^2} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^2 (a + b \csc^{-1}(cx))}{3c} + \frac{1}{3} x^3 (a + b \csc^{-1}(cx))^2 + \frac{2b(a + b \csc^{-1}(cx)) \text{ta}}{3c^3} \\ &= \frac{b^2 x}{3c^2} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^2 (a + b \csc^{-1}(cx))}{3c} + \frac{1}{3} x^3 (a + b \csc^{-1}(cx))^2 + \frac{2b(a + b \csc^{-1}(cx)) \text{ta}}{3c^3} \\ &= \frac{b^2 x}{3c^2} + \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^2 (a + b \csc^{-1}(cx))}{3c} + \frac{1}{3} x^3 (a + b \csc^{-1}(cx))^2 + \frac{2b(a + b \csc^{-1}(cx)) \text{ta}}{3c^3} \end{aligned}$$

**Mathematica [A]** time = 1.33869, size = 210, normalized size = 1.51

$$\frac{1}{3} \left( \frac{b^2 \left( i \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right) + c^3 x^3 \csc^{-1}(cx)^2 + \csc^{-1}(cx) \left( c^2 x^2 \sqrt{1 - \frac{1}{c^2 x^2}} - \log\left(1 - e^{i \csc^{-1}(cx)}\right) + \log\left(1 + e^{i \csc^{-1}(cx)}\right) \right) \right)}{c^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcCsc[c\*x])^2,x]

[Out]  $(a^2 x^3 + 2 a b x^3 \text{ArcCsc}[c x] + (a b (-c x) + c^3 x^3 + \text{Sqrt}[-1 + c^2 x^2]) \text{ArcTanh}[(c x) / \text{Sqrt}[-1 + c^2 x^2]]) / (c^4 \text{Sqrt}[1 - 1 / (c^2 x^2)] x) - (I b^2 \text{PolyLog}[2, -E^{(I \text{ArcCsc}[c x])}] / c^3 + (b^2 (c x + c^3 x^3 \text{ArcCsc}[c x])^2 + \text{ArcCsc}[c x] (c^2 \text{Sqrt}[1 - 1 / (c^2 x^2)] x^2 - \text{Log}[1 - E^{(I \text{ArcCsc}[c x])}]) + \text{Log}[1 + E^{(I \text{ArcCsc}[c x])}])) + I \text{PolyLog}[2, E^{(I \text{ArcCsc}[c x])}])) / c^3) / 3$

**Maple [B]** time = 0.352, size = 327, normalized size = 2.4

$$\frac{x^3 a^2}{3} + \frac{x^3 b^2 (\text{arccsc}(cx))^2}{3} + \frac{b^2 \text{arccsc}(cx) x^2}{3c} \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + \frac{b^2 x}{3c^2} + \frac{b^2 \text{arccsc}(cx)}{3c^3} \ln\left(1 + \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}}\right) - \frac{i b^2}{c^3} \text{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arccsc(c*x))^2,x)`

[Out]  $\frac{1}{3}x^3a^2 + \frac{1}{3}x^3b^2\arccsc(cx)^2 + \frac{1}{3}cb^2\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}\arccsc(cx)x^2 + \frac{1}{3}b^2x/c^2 + \frac{1}{3}c^3b^2\arccsc(cx)\ln\left(1+I/c/x+(1-1/c^2/x^2)^{1/2}\right) - \frac{1}{3}Ib^2\operatorname{polylog}\left(2,-I/c/x-(1-1/c^2/x^2)^{1/2}\right)/c^3 - \frac{1}{3}c^3b^2\arccsc(cx)\ln\left(1-I/c/x-(1-1/c^2/x^2)^{1/2}\right) + \frac{1}{3}Ib^2\operatorname{polylog}\left(2,I/c/x+(1-1/c^2/x^2)^{1/2}\right)/c^3 + \frac{2}{3}x^3ab\arccsc(cx) + \frac{1}{3}c^4ab\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}x^2 - \frac{1}{3}c^3ab\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2} + \frac{1}{3}c^4ab\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}/\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}/x\ln\left(cx+\left(\frac{c^2x^2-1}{c^2/x^2}\right)^{1/2}\right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{3}a^2x^3 + \frac{1}{6}(4x^3\arccsc(cx) + (2\sqrt{-1/(c^2x^2)} + 1)/(c^2(1/(c^2x^2) - 1) + c^2) + \log(\sqrt{-1/(c^2x^2)} + 1) + 1)/c^2 - \log(\sqrt{-1/(c^2x^2)} + 1) - 1)/c^2/c)ab + \frac{1}{12}(4x^3\arctan(1, \sqrt{cx+1})\sqrt{cx-1})^2 - x^3\log(c^2x^2)^2 - 2c^2(2(c^2x^3 + 3x)/c^4 - 3\log(cx+1)/c^5 + 3\log(cx-1)/c^5)\log(c)^2 + 36c^2\int(1/3x^4\log(c^2x^2)/(c^2x^2-1), x)\log(c) - 72c^2\int(1/3x^4\log(x)/(c^2x^2-1), x)\log(c) + 36c^2\int(1/3x^4\log(c^2x^2)\log(x)/(c^2x^2-1), x) - 36c^2\int(1/3x^4\log(x)^2/(c^2x^2-1), x) + 12c^2\int(1/3x^4\log(c^2x^2)/(c^2x^2-1), x) + 6(2x/c^2 - \log(cx+1)/c^3 + \log(cx-1)/c^3)\log(c)^2 - 36\int(1/3x^2\log(c^2x^2)/(c^2x^2-1), x)\log(c) + 72\int(1/3x^2\log(x)/(c^2x^2-1), x)\log(c) + 24\int(1/3\sqrt{cx+1}\sqrt{cx-1})x^2\arctan(1/(\sqrt{cx+1})\sqrt{cx-1}))/c^2x^2-1, x) - 36\int(1/3x^2\log(c^2x^2)\log(x)/(c^2x^2-1), x) + 36\int(1/3x^2\log(x)^2/(c^2x^2-1), x) - 12\int(1/3x^2\log(c^2x^2)/(c^2x^2-1), x))b^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2x^2\arccsc(cx)^2 + 2abx^2\arccsc(cx) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x^2*arccsc(c*x)^2 + 2*a*b*x^2*arccsc(c*x) + a^2*x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{acsc}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2*x^2, x)
```

### 3.17 $\int x \left( a + b \operatorname{csc}^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=55

$$\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\operatorname{csc}^{-1}(cx))}{c} + \frac{1}{2}x^2(a+b\operatorname{csc}^{-1}(cx))^2 + \frac{b^2\log(x)}{c^2}$$

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(a + b\*ArcCsc[c\*x]))/c + (x^2\*(a + b\*ArcCsc[c\*x])^2)/2 + (b^2\*Log[x])/c^2

**Rubi [A]** time = 0.0700716, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5223, 4410, 4184, 3475}

$$\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\operatorname{csc}^{-1}(cx))}{c} + \frac{1}{2}x^2(a+b\operatorname{csc}^{-1}(cx))^2 + \frac{b^2\log(x)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcCsc[c\*x])^2,x]

[Out] (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(a + b\*ArcCsc[c\*x]))/c + (x^2\*(a + b\*ArcCsc[c\*x])^2)/2 + (b^2\*Log[x])/c^2

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4410

Int[Cot[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csc[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Csc[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csc[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int x(a + b \csc^{-1}(cx))^2 dx &= -\frac{\text{Subst}\left(\int (a + bx)^2 \cot(x) \csc^2(x) dx, x, \csc^{-1}(cx)\right)}{c^2} \\
&= \frac{1}{2}x^2(a + b \csc^{-1}(cx))^2 - \frac{b \text{Subst}\left(\int (a + bx) \csc^2(x) dx, x, \csc^{-1}(cx)\right)}{c^2} \\
&= \frac{b\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))}{c} + \frac{1}{2}x^2(a + b \csc^{-1}(cx))^2 - \frac{b^2 \text{Subst}\left(\int \cot(x) dx, x, \csc^{-1}(cx)\right)}{c^2} \\
&= \frac{b\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))}{c} + \frac{1}{2}x^2(a + b \csc^{-1}(cx))^2 + \frac{b^2 \log(x)}{c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.14625, size = 89, normalized size = 1.62

$$\frac{acx \left( acx + 2b\sqrt{1 - \frac{1}{c^2x^2}} \right) + 2bcx \csc^{-1}(cx) \left( acx + b\sqrt{1 - \frac{1}{c^2x^2}} \right) + b^2c^2x^2 \csc^{-1}(cx)^2 + 2b^2 \log(cx)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcCsc[c\*x])^2,x]

[Out] (a\*c\*x\*(2\*b\*Sqrt[1 - 1/(c^2\*x^2)] + a\*c\*x) + 2\*b\*c\*x\*(b\*Sqrt[1 - 1/(c^2\*x^2)] + a\*c\*x)\*ArcCsc[c\*x] + b^2\*c^2\*x^2\*ArcCsc[c\*x]^2 + 2\*b^2\*Log[c\*x])/(2\*c^2)

**Maple [B]** time = 0.222, size = 133, normalized size = 2.4

$$\frac{a^2x^2}{2} + \frac{x^2b^2(\text{arccsc}(cx))^2}{2} + \frac{b^2\text{arccsc}(cx)x}{c} \sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b^2}{c^2} \ln\left(\frac{1}{cx}\right) + abx^2\text{arccsc}(cx) + \frac{xab}{c} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{ab}{c^3x} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsc(c\*x))^2,x)

[Out] 1/2\*a^2\*x^2+1/2\*x^2\*b^2\*arccsc(c\*x)^2+1/c\*b^2\*arccsc(c\*x)\*x\*((c^2\*x^2-1)/c^2/x^2)^(1/2)-1/c^2\*b^2\*ln(1/c/x)+a\*b\*x^2\*arccsc(c\*x)+1/c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x-1/c^3\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x

**Maxima [A]** time = 1.00184, size = 113, normalized size = 2.05

$$\frac{1}{2}b^2x^2 \text{arccsc}(cx)^2 + \frac{1}{2}a^2x^2 + \left( x^2 \text{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c} \right) ab + \left( \frac{x\sqrt{-\frac{1}{c^2x^2} + 1} \text{arccsc}(cx)}{c} + \frac{\log(x)}{c^2} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))^2,x, algorithm="maxima")

[Out] 1/2\*b^2\*x^2\*arccsc(c\*x)^2 + 1/2\*a^2\*x^2 + (x^2\*arccsc(c\*x) + x\*sqrt(-1/(c^2\*x^2) + 1)/c)\*a\*b + (x\*sqrt(-1/(c^2\*x^2) + 1)\*arccsc(c\*x)/c + log(x)/c^2)\*b^2

$\sim 2$

**Fricas [B]** time = 2.2188, size = 266, normalized size = 4.84

$$\frac{b^2 c^2 x^2 \operatorname{arccsc}(cx)^2 + a^2 c^2 x^2 - 4 abc^2 \arctan(-cx + \sqrt{c^2 x^2 - 1}) + 2 b^2 \log(x) + 2 (abc^2 x^2 - abc^2) \operatorname{arccsc}(cx) + 2 \sqrt{c^2 x^2 - 1}}{2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))^2,x, algorithm="fricas")

[Out] 1/2\*(b^2\*c^2\*x^2\*arccsc(c\*x)^2 + a^2\*c^2\*x^2 - 4\*a\*b\*c^2\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + 2\*b^2\*log(x) + 2\*(a\*b\*c^2\*x^2 - a\*b\*c^2)\*arccsc(c\*x) + 2\*sqrt(c^2\*x^2 - 1)\*(b^2\*arccsc(c\*x) + a\*b))/c^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{arccsc}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsc(c\*x))\*\*2,x)

[Out] Integral(x\*(a + b\*acsc(c\*x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2\*x, x)

### 3.18 $\int (a + b \operatorname{csc}^{-1}(cx))^2 dx$

**Optimal.** Leaf size=84

$$-\frac{2ib^2 \operatorname{PolyLog}(2, -e^{i \operatorname{csc}^{-1}(cx)})}{c} + \frac{2ib^2 \operatorname{PolyLog}(2, e^{i \operatorname{csc}^{-1}(cx)})}{c} + x(a + b \operatorname{csc}^{-1}(cx))^2 + \frac{4b \tanh^{-1}(e^{i \operatorname{csc}^{-1}(cx)})(a + b \operatorname{csc}^{-1}(cx))}{c}$$

[Out] x\*(a + b\*ArcCsc[c\*x])^2 + (4\*b\*(a + b\*ArcCsc[c\*x])\*ArcTanh[E^(I\*ArcCsc[c\*x])])/c - ((2\*I)\*b^2\*PolyLog[2, -E^(I\*ArcCsc[c\*x])])/c + ((2\*I)\*b^2\*PolyLog[2, E^(I\*ArcCsc[c\*x])])/c

**Rubi [A]** time = 0.0675351, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5217, 4410, 4183, 2279, 2391}

$$-\frac{2ib^2 \operatorname{PolyLog}(2, -e^{i \operatorname{csc}^{-1}(cx)})}{c} + \frac{2ib^2 \operatorname{PolyLog}(2, e^{i \operatorname{csc}^{-1}(cx)})}{c} + x(a + b \operatorname{csc}^{-1}(cx))^2 + \frac{4b \tanh^{-1}(e^{i \operatorname{csc}^{-1}(cx)})(a + b \operatorname{csc}^{-1}(cx))}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^2, x]

[Out] x\*(a + b\*ArcCsc[c\*x])^2 + (4\*b\*(a + b\*ArcCsc[c\*x])\*ArcTanh[E^(I\*ArcCsc[c\*x])])/c - ((2\*I)\*b^2\*PolyLog[2, -E^(I\*ArcCsc[c\*x])])/c + ((2\*I)\*b^2\*PolyLog[2, E^(I\*ArcCsc[c\*x])])/c

#### Rule 5217

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))^n, x\_Symbol] := -Dist[c^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[n, 0]

#### Rule 4410

Int[Cot[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csc[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(((c + d\*x)^m\*Csc[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csc[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 4183

Int[csc[(e\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))]^(n\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int (a + b \csc^{-1}(cx))^2 dx &= -\frac{\text{Subst}\left(\int (a + bx)^2 \cot(x) \csc(x) dx, x, \csc^{-1}(cx)\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^2 - \frac{(2b) \text{Subst}\left(\int (a + bx) \csc(x) dx, x, \csc^{-1}(cx)\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^2 + \frac{4b(a + b \csc^{-1}(cx)) \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right)}{c} + \frac{(2b^2) \text{Subst}\left(\int \log(1 - e^{ix}) dx, x, \csc^{-1}(cx)\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^2 + \frac{4b(a + b \csc^{-1}(cx)) \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right)}{c} - \frac{(2ib^2) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, \csc^{-1}(cx)\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^2 + \frac{4b(a + b \csc^{-1}(cx)) \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right)}{c} - \frac{2ib^2 \text{Li}_2\left(-e^{i \csc^{-1}(cx)}\right)}{c} + \frac{2ib^2 \text{Li}_2\left(-e^{-i \csc^{-1}(cx)}\right)}{c} \end{aligned}$$

**Mathematica [A]** time = 0.202161, size = 147, normalized size = 1.75

$$\frac{-2ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right) + 2ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right) + a^2 cx + 2abcx \csc^{-1}(cx) + 2ab \log\left(\cos\left(\frac{1}{2} \csc^{-1}(cx)\right)\right) - 2a^2 \csc^{-1}(cx)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])^2, x]

[Out] (a^2\*c\*x + 2\*a\*b\*c\*x\*ArcCsc[c\*x] + b^2\*c\*x\*ArcCsc[c\*x]^2 - 2\*b^2\*ArcCsc[c\*x]\*Log[1 - E^(I\*ArcCsc[c\*x])] + 2\*b^2\*ArcCsc[c\*x]\*Log[1 + E^(I\*ArcCsc[c\*x])] + 2\*a\*b\*Log[Cos[ArcCsc[c\*x]/2]] - 2\*a\*b\*Log[Sin[ArcCsc[c\*x]/2]] - (2\*I)\*b^2\*PolyLog[2, -E^(I\*ArcCsc[c\*x])] + (2\*I)\*b^2\*PolyLog[2, E^(I\*ArcCsc[c\*x])]) /c

**Maple [A]** time = 0.266, size = 196, normalized size = 2.3

$$xb^2 (\text{arccsc}(cx))^2 + 2xab \text{arccsc}(cx) - 2 \frac{b^2 \text{arccsc}(cx)}{c} \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2 x^2}}\right) + 2 \frac{b^2 \text{arccsc}(cx)}{c} \ln\left(1 + \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^2, x)

[Out] x\*b^2\*arccsc(c\*x)^2+2\*x\*a\*b\*arccsc(c\*x)-2/c\*b^2\*arccsc(c\*x)\*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2/c\*b^2\*arccsc(c\*x)\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2\*I\*b^2\*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+2\*I\*b^2\*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c+a^2\*x+2/c\*ln(c\*x+c\*x\*(1-1/c^2/x^2)^(1/2))\*a\*b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4} \left( 2c^2 \left( \frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3} \right) \log(c)^2 - 4c^2 \int \frac{x^2 \log(c^2 x^2)}{c^2 x^2 - 1} dx \log(c) + 8c^2 \int \frac{x^2 \log(x)}{c^2 x^2 - 1} dx \log(c) - 4x a \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2,x, algorithm="maxima")

[Out]  $-1/4*(2*c^2*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3)*\log(c)^2 - 4*c^2*\int(x^2*\log(c^2*x^2)/(c^2*x^2 - 1), x)*\log(c) + 8*c^2*\int(x^2*\log(x)/(c^2*x^2 - 1), x)*\log(c) - 4*x*\arctan(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - 4*c^2*\int(x^2*\log(c^2*x^2)*\log(x)/(c^2*x^2 - 1), x) + 4*c^2*\int(x^2*\log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*\int(x^2*\log(c^2*x^2)/(c^2*x^2 - 1), x) + x*\log(c^2*x^2)^2 + 2*(\log(c*x + 1)/c - \log(c*x - 1)/c)*\log(c)^2 + 4*\int(\log(c^2*x^2)/(c^2*x^2 - 1), x)*\log(c) - 8*\int(\log(x)/(c^2*x^2 - 1), x)*\log(c) - 8*\int(\sqrt{c*x + 1})*\sqrt{c*x - 1}*\arctan(1/(\sqrt{c*x + 1})*\sqrt{c*x - 1}))/c^2, x) + 4*\int(\log(c^2*x^2)*\log(x)/(c^2*x^2 - 1), x) - 4*\int(\log(x)^2/(c^2*x^2 - 1), x) + 4*\int(\log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arccsc(c*x) + \log(\sqrt{-1/(c^2*x^2) + 1} + 1) - \log(-\sqrt{-1/(c^2*x^2) + 1} + 1))*a*b/c$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int (b^2 \operatorname{arccsc}(cx)^2 + 2ab \operatorname{arccsc}(cx) + a^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2,x, algorithm="fricas")

[Out] integral(b^2\*arccsc(c\*x)^2 + 2\*a\*b\*arccsc(c\*x) + a^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*\*2,x)

[Out] Integral((a + b\*acsc(c\*x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2, x)

$$3.19 \quad \int \frac{(a+b \csc^{-1}(cx))^2}{x} dx$$

**Optimal.** Leaf size=91

$$ibPolyLog(2, e^{2i \csc^{-1}(cx)}) (a + b \csc^{-1}(cx)) - \frac{1}{2} b^2 PolyLog(3, e^{2i \csc^{-1}(cx)}) + \frac{i (a + b \csc^{-1}(cx))^3}{3b} - \log(1 - e^{2i \csc^{-1}(cx)}) (a$$

[Out] ((I/3)\*(a + b\*ArcCsc[c\*x])^3)/b - (a + b\*ArcCsc[c\*x])^2\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + I\*b\*(a + b\*ArcCsc[c\*x])\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])] - (b^2\*PolyLog[3, E^((2\*I)\*ArcCsc[c\*x])])/2

**Rubi [A]** time = 0.123399, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5223, 3717, 2190, 2531, 2282, 6589}

$$ibPolyLog(2, e^{2i \csc^{-1}(cx)}) (a + b \csc^{-1}(cx)) - \frac{1}{2} b^2 PolyLog(3, e^{2i \csc^{-1}(cx)}) + \frac{i (a + b \csc^{-1}(cx))^3}{3b} - \log(1 - e^{2i \csc^{-1}(cx)}) (a$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^2/x, x]

[Out] ((I/3)\*(a + b\*ArcCsc[c\*x])^3)/b - (a + b\*ArcCsc[c\*x])^2\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + I\*b\*(a + b\*ArcCsc[c\*x])\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])] - (b^2\*PolyLog[3, E^((2\*I)\*ArcCsc[c\*x])])/2

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m, x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 3717

Int[((c\_.) + (d\_.)\*(x\_)^m)\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m \* E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^n)\*((c\_) + (d\_.)\*(x\_)^m)/((a\_) + (b\_.)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^n), x\_Symbol] :> Simp[((c + d\*x)^m \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1) \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_)\*((a\_.) + (b\_.)\*(x\_)))^n]\*((f\_.) + (g\_.)\*(x\_)^m), x\_Symbol] :> -Simp[((f + g\*x)^m \* PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1) \* PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f

, g, n}, x] && GtQ[m, 0]

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \csc^{-1}(cx))^2}{x} dx &= -\text{Subst} \left( \int (a + bx)^2 \cot(x) dx, x, \csc^{-1}(cx) \right) \\ &= \frac{i(a + b \csc^{-1}(cx))^3}{3b} + 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)^2}{1 - e^{2ix}} dx, x, \csc^{-1}(cx) \right) \\ &= \frac{i(a + b \csc^{-1}(cx))^3}{3b} - (a + b \csc^{-1}(cx))^2 \log(1 - e^{2i \csc^{-1}(cx)}) + (2b) \text{Subst} \left( \int (a + bx) \log \right) \\ &= \frac{i(a + b \csc^{-1}(cx))^3}{3b} - (a + b \csc^{-1}(cx))^2 \log(1 - e^{2i \csc^{-1}(cx)}) + ib(a + b \csc^{-1}(cx)) \text{Li}_2(e^{2i \csc^{-1}(cx)}) \\ &= \frac{i(a + b \csc^{-1}(cx))^3}{3b} - (a + b \csc^{-1}(cx))^2 \log(1 - e^{2i \csc^{-1}(cx)}) + ib(a + b \csc^{-1}(cx)) \text{Li}_2(e^{2i \csc^{-1}(cx)}) \\ &= \frac{i(a + b \csc^{-1}(cx))^3}{3b} - (a + b \csc^{-1}(cx))^2 \log(1 - e^{2i \csc^{-1}(cx)}) + ib(a + b \csc^{-1}(cx)) \text{Li}_2(e^{2i \csc^{-1}(cx)}) \end{aligned}$$

**Mathematica [A]** time = 0.14246, size = 137, normalized size = 1.51

$$iab(\csc^{-1}(cx)^2 + \text{PolyLog}(2, e^{2i \csc^{-1}(cx)})) + \frac{1}{24}ib^2(-24 \csc^{-1}(cx)\text{PolyLog}(2, e^{-2i \csc^{-1}(cx)}) + 12i\text{PolyLog}(3, e^{-2i \csc^{-1}(cx)}))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x, x]
```

```
[Out] -2*a*b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a^2*Log[c*x] + I*a*b*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/24)*b^2*(Pi^3 - 8*ArcCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 24*ArcCsc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*ArcCsc[c*x])])
```

**Maple [B]** time = 0.295, size = 361, normalized size = 4.

$$a^2 \ln(cx) + \frac{i}{3}b^2(\text{arccsc}(cx))^3 - b^2(\text{arccsc}(cx))^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2ib^2 \text{arccsc}(cx) \text{polylog}\left(2, \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^2/x,x)

[Out]  $a^2 \ln(cx) + \frac{1}{3} I b^2 \arccsc(cx)^3 - b^2 \arccsc(cx)^2 \ln(1 - I/c/x - (1 - 1/c^2/x^2)^{1/2}) + 2 I b^2 \arccsc(cx) \operatorname{polylog}(2, I/c/x + (1 - 1/c^2/x^2)^{1/2}) - 2 b^2 \operatorname{polylog}(3, I/c/x + (1 - 1/c^2/x^2)^{1/2}) - b^2 \arccsc(cx)^2 \ln(1 + I/c/x + (1 - 1/c^2/x^2)^{1/2}) + 2 I b^2 \arccsc(cx) \operatorname{polylog}(2, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 2 b^2 \operatorname{polylog}(3, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) + I a b \arccsc(cx)^2 + 2 I a b \operatorname{polylog}(2, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 2 a b \arccsc(cx) \ln(1 - I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 2 a b \arccsc(cx) \ln(1 + I/c/x + (1 - 1/c^2/x^2)^{1/2}) + 2 I a b \operatorname{polylog}(2, I/c/x + (1 - 1/c^2/x^2)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} b^2 c^2 \left( \frac{\log(cx+1)}{c^2} + \frac{\log(cx-1)}{c^2} \right) \log(c)^2 + b^2 c^2 \int \frac{x^2 \log(c^2 x^2)}{c^2 x^3 - x} dx \log(c) - 2 b^2 c^2 \int \frac{x^2 \log(x)}{c^2 x^3 - x} dx \log(c) + 2 b^2 c^2 \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x,x, algorithm="maxima")

[Out]  $-1/2 b^2 c^2 (\log(cx+1)/c^2 + \log(cx-1)/c^2) \log(c)^2 + b^2 c^2 \operatorname{integrate}(x^2 \log(c^2 x^2)/(c^2 x^3 - x), x) \log(c) - 2 b^2 c^2 \operatorname{integrate}(x^2 \log(x)/(c^2 x^3 - x), x) \log(c) + 2 b^2 c^2 \operatorname{integrate}(x^2 \log(c^2 x^2) \log(x)/(c^2 x^3 - x), x) - b^2 c^2 \operatorname{integrate}(x^2 \log(x)^2/(c^2 x^3 - x), x) + 2 a b c^2 \operatorname{integrate}(x^2 \arctan(1/(\sqrt{cx+1}) \sqrt{cx-1}))/ (c^2 x^3 - x), x) + 1/2 b^2 (\log(cx+1) + \log(cx-1) - 2 \log(x)) \log(c)^2 + b^2 \arctan^2(1, \sqrt{cx+1} \sqrt{cx-1})^2 \log(x) - 1/4 b^2 \log(c^2 x^2)^2 \log(x) - b^2 \operatorname{integrate}(\log(c^2 x^2)/(c^2 x^3 - x), x) \log(c) + 2 b^2 \operatorname{integrate}(\log(x)/(c^2 x^3 - x), x) \log(c) + 2 b^2 \operatorname{integrate}(\sqrt{cx+1} \sqrt{cx-1} \arctan(1/(\sqrt{cx+1}) \sqrt{cx-1})) \log(x)/(c^2 x^3 - x), x) - 2 b^2 \operatorname{integrate}(\log(c^2 x^2) \log(x)/(c^2 x^3 - x), x) + b^2 \operatorname{integrate}(\log(x)^2/(c^2 x^3 - x), x) - 2 a b \operatorname{integrate}(\arctan(1/(\sqrt{cx+1}) \sqrt{cx-1}))/ (c^2 x^3 - x), x) + a^2 \log(x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^2 \arccsc(cx)^2 + 2 ab \arccsc(cx) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x,x, algorithm="fricas")

[Out] integral((b^2\*arccsc(c\*x))^2 + 2\*a\*b\*arccsc(c\*x) + a^2)/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*\*2/x,x)

[Out] Integral((a + b\*acsc(c\*x))\*\*2/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2/x, x)

$$3.20 \quad \int \frac{(a+b \csc^{-1}(cx))^2}{x^2} dx$$

**Optimal.** Leaf size=50

$$-2bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx)) - \frac{(a + b \csc^{-1}(cx))^2}{x} + \frac{2b^2}{x}$$

[Out] (2\*b^2)/x - 2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]) - (a + b\*ArcCsc[c\*x])^2/x

**Rubi [A]** time = 0.0608282, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5223, 3296, 2637}

$$-2bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx)) - \frac{(a + b \csc^{-1}(cx))^2}{x} + \frac{2b^2}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^2/x^2, x]

[Out] (2\*b^2)/x - 2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]) - (a + b\*ArcCsc[c\*x])^2/x

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_.)]\*(b\_.))^ (n\_.)\*(x\_)^(m\_.), x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{(a + b \csc^{-1}(cx))^2}{x^2} dx &= -\left(c \operatorname{Subst}\left(\int (a + bx)^2 \cos(x) dx, x, \csc^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \csc^{-1}(cx))^2}{x} + (2bc) \operatorname{Subst}\left(\int (a + bx) \sin(x) dx, x, \csc^{-1}(cx)\right) \\ &= -2bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx)) - \frac{(a + b \csc^{-1}(cx))^2}{x} + (2b^2c) \operatorname{Subst}\left(\int \cos(x) dx, x, \csc^{-1}(cx)\right) \\ &= \frac{2b^2}{x} - 2bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx)) - \frac{(a + b \csc^{-1}(cx))^2}{x} \end{aligned}$$

**Mathematica [A]** time = 0.146587, size = 71, normalized size = 1.42

$$\frac{a^2 + 2abcx\sqrt{1 - \frac{1}{c^2x^2}} + 2b \operatorname{csc}^{-1}(cx) \left( a + bcx\sqrt{1 - \frac{1}{c^2x^2}} \right) + b^2 \operatorname{csc}^{-1}(cx)^2 - 2b^2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])^2/x^2,x]

[Out] -((a^2 - 2\*b^2 + 2\*a\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + 2\*b\*(a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)\*ArcCsc[c\*x] + b^2\*ArcCsc[c\*x]^2)/x)

**Maple [B]** time = 0.219, size = 118, normalized size = 2.4

$$c \left( -\frac{a^2}{cx} + b^2 \left( -\frac{(\operatorname{arccsc}(cx))^2}{cx} + 2 \frac{1}{cx} - 2 \sqrt{\frac{c^2x^2-1}{c^2x^2}} \operatorname{arccsc}(cx) \right) + 2ab \left( -\frac{\operatorname{arccsc}(cx)}{cx} - \frac{c^2x^2-1}{c^2x^2} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^2/x^2,x)

[Out] c\*(-a^2/c/x+b^2\*(-1/c/x\*arccsc(c\*x)^2+2/c/x-2\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*arccsc(c\*x))+2\*a\*b\*(-1/c/x\*arccsc(c\*x)-1/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2\*(c^2\*x^2-1)))

**Maxima [A]** time = 1.00159, size = 107, normalized size = 2.14

$$-2 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} + \frac{\operatorname{arccsc}(cx)}{x} \right) ab - 2 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} \operatorname{arccsc}(cx) - \frac{1}{x} \right) b^2 - \frac{b^2 \operatorname{arccsc}(cx)^2}{x} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^2,x, algorithm="maxima")

[Out] -2\*(c\*sqrt(-1/(c^2\*x^2) + 1) + arccsc(c\*x)/x)\*a\*b - 2\*(c\*sqrt(-1/(c^2\*x^2) + 1)\*arccsc(c\*x) - 1/x)\*b^2 - b^2\*arccsc(c\*x)^2/x - a^2/x

**Fricas [A]** time = 1.8743, size = 140, normalized size = 2.8

$$\frac{b^2 \operatorname{arccsc}(cx)^2 + 2ab \operatorname{arccsc}(cx) + a^2 - 2b^2 + 2\sqrt{c^2x^2-1}(b^2 \operatorname{arccsc}(cx) + ab)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^2,x, algorithm="fricas")

[Out] -(b^2\*arccsc(c\*x)^2 + 2\*a\*b\*arccsc(c\*x) + a^2 - 2\*b^2 + 2\*sqrt(c^2\*x^2 - 1)\*(b^2\*arccsc(c\*x) + a\*b))/x

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2/x^2, x)
```



$$3.21 \quad \int \frac{(a+b \csc^{-1}(cx))^2}{x^3} dx$$

**Optimal.** Leaf size=88

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{2x} + \frac{1}{2}abc^2 \csc^{-1}(cx) - \frac{(a+b \csc^{-1}(cx))^2}{2x^2} + \frac{1}{4}b^2c^2 \csc^{-1}(cx)^2 + \frac{b^2}{4x^2}$$

[Out] b^2/(4\*x^2) + (a\*b\*c^2\*ArcCsc[c\*x])/2 + (b^2\*c^2\*ArcCsc[c\*x]^2)/4 - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(2\*x) - (a + b\*ArcCsc[c\*x])^2/(2\*x^2)

**Rubi [A]** time = 0.0779932, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5223, 4404, 3310}

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{2x} + \frac{1}{2}abc^2 \csc^{-1}(cx) - \frac{(a+b \csc^{-1}(cx))^2}{2x^2} + \frac{1}{4}b^2c^2 \csc^{-1}(cx)^2 + \frac{b^2}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^2/x^3, x]

[Out] b^2/(4\*x^2) + (a\*b\*c^2\*ArcCsc[c\*x])/2 + (b^2\*c^2\*ArcCsc[c\*x]^2)/4 - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(2\*x) - (a + b\*ArcCsc[c\*x])^2/(2\*x^2)

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m, x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^m\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4404

Int[Cos[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^m\*Sin[(a\_.) + (b\_.)\*(x\_)]^n, x\_Symbol] := Simp[((c + d\*x)^m\*Sin[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sin[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^n, x\_Symbol] := Simp[(d\*(b\*Sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{csc}^{-1}(cx))^2}{x^3} dx &= -\left(c^2 \operatorname{Subst}\left(\int (a + bx)^2 \cos(x) \sin(x) dx, x, \operatorname{csc}^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \operatorname{csc}^{-1}(cx))^2}{2x^2} + (bc^2) \operatorname{Subst}\left(\int (a + bx) \sin^2(x) dx, x, \operatorname{csc}^{-1}(cx)\right) \\
&= \frac{b^2}{4x^2} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))}{2x} - \frac{(a + b \operatorname{csc}^{-1}(cx))^2}{2x^2} + \frac{1}{2}(bc^2) \operatorname{Subst}\left(\int (a + bx) dx, x, c\right) \\
&= \frac{b^2}{4x^2} + \frac{1}{2}abc^2 \operatorname{csc}^{-1}(cx) + \frac{1}{4}b^2c^2 \operatorname{csc}^{-1}(cx)^2 - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))}{2x} - \frac{(a + b \operatorname{csc}^{-1}(cx))^2}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.116418, size = 102, normalized size = 1.16

$$\frac{-2a^2 - 2abcx\sqrt{1 - \frac{1}{c^2x^2}} + 2abc^2x^2 \sin^{-1}\left(\frac{1}{cx}\right) - 2b \operatorname{csc}^{-1}(cx)\left(2a + bcx\sqrt{1 - \frac{1}{c^2x^2}}\right) + b^2(c^2x^2 - 2) \operatorname{csc}^{-1}(cx)^2 + b^2}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])^2/x^3, x]

[Out] (-2\*a^2 + b^2 - 2\*a\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x - 2\*b\*(2\*a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)\*ArcCsc[c\*x] + b^2\*(-2 + c^2\*x^2)\*ArcCsc[c\*x]^2 + 2\*a\*b\*c^2\*x^2\*ArcSin[1/(c\*x)])/(4\*x^2)

**Maple [B]** time = 0.214, size = 191, normalized size = 2.2

$$-\frac{a^2}{2x^2} + \frac{b^2c^2(\operatorname{arccsc}(cx))^2}{4} - \frac{b^2(\operatorname{arccsc}(cx))^2}{2x^2} - \frac{cb^2\operatorname{arccsc}(cx)}{2x} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + \frac{b^2}{4x^2} - \frac{ab\operatorname{arccsc}(cx)}{x^2} + \frac{acb}{2x} \sqrt{c^2x^2 - 1} \operatorname{arctan}\left(\frac{1}{c^2x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^2/x^3, x)

[Out] -1/2\*a^2/x^2+1/4\*b^2\*c^2\*arccsc(c\*x)^2-1/2\*b^2/x^2\*arccsc(c\*x)^2-1/2\*c\*b^2\*arccsc(c\*x)/x\*((c^2\*x^2-1)/c^2/x^2)^(1/2)+1/4\*b^2/x^2-a\*b/x^2\*arccsc(c\*x)+1/2\*c\*a\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*arctan(1/(c^2\*x^2-1)^(1/2))-1/2\*c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x+1/2/c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^3, x, algorithm="maxima")

[Out] 1/2\*a\*b\*((c^4\*x\*sqrt(-1/(c^2\*x^2) + 1)/(c^2\*x^2\*(1/(c^2\*x^2) - 1) - 1) - c^3\*arctan(c\*x\*sqrt(-1/(c^2\*x^2) + 1)))/c - 2\*arccsc(c\*x)/x^2) - 1/8\*(4\*(c^2\*

$(\log(cx + 1) + \log(cx - 1) - 2\log(x))\log(c)^2 - 4c^2\int \frac{1/2x^2 \log(c^2x^2)}{(c^2x^5 - x^3), x} \log(c) + 8c^2\int \frac{1/2x^2 \log(x)}{(c^2x^5 - x^3), x} \log(c) - 4c^2\int \frac{1/2x^2 \log(c^2x^2) \log(x)}{(c^2x^5 - x^3), x} + 4c^2\int \frac{1/2x^2 \log(x)^2}{(c^2x^5 - x^3), x} + 2c^2\int \frac{1/2x^2 \log(c^2x^2)}{(c^2x^5 - x^3), x} - (c^2\log(cx + 1) + c^2\log(cx - 1) - 2c^2\log(x) + 1/x^2)\log(c)^2 + 4\int \frac{1/2 \log(c^2x^2)}{(c^2x^5 - x^3), x} \log(c) - 8\int \frac{1/2 \log(x)}{(c^2x^5 - x^3), x} \log(c) + 4\int \frac{1/2 \sqrt{cx + 1} \sqrt{cx - 1} \arctan(1/(\sqrt{cx + 1} \sqrt{cx - 1}))}{(c^2x^5 - x^3), x} + 4\int \frac{1/2 \log(c^2x^2) \log(x)}{(c^2x^5 - x^3), x} - 4\int \frac{1/2 \log(x)^2}{(c^2x^5 - x^3), x} - 2\int \frac{1/2 \log(c^2x^2)}{(c^2x^5 - x^3), x} x^2 + 4\arctan^2(1, \sqrt{cx + 1} \sqrt{cx - 1})^2 - \log(c^2x^2)^2 b^2/x^2 - 1/2a^2/x^2$

**Fricas [A]** time = 2.03099, size = 196, normalized size = 2.23

$$\frac{(b^2c^2x^2 - 2b^2)\operatorname{arccsc}(cx)^2 - 2a^2 + b^2 + 2(abc^2x^2 - 2ab)\operatorname{arccsc}(cx) - 2\sqrt{c^2x^2 - 1}(b^2\operatorname{arccsc}(cx) + ab)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^3,x, algorithm="fricas")

[Out] 1/4\*((b^2\*c^2\*x^2 - 2\*b^2)\*arccsc(c\*x)^2 - 2\*a^2 + b^2 + 2\*(a\*b\*c^2\*x^2 - 2\*a\*b)\*arccsc(c\*x) - 2\*sqrt(c^2\*x^2 - 1)\*(b^2\*arccsc(c\*x) + a\*b))/x^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*\*2/x\*\*3,x)

[Out] Integral((a + b\*arccsc(c\*x))\*\*2/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2/x^3, x)

$$3.22 \quad \int \frac{(a+b \csc^{-1}(cx))^2}{x^4} dx$$

**Optimal.** Leaf size=102

$$-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx)) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{9x^2} - \frac{(a+b \csc^{-1}(cx))^2}{3x^3} + \frac{4b^2c^2}{9x} + \frac{2b^2}{27x^3}$$

[Out] (2\*b^2)/(27\*x^3) + (4\*b^2\*c^2)/(9\*x) - (4\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/9 - (2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(9\*x^2) - (a + b\*ArcCsc[c\*x])^2/(3\*x^3)

**Rubi [A]** time = 0.0963057, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5223, 4404, 3310, 3296, 2637}

$$-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx)) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{9x^2} - \frac{(a+b \csc^{-1}(cx))^2}{3x^3} + \frac{4b^2c^2}{9x} + \frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^2/x^4, x]

[Out] (2\*b^2)/(27\*x^3) + (4\*b^2\*c^2)/(9\*x) - (4\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/9 - (2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(9\*x^2) - (a + b\*ArcCsc[c\*x])^2/(3\*x^3)

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_.)]\*(b\_.))^n\*(x\_)^m, x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4404

Int[Cos[(a\_.) + (b\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^m\*Sin[(a\_.) + (b\_.)\*(x\_.)]^n, x\_Symbol] :> Simp[((c + d\*x)^m\*Sin[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sin[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_.))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^n, x\_Symbol] :> Simp[(d\*(b\*Ssin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Ssin[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^m\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] :> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

**Rule 2637**

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

**Rubi steps**

$$\begin{aligned} \int \frac{(a + b \csc^{-1}(cx))^2}{x^4} dx &= -\left(c^3 \operatorname{Subst}\left(\int (a + bx)^2 \cos(x) \sin^2(x) dx, x, \csc^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \csc^{-1}(cx))^2}{3x^3} + \frac{1}{3}(2bc^3) \operatorname{Subst}\left(\int (a + bx) \sin^3(x) dx, x, \csc^{-1}(cx)\right) \\ &= \frac{2b^2}{27x^3} - \frac{2bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))}{9x^2} - \frac{(a + b \csc^{-1}(cx))^2}{3x^3} + \frac{1}{9}(4bc^3) \operatorname{Subst}\left(\int (a + bx) \right) \\ &= \frac{2b^2}{27x^3} - \frac{4}{9}bc^3\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx)) - \frac{2bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))}{9x^2} - \frac{(a + b \csc^{-1}(cx))^2}{3x^3} \\ &= \frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} - \frac{4}{9}bc^3\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx)) - \frac{2bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))}{9x^2} - \frac{(a + b \csc^{-1}(cx))^2}{3x^3} \end{aligned}$$

**Mathematica [A]** time = 0.225241, size = 108, normalized size = 1.06

$$\frac{9a^2 + 6abcx\sqrt{1 - \frac{1}{c^2x^2}}(2c^2x^2 + 1) + 6b \csc^{-1}(cx)\left(3a + bcx\sqrt{1 - \frac{1}{c^2x^2}}(2c^2x^2 + 1)\right) - 2b^2(6c^2x^2 + 1) + 9b^2 \csc^{-1}(cx)^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^4, x]
```

```
[Out] -(9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) - 2*b^2*(1 + 6*c^2*x^2) + 6*b*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*ArcCsc[c*x] + 9*b^2*ArcCsc[c*x]^2)/(27*x^3)
```

**Maple [A]** time = 0.219, size = 154, normalized size = 1.5

$$c^3 \left( -\frac{a^2}{3c^3x^3} + b^2 \left( -\frac{(\operatorname{arccsc}(cx))^2}{3c^3x^3} - \frac{2 \operatorname{arccsc}(cx)(2c^2x^2 + 1)}{9c^2x^2} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + \frac{2}{27c^3x^3} + \frac{4}{9cx} \right) + 2ab \left( -\frac{1}{3} \frac{\operatorname{arccsc}(cx)}{c^3x^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^4, x)
```

```
[Out] c^3*(-1/3*a^2/c^3/x^3+b^2*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*arccsc(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4)
```

**Maxima [B]** time = 2.10965, size = 266, normalized size = 2.61

$$\frac{2}{9} ab \left( \frac{c^4 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3 c^4 \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} - \frac{3 \operatorname{arccsc}(cx)}{x^3} \right) - \frac{b^2 \operatorname{arccsc}(cx)^2}{3 x^3} - \frac{a^2}{3 x^3} - \frac{2 \left( 6 c^5 x^4 \arctan \left( 1, \sqrt{cx+1} \sqrt{cx-1} \right) \right)}{27 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^4,x, algorithm="maxima")

[Out] 2/9\*a\*b\*((c^4\*(-1/(c^2\*x^2) + 1)^(3/2) - 3\*c^4\*sqrt(-1/(c^2\*x^2) + 1))/c - 3\*arccsc(c\*x)/x^3) - 1/3\*b^2\*arccsc(c\*x)^2/x^3 - 1/3\*a^2/x^3 - 2/27\*(6\*c^5\*x^4\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - 3\*c^3\*x^2\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)) - (6\*c^3\*x^2 + c)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - 3\*c\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)))\*b^2/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*c\*x^3)

**Fricas [A]** time = 2.01381, size = 224, normalized size = 2.2

$$\frac{12 b^2 c^2 x^2 - 9 b^2 \operatorname{arccsc}(cx)^2 - 18 ab \operatorname{arccsc}(cx) - 9 a^2 + 2 b^2 - 6 \left( 2 abc^2 x^2 + ab + (2 b^2 c^2 x^2 + b^2) \operatorname{arccsc}(cx) \right) \sqrt{c^2 x^2 - 1}}{27 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^4,x, algorithm="fricas")

[Out] 1/27\*(12\*b^2\*c^2\*x^2 - 9\*b^2\*arccsc(c\*x)^2 - 18\*a\*b\*arccsc(c\*x) - 9\*a^2 + 2\*b^2 - 6\*(2\*a\*b\*c^2\*x^2 + a\*b + (2\*b^2\*c^2\*x^2 + b^2)\*arccsc(c\*x))\*sqrt(c^2\*x^2 - 1))/x^3

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x\*\*4,x)

[Out] Integral((a + b\*arccsc(c\*x))^2/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^4,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2/x^4, x)

$$3.23 \quad \int \frac{(a+b \csc^{-1}(cx))^2}{x^5} dx$$

**Optimal.** Leaf size=134

$$\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{16x} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{8x^3} + \frac{3}{16}abc^4 \csc^{-1}(cx) - \frac{(a+b \csc^{-1}(cx))^2}{4x^4} + \frac{3b^2c^2}{32x^2} +$$

[Out] b^2/(32\*x^4) + (3\*b^2\*c^2)/(32\*x^2) + (3\*a\*b\*c^4\*ArcCsc[c\*x])/16 + (3\*b^2\*c^4\*ArcCsc[c\*x]^2)/32 - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(8\*x^3) - (3\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(16\*x) - (a + b\*ArcCsc[c\*x])^2/(4\*x^4)

**Rubi [A]** time = 0.112284, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5223, 4404, 3310}

$$\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{16x} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))}{8x^3} + \frac{3}{16}abc^4 \csc^{-1}(cx) - \frac{(a+b \csc^{-1}(cx))^2}{4x^4} + \frac{3b^2c^2}{32x^2} +$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^2/x^5, x]

[Out] b^2/(32\*x^4) + (3\*b^2\*c^2)/(32\*x^2) + (3\*a\*b\*c^4\*ArcCsc[c\*x])/16 + (3\*b^2\*c^4\*ArcCsc[c\*x]^2)/32 - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(8\*x^3) - (3\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x]))/(16\*x) - (a + b\*ArcCsc[c\*x])^2/(4\*x^4)

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)]^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4404

Int[Cos[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sin[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sin[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(d\*(b\*Ssin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Ssin[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{csc}^{-1}(cx))^2}{x^5} dx &= -\left(c^4 \operatorname{Subst}\left(\int (a + bx)^2 \cos(x) \sin^3(x) dx, x, \operatorname{csc}^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \operatorname{csc}^{-1}(cx))^2}{4x^4} + \frac{1}{2}(bc^4) \operatorname{Subst}\left(\int (a + bx) \sin^4(x) dx, x, \operatorname{csc}^{-1}(cx)\right) \\
&= \frac{b^2}{32x^4} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))}{8x^3} - \frac{(a + b \operatorname{csc}^{-1}(cx))^2}{4x^4} + \frac{1}{8}(3bc^4) \operatorname{Subst}\left(\int (a + bx) \sin^2(x) dx, x, \operatorname{csc}^{-1}(cx)\right) \\
&= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))}{8x^3} - \frac{3bc^3\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))}{16x} - \frac{(a + b \operatorname{csc}^{-1}(cx))^2}{4x^4} \\
&= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4 \operatorname{csc}^{-1}(cx) + \frac{3}{32}b^2c^4 \operatorname{csc}^{-1}(cx)^2 - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))}{8x^3} - \frac{3bc^3\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))}{16x}
\end{aligned}$$

**Mathematica [A]** time = 0.174765, size = 148, normalized size = 1.1

$$\frac{-8a^2 - 6abc^3x^3\sqrt{1 - \frac{1}{c^2x^2}} - 4abcx\sqrt{1 - \frac{1}{c^2x^2}} + 6abc^4x^4 \sin^{-1}\left(\frac{1}{cx}\right) - 2b \operatorname{csc}^{-1}(cx) \left(8a + bcx\sqrt{1 - \frac{1}{c^2x^2}}(3c^2x^2 + 2)\right) + 3b^2c^2}{32x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])^2/x^5, x]

[Out] (-8\*a^2 + b^2 - 4\*a\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + 3\*b^2\*c^2\*x^2 - 6\*a\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3 - 2\*b\*(8\*a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(2 + 3\*c^2\*x^2))\*ArcCsc[c\*x] + b^2\*(-8 + 3\*c^4\*x^4)\*ArcCsc[c\*x]^2 + 6\*a\*b\*c^4\*x^4\*ArcSin[1/(c\*x)])/(32\*x^4)

**Maple [B]** time = 0.228, size = 265, normalized size = 2.

$$\frac{a^2}{4x^4} - \frac{b^2(\operatorname{arccsc}(cx))^2}{4x^4} + \frac{3b^2c^4(\operatorname{arccsc}(cx))^2}{32} - \frac{3c^3b^2\operatorname{arccsc}(cx)}{16x} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} - \frac{cb^2\operatorname{arccsc}(cx)}{8x^3} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + \frac{b^2}{32x^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^2/x^5, x)

[Out] -1/4\*a^2/x^4 - 1/4\*b^2/x^4\*arccsc(c\*x)^2 + 3/32\*b^2\*c^4\*arccsc(c\*x)^2 - 3/16\*c^3\*b^2\*arccsc(c\*x)/x\*((c^2\*x^2-1)/c^2/x^2)^(1/2) - 1/8\*c\*b^2\*arccsc(c\*x)/x^3\*((c^2\*x^2-1)/c^2/x^2)^(1/2) + 1/32\*b^2/x^4 + 3/32\*b^2\*c^2/x^2 - 1/2\*a\*b/x^4\*arccsc(c\*x) + 3/16\*c^3\*a\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*arctan(1/(c^2\*x^2-1)^(1/2)) - 3/16\*c^3\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x + 1/16\*c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^3 + 1/8\*c\*a\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x^5

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*arccsc(c\*x))^2/x^5,x, algorithm="maxima")

[Out] 
$$-1/16*a*b*((3*c^5*\arctan(c*x*\sqrt{-1/(c^2*x^2)+1})+(3*c^8*x^3*(-1/(c^2*x^2)+1)^{(3/2)}+5*c^6*x*\sqrt{-1/(c^2*x^2)+1}))/c^4*x^4*(1/(c^2*x^2)-1)^2-2*c^2*x^2*(1/(c^2*x^2)-1)+1)/c+8*\arccsc(c*x)/x^4-1/16*(4*(2*(c^2*\log(c*x+1)+c^2*\log(c*x-1)-2*c^2*\log(x)+1/x^2)*c^2*\log(c)^2-16*c^2*\int(1/4*x^2*\log(c^2*x^2)/(c^2*x^7-x^5),x)*\log(c)+32*c^2*\int(1/4*x^2*\log(x)/(c^2*x^7-x^5),x)*\log(c)-16*c^2*\int(1/4*x^2*\log(c^2*x^2)*\log(x)/(c^2*x^7-x^5),x)+16*c^2*\int(1/4*x^2*\log(x)^2/(c^2*x^7-x^5),x)+4*c^2*\int(1/4*x^2*\log(c^2*x^2)/(c^2*x^7-x^5),x)-(2*c^4*\log(c*x+1)+2*c^4*\log(c*x-1)-4*c^4*\log(x)+(2*c^2*x^2+1)/x^4)*\log(c)^2+16*\int(1/4*\log(c^2*x^2)/(c^2*x^7-x^5),x)*\log(c)-32*\int(1/4*\log(x)/(c^2*x^7-x^5),x)*\log(c)+8*\int(1/4*\sqrt{c*x+1}*\sqrt{c*x-1}*\arctan(1/(\sqrt{c*x+1}*\sqrt{c*x-1}))/c^2*x^7-x^5,x)+16*\int(1/4*\log(c^2*x^2)*\log(x)/(c^2*x^7-x^5),x)-16*\int(1/4*\log(x)^2/(c^2*x^7-x^5),x)-4*\int(1/4*\log(c^2*x^2)/(c^2*x^7-x^5),x))*x^4+4*\arctan^2(1,\sqrt{c*x+1}*\sqrt{c*x-1})^2-\log(c^2*x^2)^2)*b^2/x^4-1/4*a^2/x^4$$

**Fricas [A]** time = 1.88259, size = 275, normalized size = 2.05

$$\frac{3b^2c^2x^2 + (3b^2c^4x^4 - 8b^2)\arccsc(cx)^2 - 8a^2 + b^2 + 2(3abc^4x^4 - 8ab)\arccsc(cx) - 2(3abc^2x^2 + 2ab + (3b^2c^2x^2 - 8ab)\arccsc(cx))\sqrt{c^2x^2 - 1}}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^5,x, algorithm="fricas")

[Out] 
$$1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*\arccsc(c*x)^2 - 8*a^2 + b^2 + 2*(3*a*b*c^4*x^4 - 8*a*b)*\arccsc(c*x) - 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^2*c^2*x^2 - 8*a*b)*\arccsc(c*x))*\sqrt{c^2*x^2 - 1})/x^4$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*\*2/x\*\*5,x)

[Out] Integral((a + b\*acsc(c\*x))\*\*2/x\*\*5, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^2/x^5,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2/x^5, x)

### 3.24 $\int x^3 \left( a + b \operatorname{csc}^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=207

$$\frac{ib^3 \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2c^4} + \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} - \frac{b^2 \log\left(1 - e^{2i \operatorname{csc}^{-1}(cx)}\right) (a + b \operatorname{csc}^{-1}(cx))}{c^4} + \frac{bx^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \operatorname{csc}^{-1}(cx))}{4c}$$

[Out]  $(b^3 \operatorname{Sqrt}[1 - 1/(c^2 x^2)] x)/(4c^3) + (b^2 x^2 (a + b \operatorname{ArcCsc}[c x]))/(4c^2) + ((I/2) b (a + b \operatorname{ArcCsc}[c x])^2)/c^4 + (b \operatorname{Sqrt}[1 - 1/(c^2 x^2)] x (a + b \operatorname{ArcCsc}[c x])^2)/(2c^3) + (b \operatorname{Sqrt}[1 - 1/(c^2 x^2)] x^3 (a + b \operatorname{ArcCsc}[c x])^2)/(4c) + (x^4 (a + b \operatorname{ArcCsc}[c x])^3)/4 - (b^2 (a + b \operatorname{ArcCsc}[c x]) \operatorname{Log}[1 - E^{((2I) \operatorname{ArcCsc}[c x])}])/c^4 + ((I/2) b^3 \operatorname{PolyLog}[2, E^{((2I) \operatorname{ArcCsc}[c x])}])/c^4$

**Rubi [A]** time = 0.217142, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5223, 4410, 4186, 3767, 8, 4184, 3717, 2190, 2279, 2391}

$$\frac{ib^3 \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2c^4} + \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} - \frac{b^2 \log\left(1 - e^{2i \operatorname{csc}^{-1}(cx)}\right) (a + b \operatorname{csc}^{-1}(cx))}{c^4} + \frac{bx^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \operatorname{csc}^{-1}(cx))}{4c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3 (a + b \operatorname{ArcCsc}[c x])^3, x]$

[Out]  $(b^3 \operatorname{Sqrt}[1 - 1/(c^2 x^2)] x)/(4c^3) + (b^2 x^2 (a + b \operatorname{ArcCsc}[c x]))/(4c^2) + ((I/2) b (a + b \operatorname{ArcCsc}[c x])^2)/c^4 + (b \operatorname{Sqrt}[1 - 1/(c^2 x^2)] x (a + b \operatorname{ArcCsc}[c x])^2)/(2c^3) + (b \operatorname{Sqrt}[1 - 1/(c^2 x^2)] x^3 (a + b \operatorname{ArcCsc}[c x])^2)/(4c) + (x^4 (a + b \operatorname{ArcCsc}[c x])^3)/4 - (b^2 (a + b \operatorname{ArcCsc}[c x]) \operatorname{Log}[1 - E^{((2I) \operatorname{ArcCsc}[c x])}])/c^4 + ((I/2) b^3 \operatorname{PolyLog}[2, E^{((2I) \operatorname{ArcCsc}[c x])}])/c^4$

#### Rule 5223

$\operatorname{Int}[(a + \operatorname{ArcCsc}[c x])^n (x)^m, x] \rightarrow -\operatorname{Dist}[c^{m+1} (-1), \operatorname{Subst}[\operatorname{Int}[(a + b x)^n \operatorname{Csc}[x]^{m+1} \operatorname{Cot}[x], x], x, \operatorname{ArcCsc}[c x]], x] /;$   $\operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{LtQ}[m, -1])$

#### Rule 4410

$\operatorname{Int}[\operatorname{Cot}[(a + b x)^p \operatorname{Csc}[a + b x]^n], x] \rightarrow -\operatorname{Simp}[(c + d x)^m \operatorname{Csc}[a + b x]^n / (b n), x] + \operatorname{Dist}[(d m) / (b n), \operatorname{Int}[(c + d x)^{m-1} \operatorname{Csc}[a + b x]^n, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \operatorname{EqQ}[p, 1] \ \&\& \ \operatorname{GtQ}[m, 0]$

#### Rule 4186

$\operatorname{Int}[(e + f x)^n ((c + d x)^m), x] \rightarrow -\operatorname{Simp}[(b^2 (c + d x)^m \operatorname{Cot}[e + f x] (b \operatorname{Csc}[e + f x])^{n-2}) / (f (n-1)), x] + (\operatorname{Dist}[(b^2 d^2 m (m-1)) / (f^2 (n-1) (n-2)), \operatorname{Int}[(c + d x)^{m-2} (b \operatorname{Csc}[e + f x])^{n-2}, x], x] + \operatorname{Dist}[(b^2 (n-2)) / (n-1), \operatorname{Int}[(c + d x)^m (b \operatorname{Csc}[e + f x])^{n-2}, x], x] - \operatorname{Simp}[(b^2 d m (c + d x)^{m-1} (b \operatorname{Csc}[e + f x])^{n-2}) / (f^2 (n-1) (n-2)), x]) /;$   $\operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\& \ \operatorname{NeQ}[n, 2] \ \&\& \ \operatorname{GtQ}[m, 1]$

Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3717

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int x^3 (a + b \operatorname{csc}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \cot(x) \operatorname{csc}^4(x) dx, x, \operatorname{csc}^{-1}(cx)\right)}{c^4} \\
&= \frac{1}{4}x^4 (a + b \operatorname{csc}^{-1}(cx))^3 - \frac{(3b) \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{csc}^4(x) dx, x, \operatorname{csc}^{-1}(cx)\right)}{4c^4} \\
&= \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} + \frac{b\sqrt{1 - \frac{1}{c^2 x^2}} x^3 (a + b \operatorname{csc}^{-1}(cx))^2}{4c} + \frac{1}{4}x^4 (a + b \operatorname{csc}^{-1}(cx))^3 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{csc}^4(x) dx, x, \operatorname{csc}^{-1}(cx)\right)}{4c^4} \\
&= \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} + \frac{b\sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \operatorname{csc}^{-1}(cx))^2}{2c^3} + \frac{b\sqrt{1 - \frac{1}{c^2 x^2}} x^3 (a + b \operatorname{csc}^{-1}(cx))^2}{4c} \\
&= \frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} + \frac{ib (a + b \operatorname{csc}^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \operatorname{csc}^{-1}(cx))^2}{2c^3} \\
&= \frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} + \frac{ib (a + b \operatorname{csc}^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \operatorname{csc}^{-1}(cx))^2}{2c^3} \\
&= \frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} + \frac{ib (a + b \operatorname{csc}^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \operatorname{csc}^{-1}(cx))^2}{2c^3} \\
&= \frac{b^3 \sqrt{1 - \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csc}^{-1}(cx))}{4c^2} + \frac{ib (a + b \operatorname{csc}^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1 - \frac{1}{c^2 x^2}} x (a + b \operatorname{csc}^{-1}(cx))^2}{2c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.859724, size = 285, normalized size = 1.38

$$2ib^3 \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right) + b \operatorname{csc}^{-1}(cx) \left(cx \left(3a^2 c^3 x^3 + 2ab\sqrt{1 - \frac{1}{c^2 x^2}} (c^2 x^2 + 2) + b^2 cx\right) - 4b^2 \log\left(1 - e^{2i \operatorname{csc}^{-1}(cx)}\right)\right) + a$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(a + b\*ArcCsc[c\*x])^3,x]

[Out] (2\*a^2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + a\*b^2\*c^2\*x^2 + a^2\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3 + a^3\*c^4\*x^4 + b^2\*(3\*a\*c^4\*x^4 + b\*(2\*I + 2\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3))\*ArcCsc[c\*x]^2 + b^3\*c^4\*x^4\*ArcCsc[c\*x]^3 + b\*ArcCsc[c\*x]\*(c\*x\*(b^2\*c\*x + 3\*a^2\*c^3\*x^3 + 2\*a\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*(2 + c^2\*x^2)) - 4\*b^2\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])]) - 4\*a\*b^2\*Log[1/(c\*x)] + (2\*I)\*b^3\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/(4\*c^4)

**Maple [B]** time = 0.408, size = 510, normalized size = 2.5

$$\frac{x^4 a^3}{4} + \frac{b^3 (\operatorname{arccsc}(cx))^3 x^4}{4} + \frac{b^3 (\operatorname{arccsc}(cx))^2 x^3}{4c} \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + \frac{b^3 (\operatorname{arccsc}(cx))^2 x}{2c^3} \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} - \frac{i b^3}{4c^4} + \frac{b^3 \operatorname{arccsc}(cx) x^2}{4c^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsc(c\*x))^3,x)

[Out] 1/4\*x^4\*a^3+1/4\*b^3\*arccsc(c\*x)^3\*x^4+1/4/c\*b^3\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*arccsc(c\*x)^2\*x^3+1/2/c^3\*b^3\*arccsc(c\*x)^2\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x-1/4\*I/c^4\*b^3+1/4/c^2\*b^3\*arccsc(c\*x)\*x^2+1/4/c^3\*b^3\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x+I/c^4\*b^3\*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-1/c^4\*b^3\*arccsc(c

$$x) \ln(1+I/c/x+(1-1/c^2/x^2)^{(1/2)}) - 1/c^4*b^3*\operatorname{arccsc}(c*x) \ln(1-I/c/x-(1-1/c^2/x^2)^{(1/2)}) + 1/2*I/c^4*b^3*\operatorname{arccsc}(c*x)^2 + I/c^4*b^3*\operatorname{polylog}(2, -I/c/x-(1-1/c^2/x^2)^{(1/2)}) + 3/4*a^2*b*x^4*\operatorname{arccsc}(c*x) + 1/4/c*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^3 + 1/4/c^3*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x - 1/2/c^5*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x + 3/4*a*b^2*\operatorname{arccsc}(c*x)^2*x^4 + 1/2/c*a*b^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)}*\operatorname{arccsc}(c*x)*x^3 + 1/4/c^2*x^2*a*b^2 + 1/c^3*a*b^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)}*\operatorname{arccsc}(c*x)*x - 1/c^4*a*b^2*\ln(1/c/x)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))^3,x, algorithm="maxima")

[Out]  $3/4*a*b^2*x^4*\operatorname{arccsc}(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*\operatorname{arccsc}(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^{(3/2)} + 3*x*\sqrt{-1/(c^2*x^2) + 1})/c^3)*a^2*b + 1/16*(4*x^4*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})^3 - 3*x^4*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c^2*x^2)^2 - 16*\operatorname{integrate}(3/16*(16*c^2*x^5*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c)^2 - 16*x^3*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c)^2 + 16*(c^2*x^5*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}) - x^3*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}))*\log(x)^2 - (4*x^3*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})^2 - x^3*\log(c^2*x^2)^2)*\sqrt{c*x + 1}*\sqrt{c*x - 1} - 4*((4*c^2*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c) + c^2*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}))*x^5 - (4*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c) + \operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}))*x^3 + 4*(c^2*x^5*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}) - x^3*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}))*\log(x))*\log(c^2*x^2) + 32*(c^2*x^5*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1})*\log(c) - x^3*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}))*\log(c))*\log(x))/(c^2*x^2 - 1), x)*b^3 + 1/4*(2*c^4*x^4*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}) + 2*c^2*x^2*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}) + (c^2*x^2 + 2*\log(x^2))*\sqrt{c*x + 1}*\sqrt{c*x - 1} - 4*\operatorname{arctan2}(1, \sqrt{c*x + 1})*\sqrt{c*x - 1}))*a*b^2/(\sqrt{c*x + 1}*\sqrt{c*x - 1})*c^4)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3x^3 \operatorname{arccsc}(cx)^3 + 3ab^2x^3 \operatorname{arccsc}(cx)^2 + 3a^2bx^3 \operatorname{arccsc}(cx) + a^3x^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^3\*arccsc(c\*x)^3 + 3\*a\*b^2\*x^3\*arccsc(c\*x)^2 + 3\*a^2\*b\*x^3\*arccsc(c\*x) + a^3\*x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arccsc}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))**3, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3*x^3, x)
```

### 3.25 $\int x^2 (a + b \csc^{-1}(cx))^3 dx$

**Optimal.** Leaf size=220

$$\frac{ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c^3} + \frac{ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c^3} + \frac{b^3 \text{PolyLog}\left(3, -e^{i \csc^{-1}(cx)}\right)}{c^3}$$

[Out] (b^2\*x\*(a + b\*ArcCsc[c\*x]))/c^2 + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2\*(a + b\*ArcCsc[c\*x])^2)/(2\*c) + (x^3\*(a + b\*ArcCsc[c\*x])^3)/3 + (b\*(a + b\*ArcCsc[c\*x])^2 \* ArcTanh[E^(I\*ArcCsc[c\*x])])/c^3 + (b^3 \* ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c^3 - (I\*b^2\*(a + b\*ArcCsc[c\*x])\*PolyLog[2, -E^(I\*ArcCsc[c\*x])])/c^3 + (I\*b^2\*(a + b\*ArcCsc[c\*x])\*PolyLog[2, E^(I\*ArcCsc[c\*x])])/c^3 + (b^3 \* PolyLog[3, -E^(I\*ArcCsc[c\*x])])/c^3 - (b^3 \* PolyLog[3, E^(I\*ArcCsc[c\*x])])/c^3

**Rubi [A]** time = 0.188112, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5223, 4410, 4186, 3770, 4183, 2531, 2282, 6589}

$$\frac{ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c^3} + \frac{ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c^3} + \frac{b^3 \text{PolyLog}\left(3, -e^{i \csc^{-1}(cx)}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcCsc[c\*x])^3,x]

[Out] (b^2\*x\*(a + b\*ArcCsc[c\*x]))/c^2 + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2\*(a + b\*ArcCsc[c\*x])^2)/(2\*c) + (x^3\*(a + b\*ArcCsc[c\*x])^3)/3 + (b\*(a + b\*ArcCsc[c\*x])^2 \* ArcTanh[E^(I\*ArcCsc[c\*x])])/c^3 + (b^3 \* ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c^3 - (I\*b^2\*(a + b\*ArcCsc[c\*x])\*PolyLog[2, -E^(I\*ArcCsc[c\*x])])/c^3 + (I\*b^2\*(a + b\*ArcCsc[c\*x])\*PolyLog[2, E^(I\*ArcCsc[c\*x])])/c^3 + (b^3 \* PolyLog[3, -E^(I\*ArcCsc[c\*x])])/c^3 - (b^3 \* PolyLog[3, E^(I\*ArcCsc[c\*x])])/c^3

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n \* Csc[x]^(m + 1) \* Cot[x], x], x, ArcCsc[c[x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4410

Int[Cot[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csc[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m \* Csc[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1) \* Csc[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m \* Cot[e + f\*x] \* (b \* Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2) \* (b \* Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m \* (b \* Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1) \* (b \* Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \csc^{-1}(cx))^3 dx &= -\frac{\text{Subst}\left(\int (a + bx)^3 \cot(x) \csc^3(x) dx, x, \csc^{-1}(cx)\right)}{c^3} \\
&= \frac{1}{3}x^3 (a + b \csc^{-1}(cx))^3 - \frac{b \text{Subst}\left(\int (a + bx)^2 \csc^3(x) dx, x, \csc^{-1}(cx)\right)}{c^3} \\
&= \frac{b^2x (a + b \csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^2 (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \csc^{-1}(cx))^3 - \frac{b \text{Subst}\left(\int (a + bx) \csc^3(x) dx, x, \csc^{-1}(cx)\right)}{c^3} \\
&= \frac{b^2x (a + b \csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^2 (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \csc^{-1}(cx))^3 + \frac{b(a + b \csc^{-1}(cx))}{c} \\
&= \frac{b^2x (a + b \csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^2 (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \csc^{-1}(cx))^3 + \frac{b(a + b \csc^{-1}(cx))}{c} \\
&= \frac{b^2x (a + b \csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^2 (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \csc^{-1}(cx))^3 + \frac{b(a + b \csc^{-1}(cx))}{c} \\
&= \frac{b^2x (a + b \csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}x^2 (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3 (a + b \csc^{-1}(cx))^3 + \frac{b(a + b \csc^{-1}(cx))}{c}
\end{aligned}$$



**Mathematica [B]** time = 7.75087, size = 580, normalized size = 2.64

$$ab^2 \left( 2c^3 x^3 \left( \frac{4i \operatorname{PolyLog}\left(2, e^{i \operatorname{csc}^{-1}(cx)}\right)}{c^3 x^3} + 4 \operatorname{csc}^{-1}(cx)^2 - 2 \cos\left(2 \operatorname{csc}^{-1}(cx)\right) - \frac{3 \operatorname{csc}^{-1}(cx) \log\left(1 - e^{i \operatorname{csc}^{-1}(cx)}\right)}{cx} + \frac{3 \operatorname{csc}^{-1}(cx) \log\left(1 + e^{i \operatorname{csc}^{-1}(cx)}\right)}{cx} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcCsc[c\*x])^3,x]

[Out] (a^3\*x^3)/3 + (a^2\*b\*x^2\*sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(2\*c) + a^2\*b\*x^3\*ArcCsc[c\*x] + (a^2\*b\*Log[x\*(1 + sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])])/(2\*c^3) + (a\*b^2\*((-8\*I)\*PolyLog[2, -E^(I\*ArcCsc[c\*x])] + 2\*c^3\*x^3\*(2 + 4\*ArcCsc[c\*x]^2 - 2\*Cos[2\*ArcCsc[c\*x]] - (3\*ArcCsc[c\*x]\*Log[1 - E^(I\*ArcCsc[c\*x])]))/(c\*x) + (3\*ArcCsc[c\*x]\*Log[1 + E^(I\*ArcCsc[c\*x])])/(c\*x) + ((4\*I)\*PolyLog[2, E^(I\*ArcCsc[c\*x])])/(c^3\*x^3) + 2\*ArcCsc[c\*x]\*Sin[2\*ArcCsc[c\*x]] + ArcCsc[c\*x]\*Log[1 - E^(I\*ArcCsc[c\*x])]\*Sin[3\*ArcCsc[c\*x]] - ArcCsc[c\*x]\*Log[1 + E^(I\*ArcCsc[c\*x])]\*Sin[3\*ArcCsc[c\*x]]))/(8\*c^3) + (b^3\*(24\*ArcCsc[c\*x]\*Cot[ArcCsc[c\*x]/2] + 4\*ArcCsc[c\*x]^3\*Cot[ArcCsc[c\*x]/2] + 6\*ArcCsc[c\*x]^2\*Csc[ArcCsc[c\*x]/2]^2 + (ArcCsc[c\*x]^3\*Csc[ArcCsc[c\*x]/2]^4)/(c\*x) - 24\*ArcCsc[c\*x]^2\*Log[1 - E^(I\*ArcCsc[c\*x])] + 24\*ArcCsc[c\*x]^2\*Log[1 + E^(I\*ArcCsc[c\*x])] - 48\*Log[Tan[ArcCsc[c\*x]/2]] - (48\*I)\*ArcCsc[c\*x]\*PolyLog[2, -E^(I\*ArcCsc[c\*x])] + (48\*I)\*ArcCsc[c\*x]\*PolyLog[2, E^(I\*ArcCsc[c\*x])] + 48\*PolyLog[3, -E^(I\*ArcCsc[c\*x])] - 48\*PolyLog[3, E^(I\*ArcCsc[c\*x])] - 6\*ArcCsc[c\*x]^2\*Sec[ArcCsc[c\*x]/2]^2 + 16\*c^3\*x^3\*ArcCsc[c\*x]^3\*Sin[ArcCsc[c\*x]/2]^4 + 24\*ArcCsc[c\*x]\*Tan[ArcCsc[c\*x]/2] + 4\*ArcCsc[c\*x]^3\*Tan[ArcCsc[c\*x]/2]))/(48\*c^3)

**Maple [B]** time = 0.446, size = 648, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))^3,x)

[Out] 1/3\*a^3\*x^3+1/3\*x^3\*b^3\*arccsc(c\*x)^3+1/2/c\*b^3\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*arccsc(c\*x)^2\*x^2+1/c^2\*b^3\*arccsc(c\*x)\*x-1/2/c^3\*b^3\*arccsc(c\*x)^2\*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I/c^3\*a\*b^2\*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-b^3\*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+1/2/c^3\*b^3\*arccsc(c\*x)^2\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-I/c^3\*b^3\*arccsc(c\*x)\*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+b^3\*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3+2/c^3\*b^3\*arctanh(I/c/x+(1-1/c^2/x^2)^(1/2))+b^2\*x^3\*a\*arccsc(c\*x)^2+1/c\*a\*b^2\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*arccsc(c\*x)\*x^2+I/c^3\*b^3\*arccsc(c\*x)\*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-I/c^3\*a\*b^2\*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-1/c^3\*a\*b^2\*arccsc(c\*x)\*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1/c^3\*a\*b^2\*arccsc(c\*x)\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/c^2\*x\*a\*b^2+x^3\*a^2\*b\*arccsc(c\*x)+1/2/c\*a^2\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2-1/2/c^3\*a^2\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)+1/2/c^4\*a^2\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*ln(c\*x+(c^2\*x^2-1)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))^3,x, algorithm="maxima")

[Out]  $\frac{1}{3}b^3x^3\arctan^2(1, \sqrt{cx+1}\sqrt{cx-1})^3 - \frac{1}{4}b^3x^3\arctan^2(1, \sqrt{cx+1}\sqrt{cx-1})\log(c^2x^2)^2 - \frac{1}{2}ab^2c^2(2(c^2x^3+3x)/c^4 - 3\log(cx+1)/c^5 + 3\log(cx-1)/c^5)\log(c)^2 - 12b^3c^2\int \frac{1}{4}x^4\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))/(c^2x^2-1), x)\log(c)^2 + 12b^3c^2\int \frac{1}{4}x^4\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(c^2x^2)/(c^2x^2-1), x)\log(c) - 24b^3c^2\int \frac{1}{4}x^4\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(x)/(c^2x^2-1), x)\log(c) + 12a^2b^2c^2\int \frac{1}{4}x^4\log(c^2x^2)/(c^2x^2-1), x)\log(c) - 24a^2b^2c^2\int \frac{1}{4}x^4\log(x)/(c^2x^2-1), x)\log(c) + \frac{1}{3}a^3x^3 + 12b^3c^2\int \frac{1}{4}x^4\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(c^2x^2)\log(x)/(c^2x^2-1), x) - 12b^3c^2\int \frac{1}{4}x^4\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(x)^2/(c^2x^2-1), x) + 12a^2b^2c^2\int \frac{1}{4}x^4\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))^2/(c^2x^2-1), x) + 4b^3c^2\int \frac{1}{4}x^4\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(c^2x^2)/(c^2x^2-1), x) - 3a^2b^2c^2\int \frac{1}{4}x^4\log(c^2x^2)^2/(c^2x^2-1), x) + 12a^2b^2c^2\int \frac{1}{4}x^4\log(c^2x^2)\log(x)/(c^2x^2-1), x) - 12a^2b^2c^2\int \frac{1}{4}x^4\log(x)^2/(c^2x^2-1), x) + \frac{3}{2}a^2b^2(2x/c^2 - \log(cx+1)/c^3 + \log(cx-1)/c^3)\log(c)^2 + 12b^3\int \frac{1}{4}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))/(c^2x^2-1), x)\log(c)^2 - 12b^3\int \frac{1}{4}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(c^2x^2)/(c^2x^2-1), x)\log(c) + 24b^3\int \frac{1}{4}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(x)/(c^2x^2-1), x)\log(c) - 12a^2b^2\int \frac{1}{4}x^2\log(c^2x^2)/(c^2x^2-1), x)\log(c) + 24a^2b^2\int \frac{1}{4}x^2\log(x)/(c^2x^2-1), x)\log(c) + \frac{1}{4}(4x^3\operatorname{arccsc}(cx) + (2\sqrt{-1/(c^2x^2)+1})/(c^2(1/(c^2x^2)-1)+c^2) + \log(\sqrt{-1/(c^2x^2)+1})+1)/c^2 - \log(\sqrt{-1/(c^2x^2)+1}-1)/c^2)/c^2a^2b + 4b^3\int \frac{1}{4}\sqrt{cx+1}\sqrt{cx-1}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))^2/(c^2x^2-1), x) - b^3\int \frac{1}{4}\sqrt{cx+1}\sqrt{cx-1}x^2\log(c^2x^2)^2/(c^2x^2-1), x) - 12b^3\int \frac{1}{4}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(c^2x^2)\log(x)/(c^2x^2-1), x) + 12b^3\int \frac{1}{4}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(x)^2/(c^2x^2-1), x) - 12a^2b^2\int \frac{1}{4}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))^2/(c^2x^2-1), x) - 4b^3\int \frac{1}{4}x^2\arctan(1/(\sqrt{cx+1}\sqrt{cx-1}))\log(c^2x^2)/(c^2x^2-1), x) + 3a^2b^2\int \frac{1}{4}x^2\log(c^2x^2)^2/(c^2x^2-1), x) - 12a^2b^2\int \frac{1}{4}x^2\log(c^2x^2)\log(x)/(c^2x^2-1), x) + 12a^2b^2\int \frac{1}{4}x^2\log(x)^2/(c^2x^2-1), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3x^2\operatorname{arccsc}(cx)^3 + 3ab^2x^2\operatorname{arccsc}(cx)^2 + 3a^2bx^2\operatorname{arccsc}(cx) + a^3x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))^3,x, algorithm="fricas")

[Out]  $\operatorname{integral}(b^3x^2\operatorname{arccsc}(cx)^3 + 3a^2b^2x^2\operatorname{arccsc}(cx)^2 + 3a^2bx^2\operatorname{arccsc}(cx) + a^3x^2, x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2(a + b \operatorname{arccsc}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsc(c\*x))\*\*3,x)

[Out] Integral(x\*\*2\*(a + b\*acsc(c\*x))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^3\*x^2, x)

### 3.26 $\int x \left( a + b \operatorname{csc}^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=126

$$\frac{3ib^3 \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2c^2} - \frac{3b^2 \log\left(1 - e^{2i \operatorname{csc}^{-1}(cx)}\right) \left(a + b \operatorname{csc}^{-1}(cx)\right)}{c^2} + \frac{3bx \sqrt{1 - \frac{1}{c^2 x^2}} \left(a + b \operatorname{csc}^{-1}(cx)\right)^2}{2c} + \frac{3ib \left(a + b \operatorname{csc}^{-1}(cx)\right)}{2c}$$

[Out] (((3\*I)/2)\*b\*(a + b\*ArcCsc[c\*x])^2)/c^2 + (3\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(a + b\*ArcCsc[c\*x])^2)/(2\*c) + (x^2\*(a + b\*ArcCsc[c\*x])^3)/2 - (3\*b^2\*(a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/c^2 + (((3\*I)/2)\*b^3\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/c^2

**Rubi [A]** time = 0.147319, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {5223, 4410, 4184, 3717, 2190, 2279, 2391}

$$\frac{3ib^3 \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2c^2} - \frac{3b^2 \log\left(1 - e^{2i \operatorname{csc}^{-1}(cx)}\right) \left(a + b \operatorname{csc}^{-1}(cx)\right)}{c^2} + \frac{3bx \sqrt{1 - \frac{1}{c^2 x^2}} \left(a + b \operatorname{csc}^{-1}(cx)\right)^2}{2c} + \frac{3ib \left(a + b \operatorname{csc}^{-1}(cx)\right)}{2c}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcCsc[c\*x])^3,x]

[Out] (((3\*I)/2)\*b\*(a + b\*ArcCsc[c\*x])^2)/c^2 + (3\*b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(a + b\*ArcCsc[c\*x])^2)/(2\*c) + (x^2\*(a + b\*ArcCsc[c\*x])^3)/2 - (3\*b^2\*(a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/c^2 + (((3\*I)/2)\*b^3\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/c^2

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))^ (n\_.)\*(x\_)^ (m\_.), x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4410

Int[Cot[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csc[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Csc[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csc[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3717

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[(c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))]/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int x (a + b \csc^{-1}(cx))^3 dx &= -\frac{\text{Subst}\left(\int (a + bx)^3 \cot(x) \csc^2(x) dx, x, \csc^{-1}(cx)\right)}{c^2} \\ &= \frac{1}{2}x^2 (a + b \csc^{-1}(cx))^3 - \frac{(3b) \text{Subst}\left(\int (a + bx)^2 \csc^2(x) dx, x, \csc^{-1}(cx)\right)}{2c^2} \\ &= \frac{3b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \csc^{-1}(cx))^3 - \frac{(3b^2) \text{Subst}\left(\int (a + bx) \cot(x) dx, x, \csc^{-1}(cx)\right)}{c^2} \\ &= \frac{3ib (a + b \csc^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \csc^{-1}(cx))^3 + \frac{(6ib^2) \text{Subst}\left(\int \cot(x) dx, x, \csc^{-1}(cx)\right)}{c^2} \\ &= \frac{3ib (a + b \csc^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \csc^{-1}(cx))^3 - \frac{3b^2}{c^2} \\ &= \frac{3ib (a + b \csc^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \csc^{-1}(cx))^3 - \frac{3b^2}{c^2} \\ &= \frac{3ib (a + b \csc^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 - \frac{1}{c^2x^2}}x (a + b \csc^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \csc^{-1}(cx))^3 - \frac{3b^2}{c^2} \end{aligned}$$

**Mathematica [A]** time = 0.46718, size = 182, normalized size = 1.44

$$\frac{3ib^3 \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) + a \left( acx \left( acx + 3b\sqrt{1 - \frac{1}{c^2x^2}} \right) - 6b^2 \log\left(\frac{1}{cx}\right) \right) + 3b^2 \csc^{-1}(cx)^2 \left( ac^2x^2 + b \left( cx\sqrt{1 - \frac{1}{c^2x^2}} + \right) \right)}{2c^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcCsc[c\*x])^3, x]

[Out] (3\*b^2\*(a\*c^2\*x^2 + b\*(I + c\*Sqrt[1 - 1/(c^2\*x^2)]\*x))\*ArcCsc[c\*x]^2 + b^3\*c^2\*x^2\*ArcCsc[c\*x]^3 + 3\*b\*ArcCsc[c\*x]\*(a\*c\*x\*(2\*b\*Sqrt[1 - 1/(c^2\*x^2)] + a\*c\*x) - 2\*b^2\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])]) + a\*(a\*c\*x\*(3\*b\*Sqrt[1 - 1/(c^2\*x^2)] + a\*c\*x) - 6\*b^2\*Log[1/(c\*x)]) + (3\*I)\*b^3\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/(2\*c^2)

**Maple [B]** time = 0.33, size = 349, normalized size = 2.8

$$\frac{x^2 a^3}{2} + \frac{x^2 b^3 (\operatorname{arccsc}(cx))^3}{2} + \frac{3 b^3 (\operatorname{arccsc}(cx))^2 x}{2c} \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + \frac{\frac{3i}{2} b^3 (\operatorname{arccsc}(cx))^2}{c^2} - 3 \frac{b^3 \operatorname{arccsc}(cx)}{c^2} \ln \left( 1 + \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arccsc(c*x))^3,x)`

[Out]  $\frac{1}{2} x^2 a^3 + \frac{1}{2} x^2 b^3 \operatorname{arccsc}(cx)^3 + \frac{3}{2} \frac{b^3 \operatorname{arccsc}(cx)^2 x}{c} \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}} + \frac{3i b^3 \operatorname{arccsc}(cx)^2}{c^2} - 3 \frac{b^3 \operatorname{arccsc}(cx)}{c^2} \ln \left( 1 + \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}} \right) + 3 \frac{I}{c} \frac{1}{c^2 x^2} \operatorname{arccsc}(cx)^2 - 3 \frac{I}{c} \frac{1}{c^2 x^2} \operatorname{arccsc}(cx) \ln \left( 1 + \frac{I}{c/x + (1 - 1/c^2/x^2)^{1/2}} \right) - 3 \frac{I}{c} \frac{1}{c^2 x^2} \operatorname{arccsc}(cx) \ln \left( 1 - \frac{I}{c/x - (1 - 1/c^2/x^2)^{1/2}} \right) + 3 \frac{I}{c} \frac{1}{c^2 x^2} \operatorname{polylog}(2, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) + 3 \frac{I}{c} \frac{1}{c^2 x^2} \operatorname{polylog}(2, I/c/x + (1 - 1/c^2/x^2)^{1/2}) + \frac{3}{2} x^2 a^2 b^2 \operatorname{arccsc}(cx)^2 + \frac{3}{2} \frac{c a^2 b^2 x^2 ((c^2 x^2 - 1)/c^2 x^2)^{1/2} \operatorname{arccsc}(cx) x - 3/c^2 a^2 b^2 \ln(1/c/x) + 3/2 a^2 b^2 x^2 \operatorname{arccsc}(cx) + 3/2 c a^2 b^2 ((c^2 x^2 - 1)/c^2 x^2)^{1/2} x - 3/2 c^3 a^2 b^2 ((c^2 x^2 - 1)/c^2 x^2)^{1/2} / x}{c^2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{2} a b^2 x^2 \operatorname{arccsc}(cx)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{2} \left( x^2 \operatorname{arccsc}(cx) + \frac{x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} \right) a^2 b + 3 \left( \frac{x \sqrt{-\frac{1}{c^2 x^2} + 1} \operatorname{arccsc}(cx)}{c} + \frac{\log(x)}{c^2} \right) a b^2 + \frac{1}{8} \left( \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="maxima")`

[Out]  $\frac{3}{2} a^2 b^2 x^2 \operatorname{arccsc}(cx)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{2} (x^2 \operatorname{arccsc}(cx) + x \sqrt{-1/(c^2 x^2) + 1}) a^2 b + 3 (x \sqrt{-1/(c^2 x^2) + 1} \operatorname{arccsc}(cx)/c + \log(x)/c^2) a b^2 + \frac{1}{8} (4 x^2 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1})^3 - 3 x^2 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1} \log(c^2 x^2)^2 - 8 \int (3/8 (8 c^2 x^3 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) \log(c)^2 - 8 x \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1} \log(c)^2 + 8 (c^2 x^3 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) - x \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) \log(x)^2 - (4 x \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1})^2 - x \log(c^2 x^2)^2) \sqrt{cx+1} \sqrt{cx-1} - 4 ((2 c^2 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) \log(c) + c^2 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1})) x^3 - (2 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) \log(c) + \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) x + 2 (c^2 x^3 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) - x \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) \log(x) \log(c^2 x^2) + 16 (c^2 x^3 \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) \log(c) - x \arctan^2(1, \sqrt{cx+1}) \sqrt{cx-1}) \log(c) \log(x) / (c^2 x^2 - 1), x) b^3$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3 x \operatorname{arccsc}(cx)^3 + 3 a b^2 x \operatorname{arccsc}(cx)^2 + 3 a^2 b x \operatorname{arccsc}(cx) + a^3 x, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="fricas")`

[Out]  $\text{integral}(b^3*x*\text{arccsc}(c*x)^3 + 3*a*b^2*x*\text{arccsc}(c*x)^2 + 3*a^2*b*x*\text{arccsc}(c*x) + a^3*x, x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{arccsc}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))**3,x)`

[Out] `Integral(x*(a + b*arccsc(c*x))**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)^3*x, x)`

### 3.27 $\int (a + b \csc^{-1}(cx))^3 dx$

**Optimal.** Leaf size=144

$$-\frac{6ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c} + \frac{6ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c} + \frac{6b^3 \text{PolyLog}\left(3, -e^{i \csc^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a + b*ArcCsc[c*x])^3 + (6*b*(a + b*ArcCsc[c*x])^2*ArcTanh[E^(I*ArcCsc[c*x])])/c - ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c + ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c + (6*b^3*PolyLog[3, -E^(I*ArcCsc[c*x])])/c - (6*b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c
```

**Rubi [A]** time = 0.112179, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {5217, 4410, 4183, 2531, 2282, 6589}

$$-\frac{6ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c} + \frac{6ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))}{c} + \frac{6b^3 \text{PolyLog}\left(3, -e^{i \csc^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3, x]
```

```
[Out] x*(a + b*ArcCsc[c*x])^3 + (6*b*(a + b*ArcCsc[c*x])^2*ArcTanh[E^(I*ArcCsc[c*x])])/c - ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c + ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c + (6*b^3*PolyLog[3, -E^(I*ArcCsc[c*x])])/c - (6*b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c
```

#### Rule 5217

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)^n_, x_Symbol] := -Dist[c^(-1), Subst[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[n, 0]
```

#### Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

#### Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f}
```



, g, n}, x] && GtQ[m, 0]

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int (a + b \csc^{-1}(cx))^3 dx &= -\frac{\text{Subst}\left(\int (a + bx)^3 \cot(x) \csc(x) dx, x, \csc^{-1}(cx)\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^3 - \frac{(3b) \text{Subst}\left(\int (a + bx)^2 \csc(x) dx, x, \csc^{-1}(cx)\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^3 + \frac{6b(a + b \csc^{-1}(cx))^2 \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right)}{c} + \frac{(6b^2) \text{Subst}\left(\int (a + bx) \csc(x) dx, x, \csc^{-1}(cx)\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^3 + \frac{6b(a + b \csc^{-1}(cx))^2 \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a + b \csc^{-1}(cx)) \text{Li}_2\left(-e^{i \csc^{-1}(cx)}\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^3 + \frac{6b(a + b \csc^{-1}(cx))^2 \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a + b \csc^{-1}(cx)) \text{Li}_2\left(-e^{i \csc^{-1}(cx)}\right)}{c} \\ &= x(a + b \csc^{-1}(cx))^3 + \frac{6b(a + b \csc^{-1}(cx))^2 \tanh^{-1}\left(e^{i \csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a + b \csc^{-1}(cx)) \text{Li}_2\left(-e^{i \csc^{-1}(cx)}\right)}{c} \end{aligned}$$

**Mathematica [A]** time = 0.293682, size = 265, normalized size = 1.84

$$-6ib^2 \text{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx)) + 6ib^2 \text{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx)) + 6b^3 \text{PolyLog}\left(3, -e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx)) - 6b^3 \text{PolyLog}\left(3, e^{i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3, x]
```

```
[Out] (a^3*c*x + 3*a^2*b*c*x*ArcCsc[c*x] + 3*a*b^2*c*x*ArcCsc[c*x]^2 + b^3*c*x*ArcCsc[c*x]^3 - 6*a*b^2*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])] - 3*b^3*ArcCsc[c*x]^2*Log[1 - E^(I*ArcCsc[c*x])] + 6*a*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])] + 3*b^3*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])] + 3*a^2*b*Log[c*(1 + Sqrt[1 - 1/(c^2*x^2)]]*x) - (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])] + (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])] + 6*b^3*PolyLog[3, -E^(I*ArcCsc[c*x])] - 6*b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c
```

**Maple [B]** time = 0.319, size = 437, normalized size = 3.

$$xb^3 (\text{arccsc}(cx))^3 + 3xab^2 (\text{arccsc}(cx))^2 - 3 \frac{b^3 (\text{arccsc}(cx))^2}{c} \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3 \frac{b^3 (\text{arccsc}(cx))^2}{c} \ln\left(1 + \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3,x)
```

```
[Out] x*b^3*arccsc(c*x)^3+3*x*a*b^2*arccsc(c*x)^2-3/c*b^3*arccsc(c*x)^2*ln(1-I/c/
x-(1-1/c^2/x^2)^(1/2))+3/c*b^3*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2)
)+6*I/c*b^3*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-6*I/c*arccsc(c
*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))*b^3+3*x*a^2*b*arccsc(c*x)-6/c*a*b
^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+6/c*a*b^2*arccsc(c*x)*ln(1+I
/c/x+(1-1/c^2/x^2)^(1/2))+6*I/c*a*b^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-
6*I/c*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))*a*b^2+x*a^3-6*b^3*polylog(3,I/c
/x+(1-1/c^2/x^2)^(1/2))/c+6*b^3*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+3/c
*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))*a^2*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 1
2*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))/(c^2*x^
2 - 1), x)*log(c)^2 + b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3/4
*b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*c^2*
integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))*log(c^2*x^2)/(c^2
*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)
*sqrt(c*x - 1))))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integrate(1
/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x
^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1))))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b
^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))*log(x)^2/(
c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*s
qrt(c*x - 1))))^2/(c^2*x^2 - 1), x) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1))))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2
*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrat
e(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(1/
4*x^2*log(x)^2/(c^2*x^2 - 1), x) - 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)
/c)*log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*
sqrt(c*x - 1))))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/
4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))*log(x)/(c^2*x^2 - 1), x)*log(c) -
12*a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*a*b^2*in
tegrate(1/4*log(x)/(c^2*x^2 - 1), x)*log(c) + a^3*x + 12*b^3*integrate(1/4*
sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))^2/(c^2*
x^2 - 1), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)
^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1))))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*integrate(1/4*arcta
n(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))*log(x)^2/(c^2*x^2 - 1), x) - 12*a*b^2*in
tegrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))^2/(c^2*x^2 - 1), x) - 1
2*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))))*log(c^2*x^2)/(c
^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^2 - 1), x) -
12*a*b^2*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*a*b^2*int
egrate(1/4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*(2*c*x*arccsc(c*x) + log(sqrt(-
1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a^2*b/c
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^3 \operatorname{arccsc}(cx)^3 + 3ab^2 \operatorname{arccsc}(cx)^2 + 3a^2b \operatorname{arccsc}(cx) + a^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*arccsc(c\*x)^3 + 3\*a\*b^2\*arccsc(c\*x)^2 + 3\*a^2\*b\*arccsc(c\*x) + a^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*\*3,x)

[Out] Integral((a + b\*arccsc(c\*x))\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^3, x)

$$3.28 \quad \int \frac{(a+b \csc^{-1}(cx))^3}{x} dx$$

**Optimal.** Leaf size=124

$$-\frac{3}{2}b^2 \text{PolyLog}\left(3, e^{2i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx)) + \frac{3}{2}ib \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))^2 - \frac{3}{4}ib^3 \text{PolyLog}\left(4, e^{2i \csc^{-1}(cx)}\right)$$

[Out] ((I/4)\*(a + b\*ArcCsc[c\*x])^4)/b - (a + b\*ArcCsc[c\*x])^3\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + ((3\*I)/2)\*b\*(a + b\*ArcCsc[c\*x])^2\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])] - (3\*b^2\*(a + b\*ArcCsc[c\*x])\*PolyLog[3, E^((2\*I)\*ArcCsc[c\*x])])/2 - ((3\*I)/4)\*b^3\*PolyLog[4, E^((2\*I)\*ArcCsc[c\*x])]

**Rubi [A]** time = 0.143158, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5223, 3717, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3}{2}b^2 \text{PolyLog}\left(3, e^{2i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx)) + \frac{3}{2}ib \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) (a + b \csc^{-1}(cx))^2 - \frac{3}{4}ib^3 \text{PolyLog}\left(4, e^{2i \csc^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^3/x, x]

[Out] ((I/4)\*(a + b\*ArcCsc[c\*x])^4)/b - (a + b\*ArcCsc[c\*x])^3\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + ((3\*I)/2)\*b\*(a + b\*ArcCsc[c\*x])^2\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])] - (3\*b^2\*(a + b\*ArcCsc[c\*x])\*PolyLog[3, E^((2\*I)\*ArcCsc[c\*x])])/2 - ((3\*I)/4)\*b^3\*PolyLog[4, E^((2\*I)\*ArcCsc[c\*x])]

### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_.)]\*(b\_.))^n\_.\*(x\_)^m\_.], x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c[x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rule 3717

Int[((c\_.) + (d\_.)\*(x\_)^m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[(c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))]/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^n\_.)\*((c\_.) + (d\_.)\*(x\_)^m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/ (b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^n\_.]\*((f\_.) + (g\_.)\*(x\_)^m\_.), x\_Symbol] :> -Simp[(f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/ (b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -

1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*(a\_.) + (b\_.)\*(x\_.)))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \csc^{-1}(cx))^3}{x} dx &= -\text{Subst} \left( \int (a + bx)^3 \cot(x) dx, x, \csc^{-1}(cx) \right) \\
 &= \frac{i(a + b \csc^{-1}(cx))^4}{4b} + 2i \text{Subst} \left( \int \frac{e^{2ix}(a + bx)^3}{1 - e^{2ix}} dx, x, \csc^{-1}(cx) \right) \\
 &= \frac{i(a + b \csc^{-1}(cx))^4}{4b} - (a + b \csc^{-1}(cx))^3 \log(1 - e^{2i \csc^{-1}(cx)}) + (3b) \text{Subst} \left( \int (a + bx)^2 \log \right. \\
 &= \frac{i(a + b \csc^{-1}(cx))^4}{4b} - (a + b \csc^{-1}(cx))^3 \log(1 - e^{2i \csc^{-1}(cx)}) + \frac{3}{2} ib (a + b \csc^{-1}(cx))^2 \text{Li}_2 \left( \right. \\
 &= \frac{i(a + b \csc^{-1}(cx))^4}{4b} - (a + b \csc^{-1}(cx))^3 \log(1 - e^{2i \csc^{-1}(cx)}) + \frac{3}{2} ib (a + b \csc^{-1}(cx))^2 \text{Li}_2 \left( \right. \\
 &= \frac{i(a + b \csc^{-1}(cx))^4}{4b} - (a + b \csc^{-1}(cx))^3 \log(1 - e^{2i \csc^{-1}(cx)}) + \frac{3}{2} ib (a + b \csc^{-1}(cx))^2 \text{Li}_2 \left( \right. \\
 &= \frac{i(a + b \csc^{-1}(cx))^4}{4b} - (a + b \csc^{-1}(cx))^3 \log(1 - e^{2i \csc^{-1}(cx)}) + \frac{3}{2} ib (a + b \csc^{-1}(cx))^2 \text{Li}_2 \left( \right.
 \end{aligned}$$

**Mathematica [A]** time = 0.213212, size = 242, normalized size = 1.95

$$\frac{3}{2} ia^2 b \left( \text{PolyLog} \left( 2, e^{2i \csc^{-1}(cx)} \right) + \csc^{-1}(cx) \left( \csc^{-1}(cx) + 2i \log \left( 1 - e^{2i \csc^{-1}(cx)} \right) \right) \right) + \frac{1}{8} iab^2 \left( -24 \csc^{-1}(cx) \text{PolyLog} \left( 2, \right. \right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])^3/x, x]

[Out] a^3\*Log[c\*x] + ((3\*I)/2)\*a^2\*b\*(ArcCsc[c\*x]\*(ArcCsc[c\*x] + (2\*I)\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])]) + PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]) + (I/8)\*a\*b^2\*(P

$$i^3 - 8 \operatorname{ArcCsc}[c*x]^3 + (24*I) \operatorname{ArcCsc}[c*x]^2 \operatorname{Log}[1 - E^{((-2*I) \operatorname{ArcCsc}[c*x])}] - 24 \operatorname{ArcCsc}[c*x] \operatorname{PolyLog}[2, E^{((-2*I) \operatorname{ArcCsc}[c*x])}] + (12*I) \operatorname{PolyLog}[3, E^{((-2*I) \operatorname{ArcCsc}[c*x])}] + (I/64) b^3 (\pi^4 - 16 \operatorname{ArcCsc}[c*x]^4 + (64*I) \operatorname{ArcCsc}[c*x]^3 \operatorname{Log}[1 - E^{((-2*I) \operatorname{ArcCsc}[c*x])}] - 96 \operatorname{ArcCsc}[c*x]^2 \operatorname{PolyLog}[2, E^{((-2*I) \operatorname{ArcCsc}[c*x])}] + (96*I) \operatorname{ArcCsc}[c*x] \operatorname{PolyLog}[3, E^{((-2*I) \operatorname{ArcCsc}[c*x])}] + 48 \operatorname{PolyLog}[4, E^{((-2*I) \operatorname{ArcCsc}[c*x])}])$$

**Maple [B]** time = 0.333, size = 666, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsc(c*x))^3/x,x)`

[Out]  $a^3 \ln(c*x) + 3/2 I a^2 b \operatorname{arccsc}(c*x)^2 - b^3 \operatorname{arccsc}(c*x)^3 \ln(1 + I/c/x + (1 - 1/c^2/x^2)^{1/2}) + 3 I a^2 b \operatorname{polylog}(2, I/c/x + (1 - 1/c^2/x^2)^{1/2}) - 6 b^3 \operatorname{arccsc}(c*x) \operatorname{polylog}(3, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 6 I b^3 \operatorname{polylog}(4, I/c/x + (1 - 1/c^2/x^2)^{1/2}) - b^3 \operatorname{arccsc}(c*x)^3 \ln(1 - I/c/x - (1 - 1/c^2/x^2)^{1/2}) + 3 I b^3 \operatorname{arccsc}(c*x)^2 \operatorname{polylog}(2, I/c/x + (1 - 1/c^2/x^2)^{1/2}) - 6 b^3 \operatorname{arccsc}(c*x) \operatorname{polylog}(3, I/c/x + (1 - 1/c^2/x^2)^{1/2}) + 3 I b^3 \operatorname{arccsc}(c*x)^2 \operatorname{polylog}(2, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) + 1/4 I b^3 \operatorname{arccsc}(c*x)^4 + 6 I a b^2 \operatorname{arccsc}(c*x) \operatorname{polylog}(2, I/c/x + (1 - 1/c^2/x^2)^{1/2}) - 3 a b^2 \operatorname{arccsc}(c*x)^2 \ln(1 - I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 3 a b^2 \operatorname{arccsc}(c*x)^2 \ln(1 + I/c/x + (1 - 1/c^2/x^2)^{1/2}) + 6 I a b^2 \operatorname{arccsc}(c*x) \operatorname{polylog}(2, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 6 a b^2 \operatorname{polylog}(3, I/c/x + (1 - 1/c^2/x^2)^{1/2}) - 6 a b^2 \operatorname{polylog}(3, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) + 3 I a^2 b \operatorname{polylog}(2, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 3 a^2 b \operatorname{arccsc}(c*x) \ln(1 - I/c/x - (1 - 1/c^2/x^2)^{1/2}) - 3 a^2 b \operatorname{arccsc}(c*x) \ln(1 + I/c/x + (1 - 1/c^2/x^2)^{1/2}) + I a b^2 \operatorname{arccsc}(c*x)^3 - 6 I b^3 \operatorname{polylog}(4, -I/c/x - (1 - 1/c^2/x^2)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))^3/x,x, algorithm="maxima")`

[Out]  $-3/2 a b^2 c^2 (\log(c*x + 1)/c^2 + \log(c*x - 1)/c^2) \log(c)^2 - 12 b^3 c^2 \operatorname{integrate}(1/4 x^2 \arctan(1/(\sqrt{c*x + 1} \sqrt{c*x - 1}))/ (c^2 x^3 - x), x) \log(c)^2 + 12 b^3 c^2 \operatorname{integrate}(1/4 x^2 \arctan(1/(\sqrt{c*x + 1} \sqrt{c*x - 1})) \log(c^2 x^2) / (c^2 x^3 - x), x) \log(c) - 24 b^3 c^2 \operatorname{integrate}(1/4 x^2 \arctan(1/(\sqrt{c*x + 1} \sqrt{c*x - 1})) \log(x) / (c^2 x^3 - x), x) \log(c) + 12 a b^2 c^2 \operatorname{integrate}(1/4 x^2 \log(c^2 x^2) / (c^2 x^3 - x), x) \log(c) - 24 a b^2 c^2 \operatorname{integrate}(1/4 x^2 \log(x) / (c^2 x^3 - x), x) \log(c) + b^3 \arctan^2(1, \sqrt{c*x + 1} \sqrt{c*x - 1})^3 \log(x) - 3/4 b^3 \arctan^2(1, \sqrt{c*x + 1} \sqrt{c*x - 1}) \log(c^2 x^2)^2 \log(x) + 24 b^3 c^2 \operatorname{integrate}(1/4 x^2 \arctan(1/(\sqrt{c*x + 1} \sqrt{c*x - 1})) \log(c^2 x^2) \log(x) / (c^2 x^3 - x), x) - 12 b^3 c^2 \operatorname{integrate}(1/4 x^2 \arctan(1/(\sqrt{c*x + 1} \sqrt{c*x - 1})) \log(x)^2 / (c^2 x^3 - x), x) + 12 a b^2 c^2 \operatorname{integrate}(1/4 x^2 \arctan(1/(\sqrt{c*x + 1} \sqrt{c*x - 1}))^2 / (c^2 x^3 - x), x) - 3 a b^2 c^2 \operatorname{integrate}(1/4 x^2 \log(c^2 x^2)^2 / (c^2 x^3 - x), x) + 12 a b^2 c^2 \operatorname{integrate}(1/4 x^2 \log(c^2 x^2) \log(x) / (c^2 x^3 - x), x) - 12 a b^2 c^2 \operatorname{integrate}(1/4 x^2 \log(x)^2 / (c^2 x^3 - x), x) + 12 a^2 b c^2 \operatorname{integrate}(1/4 x^2 \arctan(1/(\sqrt{c*x + 1} \sqrt{c*x - 1})))$

```

)))/(c^2*x^3 - x), x) + 3/2*a*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*
log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^
2*x^3 - x), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt
(c*x - 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*b^3*integrate(1/4*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c) - 12*
a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*a*b^2*integr
ate(1/4*log(x)/(c^2*x^3 - x), x)*log(c) + 12*b^3*integrate(1/4*sqrt(c*x + 1
)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2*log(x)/(c^2*x^3 -
x), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2*lo
g(x)/(c^2*x^3 - x), x) - 24*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*b^3*integrate(1/4*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^3 - x), x) - 12*a*b^2*
integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^3 - x), x) +
3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^3 - x), x) - 12*a*b^2*integrat
e(1/4*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*a*b^2*integrate(1/4*log(x)
^2/(c^2*x^3 - x), x) - 12*a^2*b*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))/(c^2*x^3 - x), x) + a^3*log(x)

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arccsc}(cx)^3 + 3ab^2 \operatorname{arccsc}(cx)^2 + 3a^2b \operatorname{arccsc}(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
a^3)/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3/x, x)
```

$$3.29 \quad \int \frac{(a+b \csc^{-1}(cx))^3}{x^2} dx$$

**Optimal.** Leaf size=80

$$\frac{6b^2(a+b \csc^{-1}(cx))}{x} - 3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2 - \frac{(a+b \csc^{-1}(cx))^3}{x} + 6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

[Out] 6\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)] + (6\*b^2\*(a + b\*ArcCsc[c\*x]))/x - 3\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2 - (a + b\*ArcCsc[c\*x])^3/x

**Rubi [A]** time = 0.0857982, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5223, 3296, 2638}

$$\frac{6b^2(a+b \csc^{-1}(cx))}{x} - 3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2 - \frac{(a+b \csc^{-1}(cx))^3}{x} + 6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^3/x^2,x]

[Out] 6\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)] + (6\*b^2\*(a + b\*ArcCsc[c\*x]))/x - 3\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2 - (a + b\*ArcCsc[c\*x])^3/x

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m, x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \csc^{-1}(cx))^3}{x^2} dx &= -\left(c \operatorname{Subst}\left(\int (a + bx)^3 \cos(x) dx, x, \csc^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \csc^{-1}(cx))^3}{x} + (3bc) \operatorname{Subst}\left(\int (a + bx)^2 \sin(x) dx, x, \csc^{-1}(cx)\right) \\
&= -3bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))^2 - \frac{(a + b \csc^{-1}(cx))^3}{x} + (6b^2c) \operatorname{Subst}\left(\int (a + bx) \cos(x) dx, x, \csc^{-1}(cx)\right) \\
&= \frac{6b^2(a + b \csc^{-1}(cx))}{x} - 3bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))^2 - \frac{(a + b \csc^{-1}(cx))^3}{x} - (6b^3c) \operatorname{Subst}\left(\int \cos(x) dx, x, \csc^{-1}(cx)\right) \\
&= 6b^3c\sqrt{1 - \frac{1}{c^2x^2}} + \frac{6b^2(a + b \csc^{-1}(cx))}{x} - 3bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))^2 - \frac{(a + b \csc^{-1}(cx))^3}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.200749, size = 135, normalized size = 1.69

$$\frac{3b \csc^{-1}(cx) \left( a^2 + 2abcx\sqrt{1 - \frac{1}{c^2x^2}} - 2b^2 \right) + 3a^2bcx\sqrt{1 - \frac{1}{c^2x^2}} + a^3 + 3b^2 \csc^{-1}(cx)^2 \left( a + bcx\sqrt{1 - \frac{1}{c^2x^2}} \right) - 6ab^2 - 6b^3}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])^3/x^2, x]

[Out] -((a^3 - 6\*a\*b^2 + 3\*a^2\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x - 6\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + 3\*b\*(a^2 - 2\*b^2 + 2\*a\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)\*ArcCsc[c\*x] + 3\*b^2\*(a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)\*ArcCsc[c\*x]^2 + b^3\*ArcCsc[c\*x]^3)/x)

**Maple [B]** time = 0.25, size = 199, normalized size = 2.5

$$c \left( -\frac{a^3}{cx} + b^3 \left( -\frac{(\operatorname{arccsc}(cx))^3}{cx} - 3(\operatorname{arccsc}(cx))^2 \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + 6\sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + 6\frac{\operatorname{arccsc}(cx)}{cx} \right) + 3ab^2 \left( -\frac{(\operatorname{arccsc}(cx))^2}{cx} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^3/x^2, x)

[Out] c\*(-a^3/c/x+b^3\*(-1/c/x\*arccsc(c\*x)^3-3\*arccsc(c\*x)^2\*((c^2\*x^2-1)/c^2/x^2)^(1/2)+6\*((c^2\*x^2-1)/c^2/x^2)^(1/2)+6/c/x\*arccsc(c\*x))+3\*a\*b^2\*(-1/c/x\*arccsc(c\*x)^2+2/c/x-2\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*arccsc(c\*x))+3\*a^2\*b\*(-1/c/x\*arccsc(c\*x)-1/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2\*(c^2\*x^2-1)))

**Maxima [A]** time = 1.02653, size = 198, normalized size = 2.48

$$-\frac{b^3 \operatorname{arccsc}(cx)^3}{x} - 3 \left( c\sqrt{-\frac{1}{c^2x^2}} + 1 + \frac{\operatorname{arccsc}(cx)}{x} \right) a^2 b - 6 \left( c\sqrt{-\frac{1}{c^2x^2}} + 1 \operatorname{arccsc}(cx) - \frac{1}{x} \right) ab^2 - 3 \left( c\sqrt{-\frac{1}{c^2x^2}} + 1 \operatorname{arccsc}(cx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^2, x, algorithm="maxima")

```
[Out] -b^3*arccsc(c*x)^3/x - 3*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a^2*b -
6*(c*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x) - 1/x)*a*b^2 - 3*(c*sqrt(-1/(c^2*x
^2) + 1)*arccsc(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) - 2*arccsc(c*x)/x)*b^3
- 3*a*b^2*arccsc(c*x)^2/x - a^3/x
```

**Fricas [A]** time = 2.37818, size = 238, normalized size = 2.98

$$\frac{b^3 \operatorname{arccsc}(cx)^3 + 3ab^2 \operatorname{arccsc}(cx)^2 + a^3 - 6ab^2 + 3(a^2b - 2b^3) \operatorname{arccsc}(cx) + 3(b^3 \operatorname{arccsc}(cx)^2 + 2ab^2 \operatorname{arccsc}(cx) + a^2b - 2b^3) \operatorname{arccsc}(cx) + 3(b^3 \operatorname{arccsc}(cx)^2 + 2ab^2 \operatorname{arccsc}(cx) + a^2b - 2b^3) \operatorname{arccsc}(cx) + a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2*
b^3)*arccsc(c*x) + 3*(b^3*arccsc(c*x)^2 + 2*a*b^2*arccsc(c*x) + a^2*b - 2*b
^3)*sqrt(c^2*x^2 - 1))/x
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3/x^2, x)
```

$$3.30 \quad \int \frac{(a+b \csc^{-1}(cx))^3}{x^3} dx$$

**Optimal.** Leaf size=125

$$\frac{3b^2(a+b \csc^{-1}(cx))}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{4x} + \frac{1}{4}c^2(a+b \csc^{-1}(cx))^3 - \frac{(a+b \csc^{-1}(cx))^3}{2x^2} + \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}$$

[Out] (3\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)])/(8\*x) - (3\*b^3\*c^2\*ArcCsc[c\*x])/8 + (3\*b^2\*(a + b\*ArcCsc[c\*x]))/(4\*x^2) - (3\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(4\*x) + (c^2\*(a + b\*ArcCsc[c\*x])^3)/4 - (a + b\*ArcCsc[c\*x])^3/(2\*x^2)

**Rubi [A]** time = 0.104995, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5223, 4404, 3311, 32, 2635, 8}

$$\frac{3b^2(a+b \csc^{-1}(cx))}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{4x} + \frac{1}{4}c^2(a+b \csc^{-1}(cx))^3 - \frac{(a+b \csc^{-1}(cx))^3}{2x^2} + \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^3/x^3, x]

[Out] (3\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)])/(8\*x) - (3\*b^3\*c^2\*ArcCsc[c\*x])/8 + (3\*b^2\*(a + b\*ArcCsc[c\*x]))/(4\*x^2) - (3\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(4\*x) + (c^2\*(a + b\*ArcCsc[c\*x])^3)/4 - (a + b\*ArcCsc[c\*x])^3/(2\*x^2)

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))^n\*(x\_)^m, x\_Symbol] := -Dist[(c^(m+1))^(-1), Subst[Int[(a+b\*x)^n\*Csc[x]^(m+1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4404

Int[Cos[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^m\*Sin[(a\_.) + (b\_.)\*(x\_)]^n, x\_Symbol] := Simp[((c+d\*x)^m\*Sin[a+b\*x]^(n+1))/(b\*(n+1)), x] - Dist[(d\*m)/(b\*(n+1)), Int[(c+d\*x)^(m-1)\*Sin[a+b\*x]^(n+1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^m\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^n, x\_Symbol] := Simp[(d\*m\*(c+d\*x)^(m-1)\*(b\*Sin[e+f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n-1))/n, Int[(c+d\*x)^m\*(b\*Sin[e+f\*x])^(n-2), x], x] - Dist[(d^2\*m\*(m-1))/(f^2\*n^2), Int[(c+d\*x)^(m-2)\*(b\*Sin[e+f\*x])^n, x], x] - Simp[(b\*(c+d\*x)^m\*Cos[e+f\*x]\*(b\*Sin[e+f\*x])^(n-1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 32

Int[(a\_.) + (b\_.)\*(x\_)]^m, x\_Symbol] := Simp[(a+b\*x)^(m+1)/(b\*(m+1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{csc}^{-1}(cx))^3}{x^3} dx &= -\left(c^2 \operatorname{Subst}\left(\int (a + bx)^3 \cos(x) \sin(x) dx, x, \operatorname{csc}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{csc}^{-1}(cx))^3}{2x^2} + \frac{1}{2}(3bc^2) \operatorname{Subst}\left(\int (a + bx)^2 \sin^2(x) dx, x, \operatorname{csc}^{-1}(cx)\right) \\ &= \frac{3b^2(a + b \operatorname{csc}^{-1}(cx))}{4x^2} - \frac{3bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))^2}{4x} - \frac{(a + b \operatorname{csc}^{-1}(cx))^3}{2x^2} + \frac{1}{4}(3bc^2) \operatorname{Subst}\left(\int (a + bx) \sin^2(x) dx, x, \operatorname{csc}^{-1}(cx)\right) \\ &= \frac{3b^3c\sqrt{1 - \frac{1}{c^2x^2}}}{8x} + \frac{3b^2(a + b \operatorname{csc}^{-1}(cx))}{4x^2} - \frac{3bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))^2}{4x} + \frac{1}{4}c^2(a + b \operatorname{csc}^{-1}(cx)) \operatorname{Subst}\left(\int \sin^2(x) dx, x, \operatorname{csc}^{-1}(cx)\right) \\ &= \frac{3b^3c\sqrt{1 - \frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2 \operatorname{csc}^{-1}(cx) + \frac{3b^2(a + b \operatorname{csc}^{-1}(cx))}{4x^2} - \frac{3bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \operatorname{csc}^{-1}(cx))^2}{4x} + \frac{1}{4}c^2(a + b \operatorname{csc}^{-1}(cx)) \operatorname{Subst}\left(\int \sin^2(x) dx, x, \operatorname{csc}^{-1}(cx)\right) \end{aligned}$$

**Mathematica [A]** time = 0.210623, size = 186, normalized size = 1.49

$$\frac{-3bc^2x^2(b^2 - 2a^2)\sin^{-1}\left(\frac{1}{cx}\right) + 6b \operatorname{csc}^{-1}(cx)\left(-2a^2 - 2abcx\sqrt{1 - \frac{1}{c^2x^2}} + b^2\right) - 6a^2bcx\sqrt{1 - \frac{1}{c^2x^2}} - 4a^3 - 6b^2 \operatorname{csc}^{-1}(cx)^2(a + b \operatorname{csc}^{-1}(cx))}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^3,x]
```

```
[Out] (-4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^3*c*Sqrt[1 - 1/
(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[
c*x] - 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(2 - c^2*x^2))*ArcCsc[c*x]^2
+ 2*b^3*(-2 + c^2*x^2)*ArcCsc[c*x]^3 - 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcSin[1/
(c*x)])/(8*x^2)
```

**Maple [B]** time = 0.26, size = 315, normalized size = 2.5

$$-\frac{a^3}{2x^2} + \frac{c^2b^3(\operatorname{arccsc}(cx))^3}{4} - \frac{b^3(\operatorname{arccsc}(cx))^3}{2x^2} - \frac{3cb^3(\operatorname{arccsc}(cx))^2}{4x} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} - \frac{3b^3c^2 \operatorname{arccsc}(cx)}{8} + \frac{3b^3 \operatorname{arccsc}(cx)}{4x^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2+1/4*c^2*b^3*arccsc(c*x)^3-1/2*b^3*arccsc(c*x)^3/x^2-3/4*c*b^3*
arccsc(c*x)^2/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-3/8*b^3*c^2*arccsc(c*x)+3/4*b^3
```

$$\frac{1}{x^2} \operatorname{arccsc}(cx) + 3/8 * c * b^3 * ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / x + 3/4 * c^2 * a * b^2 * \operatorname{arccsc}(cx)^2 - 3/2 * a * b^2 / x^2 * \operatorname{arccsc}(cx)^2 - 3/2 * c * a * b^2 * \operatorname{arccsc}(cx) / x * ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} + 3/4 * a * b^2 / x^2 - 3/2 * a^2 * b / x^2 * \operatorname{arccsc}(cx) + 3/4 * c * a^2 * b * (c^2 * x^2 - 1)^{(1/2)} / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / x * \arctan(1 / (c^2 * x^2 - 1)^{(1/2)}) - 3/4 * c * a^2 * b / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / x + 3/4 * c * a^2 * b / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / x^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^3,x, algorithm="maxima")

[Out]  $3/4 * a^2 * b * ((c^4 * x * \sqrt{-1 / (c^2 * x^2) + 1}) / (c^2 * x^2 * (1 / (c^2 * x^2) - 1) - 1) - c^3 * \arctan(c * x * \sqrt{-1 / (c^2 * x^2) + 1})) / c - 2 * \operatorname{arccsc}(c * x) / x^2 - 1/2 * a^3 / x^2 - 1/8 * (4 * b^3 * \arctan^2(1, \sqrt{c * x + 1}) * \sqrt{c * x - 1})^3 - 3 * b^3 * \arctan^2(1, \sqrt{c * x + 1}) * \sqrt{c * x - 1}) * \log(c^2 * x^2)^2 + 12 * (a * b^2 * c^2 * (\log(c * x + 1) + \log(c * x - 1) - 2 * \log(x)) * \log(c)^2 + 16 * b^3 * c^2 * \int (1/8 * x^2 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) / (c^2 * x^5 - x^3), x) * \log(c)^2 - 16 * b^3 * c^2 * \int \int (1/8 * x^2 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(c^2 * x^2) / (c^2 * x^5 - x^3), x) * \log(c) + 32 * b^3 * c^2 * \int (1/8 * x^2 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(x) / (c^2 * x^5 - x^3), x) * \log(c) - 16 * a * b^2 * c^2 * \int \int (1/8 * x^2 * \log(c^2 * x^2) / (c^2 * x^5 - x^3), x) * \log(c) + 32 * a * b^2 * c^2 * \int (1/8 * x^2 * \log(x) / (c^2 * x^5 - x^3), x) * \log(c) - 16 * b^3 * c^2 * \int (1/8 * x^2 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(c^2 * x^2) * \log(x) / (c^2 * x^5 - x^3), x) + 16 * b^3 * c^2 * \int (1/8 * x^2 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(x)^2 / (c^2 * x^5 - x^3), x) - 16 * a * b^2 * c^2 * \int (1/8 * x^2 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(c^2 * x^2) / (c^2 * x^5 - x^3), x) + 4 * a * b^2 * c^2 * \int (1/8 * x^2 * \log(c^2 * x^2)^2 / (c^2 * x^5 - x^3), x) - 16 * a * b^2 * c^2 * \int (1/8 * x^2 * \log(c^2 * x^2) * \log(x) / (c^2 * x^5 - x^3), x) + 16 * a * b^2 * c^2 * \int (1/8 * x^2 * \log(x)^2 / (c^2 * x^5 - x^3), x) - (c^2 * \log(c * x + 1) + c^2 * \log(c * x - 1) - 2 * c^2 * \log(x) + 1/x^2) * a * b^2 * \log(c)^2 - 16 * b^3 * \int (1/8 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) / (c^2 * x^5 - x^3), x) * \log(c)^2 + 16 * b^3 * \int (1/8 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(c^2 * x^2) / (c^2 * x^5 - x^3), x) * \log(c) - 32 * b^3 * \int (1/8 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(x) / (c^2 * x^5 - x^3), x) * \log(c) + 16 * a * b^2 * \int (1/8 * \log(c^2 * x^2) / (c^2 * x^5 - x^3), x) * \log(c) - 32 * a * b^2 * \int (1/8 * \log(x) / (c^2 * x^5 - x^3), x) * \log(c) + 8 * b^3 * \int (1/8 * \sqrt{c * x + 1}) * \sqrt{c * x - 1} * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1}))^2 / (c^2 * x^5 - x^3), x) - 2 * b^3 * \int (1/8 * \sqrt{c * x + 1}) * \sqrt{c * x - 1} * \log(c^2 * x^2)^2 / (c^2 * x^5 - x^3), x) + 16 * b^3 * \int (1/8 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(c^2 * x^2) * \log(x) / (c^2 * x^5 - x^3), x) - 16 * b^3 * \int (1/8 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(x)^2 / (c^2 * x^5 - x^3), x) + 16 * a * b^2 * \int (1/8 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1}))^2 / (c^2 * x^5 - x^3), x) - 8 * b^3 * \int (1/8 * \arctan(1 / (\sqrt{c * x + 1}) * \sqrt{c * x - 1})) * \log(c^2 * x^2) / (c^2 * x^5 - x^3), x) - 4 * a * b^2 * \int (1/8 * \log(c^2 * x^2)^2 / (c^2 * x^5 - x^3), x) + 16 * a * b^2 * \int (1/8 * \log(c^2 * x^2) * \log(x) / (c^2 * x^5 - x^3), x) - 16 * a * b^2 * \int (1/8 * \log(x)^2 / (c^2 * x^5 - x^3), x) * x^2 / x^2$

**Fricas [A]** time = 2.25876, size = 342, normalized size = 2.74

$$2(b^3 c^2 x^2 - 2b^3) \operatorname{arccsc}(cx)^3 - 4a^3 + 6ab^2 + 6(ab^2 c^2 x^2 - 2ab^2) \operatorname{arccsc}(cx)^2 + 3\left(\left(2a^2 b - b^3\right) c^2 x^2 - 4a^2 b + 2b^3\right) \operatorname{arccsc}(cx) + \frac{8x^2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^3,x, algorithm="fricas")

[Out]  $\frac{1}{8}(2*(b^3*c^2*x^2 - 2*b^3)*\operatorname{arccsc}(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^2*x^2 - 2*a*b^2)*\operatorname{arccsc}(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*b^3)*\operatorname{arccsc}(c*x) - 3*(2*b^3*\operatorname{arccsc}(c*x)^2 + 4*a*b^2*\operatorname{arccsc}(c*x) + 2*a^2*b - b^3)*\sqrt{c^2*x^2 - 1})/x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*\*3/x\*\*3,x)

[Out] Integral((a + b\*acsc(c\*x))\*\*3/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^3/x^3, x)

$$3.31 \quad \int \frac{(a+b \csc^{-1}(cx))^3}{x^4} dx$$

**Optimal.** Leaf size=170

$$\frac{4b^2c^2(a+b \csc^{-1}(cx))}{3x} + \frac{2b^2(a+b \csc^{-1}(cx))}{9x^3} - \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{3x^2}$$

[Out] (14\*b^3\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]/9 - (2\*b^3\*c^3\*(1 - 1/(c^2\*x^2))^(3/2))/27 + (2\*b^2\*(a + b\*ArcCsc[c\*x]))/(9\*x^3) + (4\*b^2\*c^2\*(a + b\*ArcCsc[c\*x]))/(3\*x) - (2\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/3 - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(3\*x^2) - (a + b\*ArcCsc[c\*x])^3/(3\*x^3)

**Rubi [A]** time = 0.149797, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5223, 4404, 3311, 3296, 2638, 2633}

$$\frac{4b^2c^2(a+b \csc^{-1}(cx))}{3x} + \frac{2b^2(a+b \csc^{-1}(cx))}{9x^3} - \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^3/x^4, x]

[Out] (14\*b^3\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]/9 - (2\*b^3\*c^3\*(1 - 1/(c^2\*x^2))^(3/2))/27 + (2\*b^2\*(a + b\*ArcCsc[c\*x]))/(9\*x^3) + (4\*b^2\*c^2\*(a + b\*ArcCsc[c\*x]))/(3\*x) - (2\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/3 - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(3\*x^2) - (a + b\*ArcCsc[c\*x])^3/(3\*x^3)

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m, x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*c\*x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4404

Int[Cos[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^m\*Sin[(a\_.) + (b\_.)\*(x\_)]^n, x\_Symbol] := Simp[((c + d\*x)^m\*Sin[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sin[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^m\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^n, x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sin[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \csc^{-1}(cx))^3}{x^4} dx &= -\left(c^3 \operatorname{Subst}\left(\int (a + bx)^3 \cos(x) \sin^2(x) dx, x, \csc^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \csc^{-1}(cx))^3}{3x^3} + (bc^3) \operatorname{Subst}\left(\int (a + bx)^2 \sin^3(x) dx, x, \csc^{-1}(cx)\right) \\
&= \frac{2b^2(a + b \csc^{-1}(cx))}{9x^3} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))^2}{3x^2} - \frac{(a + b \csc^{-1}(cx))^3}{3x^3} + \frac{1}{3}(2bc^3) \operatorname{Subst}\left(\int (a + bx) \sin^3(x) dx, x, \csc^{-1}(cx)\right) \\
&= \frac{2b^2(a + b \csc^{-1}(cx))}{9x^3} - \frac{2}{3}bc^3\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))^2 - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}(a + b \csc^{-1}(cx))^2}{3x^2} \\
&= \frac{2}{9}b^3c^3\sqrt{1 - \frac{1}{c^2x^2}} - \frac{2}{27}b^3c^3\left(1 - \frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a + b \csc^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a + b \csc^{-1}(cx))}{3x} \\
&= \frac{14}{9}b^3c^3\sqrt{1 - \frac{1}{c^2x^2}} - \frac{2}{27}b^3c^3\left(1 - \frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a + b \csc^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a + b \csc^{-1}(cx))}{3x}
\end{aligned}$$

**Mathematica [A]** time = 0.299556, size = 204, normalized size = 1.2

$$\frac{3b \csc^{-1}(cx) \left(-9a^2 - 6abcx\sqrt{1 - \frac{1}{c^2x^2}}(2c^2x^2 + 1) + 2b^2(6c^2x^2 + 1)\right) - 9a^2bcx\sqrt{1 - \frac{1}{c^2x^2}}(2c^2x^2 + 1) - 9a^3 + 6ab^2(6c^2x^2 + 1)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^4, x]
```

```
[Out] (-9*a^3 - 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
6*c^2*x^2) + 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a^2
- 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^2))
*ArcCsc[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*Ar
cCsc[c*x]^2 - 9*b^3*ArcCsc[c*x]^3)/(27*x^3)
```

**Maple [B]** time = 0.261, size = 299, normalized size = 1.8

$$c^3 \left( -\frac{a^3}{3c^3x^3} + b^3 \left( -\frac{(\operatorname{arccsc}(cx))^3}{3c^3x^3} - \frac{(\operatorname{arccsc}(cx))^2(2c^2x^2 + 1)}{3c^2x^2} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + \frac{4}{3} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + \frac{4 \operatorname{arccsc}(cx)}{3cx} + \frac{2 \operatorname{arccsc}(cx)}{9c^3x^3} \right) \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^3/x^4,x)

[Out]  $c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arccsc(c*x)^3-1/3*arccsc(c*x)^2*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)}+4/3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}+4/3/c/x*arccsc(c*x)+2/9/c^3/x^3*arccsc(c*x)+2/27*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)})+3*a*b^2*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*arccsc(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)}+2/27/c^3/x^3+4/9/c/x)+3*a^2*b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)^{(1/2)})/c^4/x^4)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^4,x, algorithm="maxima")

[Out]  $1/3*a^2*b*((c^4*(-1/(c^2*x^2)+1)^{(3/2)}-3*c^4*\sqrt{-1/(c^2*x^2)+1}))/c-3*arccsc(c*x)/x^3-a*b^2*arccsc(c*x)^2/x^3+1/12*(12*x^3*integrate(-1/4*(12*c^2*x^2*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c)^2-12*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c)^2+12*(c^2*x^2*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})-arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1}))*\log(x)^2+\sqrt{c*x+1}*\sqrt{c*x-1}*(4*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})^2-\log(c^2*x^2)^2-4*((3*c^2*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c)-c^2*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1}))*x^2-3*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c)+3*(c^2*x^2*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})-arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1}))*\log(x)+arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1}))*\log(c^2*x^2)+24*(c^2*x^2*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c)-arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c))*\log(x))/(c^2*x^6-x^4),x)-4*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})^3+3*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*\log(c^2*x^2)^2*b^3/x^3-1/3*a^3/x^3-2/9*(6*c^5*x^4*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})-3*c^3*x^2*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1})*(6*c^3*x^2+c)*\sqrt{c*x+1})*\sqrt{c*x-1}-3*c*arctan2(1,\sqrt{c*x+1})*\sqrt{c*x-1}))*a*b^2/(\sqrt{c*x+1})*\sqrt{c*x-1}*c*x^3)$

**Fricas [A]** time = 2.3525, size = 401, normalized size = 2.36

$$\frac{36ab^2c^2x^2 - 9b^3 \operatorname{arccsc}(cx)^3 - 27ab^2 \operatorname{arccsc}(cx)^2 - 9a^3 + 6ab^2 + 3(12b^3c^2x^2 - 9a^2b + 2b^3) \operatorname{arccsc}(cx) - (2(9a^2b - 20b^3)c^2x^2 + 9a^2b - 2b^3 + 9(2b^3c^2x^2 + b^3) \operatorname{arccsc}(cx))^2 + 18(2ab^2c^2x^2 + ab^2) \operatorname{arccsc}(cx) \sqrt{c^2x^2 - 1}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^4,x, algorithm="fricas")

[Out]  $1/27*(36*a*b^2*c^2*x^2-9*b^3*arccsc(c*x)^3-27*a*b^2*arccsc(c*x)^2-9*a^3+6*a*b^2+3*(12*b^3*c^2*x^2-9*a^2*b+2*b^3)*arccsc(c*x)-(2*(9*a^2*b-20*b^3)*c^2*x^2+9*a^2*b-2*b^3+9*(2*b^3*c^2*x^2+b^3)*arccsc(c*x))^2+18*(2*a*b^2*c^2*x^2+ab^2)*arccsc(c*x))*\sqrt{c^2*x^2-1}/x^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*\*3/x\*\*4,x)

[Out] Integral((a + b\*acsc(c\*x))\*\*3/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^4,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^3/x^4, x)

$$3.32 \quad \int \frac{(a+b \csc^{-1}(cx))^3}{x^5} dx$$

**Optimal.** Leaf size=208

$$\frac{9b^2c^2(a+b \csc^{-1}(cx))}{32x^2} + \frac{3b^2(a+b \csc^{-1}(cx))}{32x^4} - \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{32x} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{16x^3}$$

[Out] (3\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)])/(128\*x^3) + (45\*b^3\*c^3\*Sqrt[1 - 1/(c^2\*x^2)])/(256\*x) - (45\*b^3\*c^4\*ArcCsc[c\*x])/256 + (3\*b^2\*(a + b\*ArcCsc[c\*x]))/(32\*x^4) + (9\*b^2\*c^2\*(a + b\*ArcCsc[c\*x]))/(32\*x^2) - (3\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(16\*x^3) - (9\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(32\*x) + (3\*c^4\*(a + b\*ArcCsc[c\*x])^3)/32 - (a + b\*ArcCsc[c\*x])^3/(4\*x^4)

**Rubi [A]** time = 0.175948, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5223, 4404, 3311, 32, 2635, 8}

$$\frac{9b^2c^2(a+b \csc^{-1}(cx))}{32x^2} + \frac{3b^2(a+b \csc^{-1}(cx))}{32x^4} - \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{32x} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b \csc^{-1}(cx))^2}{16x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])^3/x^5, x]

[Out] (3\*b^3\*c\*Sqrt[1 - 1/(c^2\*x^2)])/(128\*x^3) + (45\*b^3\*c^3\*Sqrt[1 - 1/(c^2\*x^2)])/(256\*x) - (45\*b^3\*c^4\*ArcCsc[c\*x])/256 + (3\*b^2\*(a + b\*ArcCsc[c\*x]))/(32\*x^4) + (9\*b^2\*c^2\*(a + b\*ArcCsc[c\*x]))/(32\*x^2) - (3\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(16\*x^3) - (9\*b\*c^3\*Sqrt[1 - 1/(c^2\*x^2)]\*(a + b\*ArcCsc[c\*x])^2)/(32\*x) + (3\*c^4\*(a + b\*ArcCsc[c\*x])^3)/32 - (a + b\*ArcCsc[c\*x])^3/(4\*x^4)

### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^n\*(x\_)^m, x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rule 4404

Int[Cos[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^m\*Sin[(a\_.) + (b\_.)\*(x\_)]^n, x\_Symbol] := Simp[((c + d\*x)^m\*Sin[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sin[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^m\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^n, x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sin[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]



Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))^3/x^5,x)

[Out] 
$$-1/4*a^3/x^4-1/4*b^3/x^4*arccsc(c*x)^3+3/32*c^4*b^3*arccsc(c*x)^3-9/32*c^3*b^3*arccsc(c*x)^2/x*((c^2*x^2-1)/c^2/x^2)^{(1/2)}-3/16*c*b^3*arccsc(c*x)^2/x^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}+3/32*b^3/x^4*arccsc(c*x)+45/256*c^3*b^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x+3/128*c*b^3/x^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}-45/256*b^3*c^4*arccsc(c*x)+9/32*c^2*b^3/x^2*arccsc(c*x)-3/4*a*b^2/x^4*arccsc(c*x)^2+9/32*c^4*a*b^2*arccsc(c*x)^2-9/16*c^3*a*b^2*arccsc(c*x)/x*((c^2*x^2-1)/c^2/x^2)^{(1/2)}-3/8*c*a*b^2*arccsc(c*x)/x^3*((c^2*x^2-1)/c^2/x^2)^{(1/2)}+3/32*a*b^2/x^4+9/32*c^2*a*b^2/x^2-3/4*a^2*b/x^4*arccsc(c*x)+9/32*c^3*a^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*arctan(1/(c^2*x^2-1)^{(1/2)})-9/32*c^3*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x+3/32*c*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^3+3/16/c*a^2*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^5$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))^3/x^5,x, algorithm="maxima")

[Out] 
$$-3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2)+1))+(3*c^8*x^3*(-1/(c^2*x^2)+1)^{(3/2)}+5*c^6*x*sqrt(-1/(c^2*x^2)+1))/(c^4*x^4*(1/(c^2*x^2)-1)^2-2*c^2*x^2*(1/(c^2*x^2)-1)+1))/c+8*arccsc(c*x)/x^4)-1/4*a^3/x^4-1/16*(4*b^3*arctan(1, sqrt(c*x+1))*sqrt(c*x-1))^3-3*b^3*arctan(1, sqrt(c*x+1))*sqrt(c*x-1))*log(c^2*x^2)^2+12*(2*(c^2*log(c*x+1)+c^2*log(c*x-1)-2*c^2*log(x)+1/x^2)*a*b^2*c^2*log(c)^2+64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))/(c^2*x^7-x^5), x)*log(c)^2-64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(c^2*x^2)/(c^2*x^7-x^5), x)*log(c)+128*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1))*log(x)/(c^2*x^7-x^5), x)*log(c)-64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7-x^5), x)*log(c)+128*a*b^2*c^2*integrate(1/16*x^2*log(x)/(c^2*x^7-x^5), x)*log(c)-64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(c^2*x^2)*log(x)/(c^2*x^7-x^5), x)+64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(x)^2/(c^2*x^7-x^5), x)-64*a*b^2*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))^2/(c^2*x^7-x^5), x)+16*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(c^2*x^2)/(c^2*x^7-x^5), x)+16*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)^2/(c^2*x^7-x^5), x)-64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)*log(x)/(c^2*x^7-x^5), x)+64*a*b^2*c^2*integrate(1/16*x^2*log(x)^2/(c^2*x^7-x^5), x)-(2*c^4*log(c*x+1)+2*c^4*log(c*x-1)-4*c^4*log(x)+(2*c^2*x^2+1)/x^4)*a*b^2*log(c)^2-64*b^3*integrate(1/16*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))/(c^2*x^7-x^5), x)*log(c)^2+64*b^3*integrate(1/16*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(c^2*x^2)/(c^2*x^7-x^5), x)*log(c)-128*b^3*integrate(1/16*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(x)/(c^2*x^7-x^5), x)*log(c)+64*a*b^2*integrate(1/16*log(c^2*x^2)/(c^2*x^7-x^5), x)*log(c)-128*a*b^2*integrate(1/16*log(x)/(c^2*x^7-x^5), x)*log(c)+16*b^3*integrate(1/16*sqrt(c*x+1)*sqrt(c*x-1)*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))^2/(c^2*x^7-x^5), x)-4*b^3*integrate(1/16*sqrt(c*x+1)*sqrt(c*x-1)*log(c^2*x^2)^2/(c^2*x^7-x^5), x)+64*b^3*integrate(1/16*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(c^2*x^2)*log(x)/(c^2*x^7-x^5), x)-64*b^3*integrate(1/16*arctan(1/(sqrt(c*x+1))*sqrt(c*x-1)))*log(x)^2/(c^2*x^7-x^5), x)+64*a*b^2*integrate(1/16*arctan(1/(sqrt$$

$(c*x + 1)*\sqrt{c*x - 1})^2/(c^2*x^7 - x^5), x) - 16*b^3*\text{integrate}(1/16*\text{arctan}(1/(\sqrt{c*x + 1})*\sqrt{c*x - 1}))*\log(c^2*x^2)/(c^2*x^7 - x^5), x) - 16*a*b^2*\text{integrate}(1/16*\log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*\text{integrate}(1/16*\log(c^2*x^2)*\log(x)/(c^2*x^7 - x^5), x) - 64*a*b^2*\text{integrate}(1/16*\log(x)^2/(c^2*x^7 - x^5), x))*x^4)/x^4$

**Fricas [A]** time = 2.38485, size = 512, normalized size = 2.46

$72 ab^2 c^2 x^2 + 8 (3 b^3 c^4 x^4 - 8 b^3) \operatorname{arccsc}(cx)^3 - 64 a^3 + 24 ab^2 + 24 (3 ab^2 c^4 x^4 - 8 ab^2) \operatorname{arccsc}(cx)^2 + 3 (3 (8 a^2 b - 5 b^3) c^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="fricas")`

[Out]  $1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*\operatorname{arccsc}(c*x)^3 - 64*a^3 + 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*\operatorname{arccsc}(c*x)^2 + 3*(3*(8*a^2*b - 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*\operatorname{arccsc}(c*x) - 3*(3*(8*a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*\operatorname{arccsc}(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*\operatorname{arccsc}(c*x))*\sqrt{c^2*x^2 - 1})/x^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))**3/x**5,x)`

[Out] `Integral((a + b*acsc(c*x))**3/x**5, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)^3/x^5, x)`

$$3.33 \quad \int \frac{x}{a+b \csc^{-1}(cx)} dx$$

**Optimal.** Leaf size=14

$$\text{Unintegrable}\left(\frac{x}{a+b \csc^{-1}(cx)}, x\right)$$

[Out] Unintegrable[x/(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.0150766, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x}{a+b \csc^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int][x/(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int \frac{x}{a+b \csc^{-1}(cx)} dx = \int \frac{x}{a+b \csc^{-1}(cx)} dx$$

**Mathematica [A]** time = 2.89571, size = 0, normalized size = 0.

$$\int \frac{x}{a+b \csc^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[x/(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 1.02, size = 0, normalized size = 0.

$$\int \frac{x}{a+b \operatorname{arccsc}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*arccsc(c\*x)), x)

[Out] int(x/(a+b\*arccsc(c\*x)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b \operatorname{arccsc}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate(x/(b\*arccsc(c\*x) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x}{b \operatorname{arccsc}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral(x/(b\*arccsc(c\*x) + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b \operatorname{acsc}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*acsc(c\*x)),x)

[Out] Integral(x/(a + b\*acsc(c\*x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b \operatorname{arccsc}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate(x/(b\*arccsc(c\*x) + a), x)



$$3.34 \quad \int \frac{1}{a+b \csc^{-1}(cx)} dx$$

**Optimal.** Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{a+b \csc^{-1}(cx)}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])^(-1), x]

**Rubi [A]** time = 0.005974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{a+b \csc^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])^(-1), x]

[Out] Defer[Int][(a + b\*ArcCsc[c\*x])^(-1), x]

Rubi steps

$$\int \frac{1}{a+b \csc^{-1}(cx)} dx = \int \frac{1}{a+b \csc^{-1}(cx)} dx$$

**Mathematica [A]** time = 3.36633, size = 0, normalized size = 0.

$$\int \frac{1}{a+b \csc^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])^(-1), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])^(-1), x]

**Maple [A]** time = 0.371, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx))^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccsc(c\*x)), x)

[Out] int(1/(a+b\*arccsc(c\*x)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arccsc}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate(1/(b\*arccsc(c\*x) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b \operatorname{arccsc}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*arccsc(c\*x) + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \operatorname{acsc}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acsc(c\*x)),x)

[Out] Integral(1/(a + b\*acsc(c\*x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arccsc}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate(1/(b\*arccsc(c\*x) + a), x)

$$3.35 \quad \int \frac{1}{x(a+b \csc^{-1}(cx))} dx$$

**Optimal.** Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x(a+b \csc^{-1}(cx))}, x\right)$$

[Out] Unintegrable[1/(x\*(a + b\*ArcCsc[c\*x])), x]

**Rubi [A]** time = 0.0244585, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{x(a+b \csc^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcCsc[c\*x])), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcCsc[c\*x])), x]

Rubi steps

$$\int \frac{1}{x(a+b \csc^{-1}(cx))} dx = \int \frac{1}{x(a+b \csc^{-1}(cx))} dx$$

**Mathematica [A]** time = 0.284892, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b \csc^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcCsc[c\*x])), x]

[Out] Integrate[1/(x\*(a + b\*ArcCsc[c\*x])), x]

**Maple [A]** time = 0.729, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b \operatorname{arccsc}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arccsc(c\*x)), x)

[Out] int(1/x/(a+b\*arccsc(c\*x)), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{bx \operatorname{arccsc}(cx) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x\*arccsc(c\*x) + a\*x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(a + b \operatorname{acsc}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*acsc(c\*x)),x)

[Out] Integral(1/(x\*(a + b\*acsc(c\*x))), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x), x)

$$3.36 \quad \int \frac{1}{x^2(a+b \csc^{-1}(cx))} dx$$

**Optimal.** Leaf size=47

$$-\frac{c \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{b} - \frac{c \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{b}$$

[Out] -((c\*cos[a/b]\*CosIntegral[a/b + ArcCsc[c\*x]])/b) - (c\*sin[a/b]\*SinIntegral[a/b + ArcCsc[c\*x]])/b

**Rubi [A]** time = 0.106107, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5223, 3303, 3299, 3302}

$$-\frac{c \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{b} - \frac{c \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*ArcCsc[c\*x])),x]

[Out] -((c\*cos[a/b]\*CosIntegral[a/b + ArcCsc[c\*x]])/b) - (c\*sin[a/b]\*SinIntegral[a/b + ArcCsc[c\*x]])/b

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.), x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

#### Rule 3302

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (a + b \csc^{-1}(cx))} dx &= - \left( c \operatorname{Subst} \left( \int \frac{\cos(x)}{a + bx} dx, x, \csc^{-1}(cx) \right) \right) \\ &= - \left( \left( c \cos \left( \frac{a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\cos \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \csc^{-1}(cx) \right) \right) - \left( c \sin \left( \frac{a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\sin \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \csc^{-1}(cx) \right) \\ &= - \frac{c \cos \left( \frac{a}{b} \right) \operatorname{Ci} \left( \frac{a}{b} + \csc^{-1}(cx) \right)}{b} - \frac{c \sin \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \csc^{-1}(cx) \right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0811525, size = 43, normalized size = 0.91

$$\frac{c \left( \cos \left( \frac{a}{b} \right) \operatorname{CosIntegral} \left( \frac{a}{b} + \csc^{-1}(cx) \right) + \sin \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \csc^{-1}(cx) \right) \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*ArcCsc[c\*x])),x]

[Out] -((c\*(Cos[a/b]\*CosIntegral[a/b + ArcCsc[c\*x]] + Sin[a/b]\*SinIntegral[a/b + ArcCsc[c\*x]]))/b)

**Maple [A]** time = 0.214, size = 48, normalized size = 1.

$$c \left( -\frac{1}{b} \operatorname{Si} \left( \frac{a}{b} + \operatorname{arccsc}(cx) \right) \sin \left( \frac{a}{b} \right) - \frac{1}{b} \operatorname{Ci} \left( \frac{a}{b} + \operatorname{arccsc}(cx) \right) \cos \left( \frac{a}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arccsc(c\*x)),x)

[Out] c\*(-Si(a/b+arccsc(c\*x))\*sin(a/b)/b-Ci(a/b+arccsc(c\*x))\*cos(a/b)/b)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{1}{bx^2 \operatorname{arccsc}(cx) + ax^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x^2\*arccsc(c\*x) + a\*x^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{arccsc}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*acsc(c\*x)),x)

[Out] Integral(1/(x\*\*2\*(a + b\*acsc(c\*x))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x^2), x)

$$3.37 \quad \int \frac{1}{x^3(a+b \csc^{-1}(cx))} dx$$

**Optimal.** Leaf size=63

$$\frac{c^2 \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b} + 2 \csc^{-1}(cx)\right)}{2b} - \frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \csc^{-1}(cx)\right)}{2b}$$

[Out] (c^2\*CosIntegral[(2\*a)/b + 2\*ArcCsc[c\*x]]\*Sin[(2\*a)/b])/(2\*b) - (c^2\*Cos[(2\*a)/b]\*SinIntegral[(2\*a)/b + 2\*ArcCsc[c\*x]])/(2\*b)

**Rubi [A]** time = 0.134288, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5223, 4406, 12, 3303, 3299, 3302}

$$\frac{c^2 \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b} + 2 \csc^{-1}(cx)\right)}{2b} - \frac{c^2 \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b} + 2 \csc^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*ArcCsc[c\*x])),x]

[Out] (c^2\*CosIntegral[(2\*a)/b + 2\*ArcCsc[c\*x]]\*Sin[(2\*a)/b])/(2\*b) - (c^2\*Cos[(2\*a)/b]\*SinIntegral[(2\*a)/b + 2\*ArcCsc[c\*x]])/(2\*b)

#### Rule 5223

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(x\_)^m\_., x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csc[x]^(m + 1)\*Cot[x], x], x, ArcCsc[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rule 4406

Int[Cos[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[a + b\*x]^(n)\*Cos[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3299

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]



Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (a + b \csc^{-1}(cx))} dx &= - \left( c^2 \operatorname{Subst} \left( \int \frac{\cos(x) \sin(x)}{a + bx} dx, x, \csc^{-1}(cx) \right) \right) \\ &= - \left( c^2 \operatorname{Subst} \left( \int \frac{\sin(2x)}{2(a + bx)} dx, x, \csc^{-1}(cx) \right) \right) \\ &= - \left( \frac{1}{2} c^2 \operatorname{Subst} \left( \int \frac{\sin(2x)}{a + bx} dx, x, \csc^{-1}(cx) \right) \right) \\ &= - \left( \frac{1}{2} \left( c^2 \cos \left( \frac{2a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\sin \left( \frac{2a}{b} + 2x \right)}{a + bx} dx, x, \csc^{-1}(cx) \right) \right) + \frac{1}{2} \left( c^2 \sin \left( \frac{2a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{1}{a + bx} dx, x, \csc^{-1}(cx) \right) \\ &= \frac{c^2 \operatorname{Ci} \left( \frac{2a}{b} + 2 \csc^{-1}(cx) \right) \sin \left( \frac{2a}{b} \right)}{2b} - \frac{c^2 \cos \left( \frac{2a}{b} \right) \operatorname{Si} \left( \frac{2a}{b} + 2 \csc^{-1}(cx) \right)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.0767995, size = 56, normalized size = 0.89

$$\frac{c^2 \left( \cos \left( \frac{2a}{b} \right) \operatorname{Si} \left( \frac{2a}{b} + 2 \csc^{-1}(cx) \right) - \sin \left( \frac{2a}{b} \right) \operatorname{CosIntegral} \left( \frac{2a}{b} + 2 \csc^{-1}(cx) \right) \right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcCsc[c*x])), x]
```

```
[Out] -(c^2*(-(CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]]))/(2*b)
```

**Maple [A]** time = 0.214, size = 58, normalized size = 0.9

$$c^2 \left( -\frac{1}{2b} \operatorname{Si} \left( 2 \frac{a}{b} + 2 \operatorname{arccsc}(cx) \right) \cos \left( 2 \frac{a}{b} \right) + \frac{1}{2b} \operatorname{Ci} \left( 2 \frac{a}{b} + 2 \operatorname{arccsc}(cx) \right) \sin \left( 2 \frac{a}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arccsc(c*x)), x)
```

```
[Out] c^2*(-1/2*Si(2*a/b+2*arccsc(c*x))*cos(2*a/b)/b+1/2*Ci(2*a/b+2*arccsc(c*x))*sin(2*a/b)/b)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^3 \arccsc(cx) + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x^3\*arccsc(c\*x) + a\*x^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \operatorname{acsc}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(a+b\*acsc(c\*x)),x)

[Out] Integral(1/(x\*\*3\*(a + b\*acsc(c\*x))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x^3), x)

$$3.38 \quad \int \frac{1}{x^4(a+b \csc^{-1}(cx))} dx$$

**Optimal.** Leaf size=117

$$\frac{c^3 \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{4b} + \frac{c^3 \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b} + 3 \csc^{-1}(cx)\right)}{4b} - \frac{c^3 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{4b}$$

[Out]  $-(c^3 \cos[a/b] \text{CosIntegral}[a/b + \text{ArcCsc}[c*x]])/(4*b) + (c^3 \cos[(3*a)/b] \text{CosIntegral}[(3*a)/b + 3*\text{ArcCsc}[c*x]])/(4*b) - (c^3 \sin[a/b] \text{SinIntegral}[a/b + \text{ArcCsc}[c*x]])/(4*b) + (c^3 \sin[(3*a)/b] \text{SinIntegral}[(3*a)/b + 3*\text{ArcCsc}[c*x]])/(4*b)$

**Rubi [A]** time = 0.228755, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5223, 4406, 3303, 3299, 3302}

$$\frac{c^3 \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{4b} + \frac{c^3 \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b} + 3 \csc^{-1}(cx)\right)}{4b} - \frac{c^3 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \csc^{-1}(cx)\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*(a + b*\text{ArcCsc}[c*x])), x]$

[Out]  $-(c^3 \cos[a/b] \text{CosIntegral}[a/b + \text{ArcCsc}[c*x]])/(4*b) + (c^3 \cos[(3*a)/b] \text{CosIntegral}[(3*a)/b + 3*\text{ArcCsc}[c*x]])/(4*b) - (c^3 \sin[a/b] \text{SinIntegral}[a/b + \text{ArcCsc}[c*x]])/(4*b) + (c^3 \sin[(3*a)/b] \text{SinIntegral}[(3*a)/b + 3*\text{ArcCsc}[c*x]])/(4*b)$

#### Rule 5223

$\text{Int}[(a_. + \text{ArcCsc}[(c_.)*(x_.)]*(b_.))^{(n_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow -\text{Dist}[(c^{(m+1)})^{(-1)}, \text{Subst}[\text{Int}[(a + b*x)^n * \text{Csc}[x]^{(m+1)} * \text{Cot}[x], x], x, \text{ArcCsc}[c*x]], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{LtQ}[m, -1])$

#### Rule 4406

$\text{Int}[\text{Cos}[(a_. + (b_.)*(x_.))^{(p_.)}*((c_. + (d_.)*(x_.))^{(m_.)}*\text{Sin}[(a_. + (b_.)*(x_.))^{(n_.)}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^{(n)}*\text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 3303

$\text{Int}[\text{sin}[(e_. + (f_.)*(x_.)]/((c_. + (d_.)*(x_.))), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

#### Rule 3299

$\text{Int}[\text{sin}[(e_. + (f_.)*(x_.)]/((c_. + (d_.)*(x_.))), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 (a + b \operatorname{csc}^{-1}(cx))} dx &= - \left( c^3 \operatorname{Subst} \left( \int \frac{\cos(x) \sin^2(x)}{a + bx} dx, x, \operatorname{csc}^{-1}(cx) \right) \right) \\ &= - \left( c^3 \operatorname{Subst} \left( \int \left( \frac{\cos(x)}{4(a + bx)} - \frac{\cos(3x)}{4(a + bx)} \right) dx, x, \operatorname{csc}^{-1}(cx) \right) \right) \\ &= - \left( \frac{1}{4} c^3 \operatorname{Subst} \left( \int \frac{\cos(x)}{a + bx} dx, x, \operatorname{csc}^{-1}(cx) \right) \right) + \frac{1}{4} c^3 \operatorname{Subst} \left( \int \frac{\cos(3x)}{a + bx} dx, x, \operatorname{csc}^{-1}(cx) \right) \\ &= - \left( \frac{1}{4} \left( c^3 \cos \left( \frac{a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\cos \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \operatorname{csc}^{-1}(cx) \right) \right) + \frac{1}{4} \left( c^3 \cos \left( \frac{3a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\cos \left( \frac{3a}{b} + x \right)}{a + bx} dx, x, \operatorname{csc}^{-1}(cx) \right) \\ &= - \frac{c^3 \cos \left( \frac{a}{b} \right) \operatorname{Ci} \left( \frac{a}{b} + \operatorname{csc}^{-1}(cx) \right)}{4b} + \frac{c^3 \cos \left( \frac{3a}{b} \right) \operatorname{Ci} \left( \frac{3a}{b} + 3 \operatorname{csc}^{-1}(cx) \right)}{4b} - \frac{c^3 \sin \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \operatorname{csc}^{-1}(cx) \right)}{4b} \end{aligned}$$

**Mathematica [A]** time = 0.171639, size = 91, normalized size = 0.78

$$\frac{c^3 \left( \cos \left( \frac{a}{b} \right) \operatorname{CosIntegral} \left( \frac{a}{b} + \operatorname{csc}^{-1}(cx) \right) - \cos \left( \frac{3a}{b} \right) \operatorname{CosIntegral} \left( 3 \left( \frac{a}{b} + \operatorname{csc}^{-1}(cx) \right) \right) + \sin \left( \frac{a}{b} \right) \operatorname{Si} \left( \frac{a}{b} + \operatorname{csc}^{-1}(cx) \right) - \sin \left( \frac{3a}{b} \right) \operatorname{Si} \left( 3 \left( \frac{a}{b} + \operatorname{csc}^{-1}(cx) \right) \right) \right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -(c^3*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] - Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCsc[c*x])] + Sin[a/b]*SinIntegral[a/b + ArcCsc[c*x]] - Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCsc[c*x])]))/(4*b)
```

**Maple [A]** time = 0.218, size = 102, normalized size = 0.9

$$c^3 \left( -\frac{1}{4b} \operatorname{Si} \left( \frac{a}{b} + \operatorname{arccsc}(cx) \right) \sin \left( \frac{a}{b} \right) - \frac{1}{4b} \operatorname{Ci} \left( \frac{a}{b} + \operatorname{arccsc}(cx) \right) \cos \left( \frac{a}{b} \right) + \frac{1}{4b} \operatorname{Si} \left( 3 \frac{a}{b} + 3 \operatorname{arccsc}(cx) \right) \sin \left( 3 \frac{a}{b} \right) + \frac{1}{4b} \operatorname{Ci} \left( 3 \frac{a}{b} + 3 \operatorname{arccsc}(cx) \right) \cos \left( 3 \frac{a}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arccsc(c*x)),x)
```

```
[Out] c^3*(-1/4*Si(a/b+arccsc(c*x))*sin(a/b)/b-1/4*Ci(a/b+arccsc(c*x))*cos(a/b)/b+1/4*Si(3*a/b+3*arccsc(c*x))*sin(3*a/b)/b+1/4*Ci(3*a/b+3*arccsc(c*x))*cos(3*a/b)/b)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^4 \arccsc(cx) + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x^4\*arccsc(c\*x) + a\*x^4), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4(a + b \arccsc(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(a+b\*arccsc(c\*x)),x)

[Out] Integral(1/(x\*\*4\*(a + b\*arccsc(c\*x))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \arccsc(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate(1/((b\*arccsc(c\*x) + a)\*x^4), x)

$$\mathbf{3.39} \quad \int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^3 dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable} \left( (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^3, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^3, x]

**Rubi [A]** time = 0.0235343, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^3, x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcCsc[c\*x])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^3 dx = \int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^3 dx$$

**Mathematica [A]** time = 5.50281, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^3, x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^3, x]

**Maple [A]** time = 2.495, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{arccsc}(cx) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arccsc(c\*x))^3, x)

[Out] int((d\*x)^m\*(a+b\*arccsc(c\*x))^3, x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsc(c\*x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{arccsc}(cx)^3 + 3ab^2 \operatorname{arccsc}(cx)^2 + 3a^2b \operatorname{arccsc}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsc(c\*x))^3,x, algorithm="fricas")

[Out] integral((b^3\*arccsc(c\*x)^3 + 3\*a\*b^2\*arccsc(c\*x)^2 + 3\*a^2\*b\*arccsc(c\*x) + a^3)\*(d\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*acsc(c\*x))\*\*3,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsc(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^3\*(d\*x)^m, x)

### 3.40 $\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^2, x\right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^2, x]

**Rubi [A]** time = 0.024169, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^2,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcCsc[c\*x])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^2 dx = \int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^2 dx$$

**Mathematica [A]** time = 3.58173, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^2,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcCsc[c\*x])^2, x]

**Maple [A]** time = 2.259, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{arccsc}(cx) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arccsc(c\*x))^2,x)

[Out] int((d\*x)^m\*(a+b\*arccsc(c\*x))^2,x)



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsc(c\*x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (b^2 \operatorname{arccsc}(cx)^2 + 2ab \operatorname{arccsc}(cx) + a^2) (dx)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsc(c\*x))^2,x, algorithm="fricas")

[Out] integral((b^2\*arccsc(c\*x)^2 + 2\*a\*b\*arccsc(c\*x) + a^2)\*(d\*x)^m, x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{arccsc}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*arccsc(c\*x))\*\*2,x)

[Out] Integral((d\*x)\*\*m\*(a + b\*arccsc(c\*x))\*\*2, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsc(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)^2\*(d\*x)^m, x)

### 3.41 $\int (dx)^m \left( a + b \operatorname{csc}^{-1}(cx) \right) dx$

**Optimal.** Leaf size=66

$$\frac{(dx)^{m+1} \left( a + b \operatorname{csc}^{-1}(cx) \right)}{d(m+1)} + \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; \frac{1}{c^2 x^2}\right)}{cm(m+1)}$$

[Out] ((d\*x)^(1 + m)\*(a + b\*ArcCsc[c\*x]))/(d\*(1 + m)) + (b\*(d\*x)^m\*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(c^2\*x^2)])/(c\*m\*(1 + m))

**Rubi [A]** time = 0.043359, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5221, 339, 364}

$$\frac{(dx)^{m+1} \left( a + b \operatorname{csc}^{-1}(cx) \right)}{d(m+1)} + \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; \frac{1}{c^2 x^2}\right)}{cm(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*ArcCsc[c\*x]),x]

[Out] ((d\*x)^(1 + m)\*(a + b\*ArcCsc[c\*x]))/(d\*(1 + m)) + (b\*(d\*x)^m\*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(c^2\*x^2)])/(c\*m\*(1 + m))

#### Rule 5221

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 - 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rule 339

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Dist[((c\*x)^(m + 1)\*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/ (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \operatorname{csc}^{-1}(cx)) dx &= \frac{(dx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{d(1+m)} + \frac{(bd) \int \frac{(dx)^{-1+m}}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{c(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{d(1+m)} - \frac{\left(b \left(\frac{1}{x}\right)^m (dx)^m\right) \operatorname{Subst} \left( \int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{c(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{d(1+m)} + \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; \frac{1}{c^2x^2}\right)}{cm(1+m)} \end{aligned}$$

**Mathematica [A]** time = 0.172998, size = 83, normalized size = 1.26

$$\frac{(dx)^m \left( (m+1)x (a + b \operatorname{csc}^{-1}(cx)) + \frac{b\sqrt{1-c^2x^2} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; c^2x^2\right)}{c\sqrt{1-\frac{1}{c^2x^2}}} \right)}{(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcCsc[c\*x]), x]

[Out] ((d\*x)^m\*((1+m)\*x\*(a + b\*ArcCsc[c\*x]) + (b\*sqrt[1 - c^2\*x^2]\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/(c\*sqrt[1 - 1/(c^2\*x^2)])))/(1+m)^2

**Maple [F]** time = 2.089, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arccsc(c\*x)), x)

[Out] int((d\*x)^m\*(a+b\*arccsc(c\*x)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsc(c\*x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b \operatorname{arccsc}(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*(d*x)^m, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*acsc(c*x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccsc}(cx) + a) (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(d*x)^m, x)
```

$$3.42 \quad \int \frac{(dx)^m}{a+b \csc^{-1}(cx)} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b \csc^{-1}(cx)}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.0257448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{a+b \csc^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b \csc^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b \csc^{-1}(cx)} dx$$

**Mathematica [A]** time = 0.390922, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b \csc^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 1.878, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b \operatorname{arccsc}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arccsc(c\*x)), x)

[Out] int((d\*x)^m/(a+b\*arccsc(c\*x)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{arccsc}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*arccsc(c\*x) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b \operatorname{arccsc}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral((d\*x)^m/(b\*arccsc(c\*x) + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{acsc}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*acsc(c\*x)),x)

[Out] Integral((d\*x)\*\*m/(a + b\*acsc(c\*x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{arccsc}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arccsc(c\*x) + a), x)

$$3.43 \quad \int \frac{(dx)^m}{(a+b \csc^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable} \left( \frac{(dx)^m}{(a+b \csc^{-1}(cx))^2}, x \right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcCsc[c\*x])^2, x]

**Rubi [A]** time = 0.0255385, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{(a+b \csc^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcCsc[c\*x])^2, x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcCsc[c\*x])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{(a+b \csc^{-1}(cx))^2} dx = \int \frac{(dx)^m}{(a+b \csc^{-1}(cx))^2} dx$$

**Mathematica [A]** time = 0.777572, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a+b \csc^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcCsc[c\*x])^2, x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcCsc[c\*x])^2, x]

**Maple [A]** time = 1.842, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a+b \operatorname{arccsc}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arccsc(c\*x))^2, x)

[Out]  $\text{int}((d*x)^m/(a+b*\text{arccsc}(c*x))^2,x)$

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^m/(a+b*\text{arccsc}(c*x))^2,x, \text{algorithm}="maxima")$

[Out]  $(4*\sqrt{c*x + 1}*\sqrt{c*x - 1}*(b*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})) + a)*d^m*x^m - (4*b^3*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})^2 + b^3*\log(c^2*x^2)^2 + 4*b^3*\log(c)^2 + 8*b^3*\log(c)*\log(x) + 4*b^3*\log(x)^2 + 8*a*b^2*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}) + 4*a^2*b - 4*(b^3*\log(c) + b^3*\log(x))*\log(c^2*x^2))*\text{integrate}(4*((b*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})) + a)*d^m*m - ((b*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})) + a)*c^2*d^m*m + 2*(b*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})) + a)*c^2*d^m)*x^2 + (b*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})) + a)*d^m)*\sqrt{c*x + 1}*\sqrt{c*x - 1}*x^m / (4*b^3*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})^2 + 4*b^3*\log(c)^2 + 8*a*b^2*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}) + 4*a^2*b - 4*(b^3*c^2*\log(c)^2 + (b^3*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}))^2 + 2*a*b^2*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}) + a^2*b)*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*\log(c^2*x^2)^2 - 4*(b^3*c^2*x^2 - b^3)*\log(x)^2 + 4*(b^3*c^2*x^2*\log(c) - b^3*\log(c) + (b^3*c^2*x^2 - b^3)*\log(x))*\log(c^2*x^2) - 8*(b^3*c^2*x^2*\log(c) - b^3*\log(c))*\log(x)), x) / (4*b^3*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})^2 + b^3*\log(c^2*x^2)^2 + 4*b^3*\log(c)^2 + 8*b^3*\log(c)*\log(x) + 4*b^3*\log(x)^2 + 8*a*b^2*\arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}) + 4*a^2*b - 4*(b^3*\log(c) + b^3*\log(x))*\log(c^2*x^2))$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^2 \text{arccsc}(cx)^2 + 2ab \text{arccsc}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^m/(a+b*\text{arccsc}(c*x))^2,x, \text{algorithm}="fricas")$

[Out]  $\text{integral}((d*x)^m/(b^2*\text{arccsc}(c*x)^2 + 2*a*b*\text{arccsc}(c*x) + a^2), x)$

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \text{acsc}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)**m/(a+b*\text{acsc}(c*x))**2,x)$

[Out]  $\text{Integral}((d*x)**m/(a + b*\text{acsc}(c*x))**2, x)$



---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \operatorname{arccsc}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsc(c\*x))^2,x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arccsc(c\*x) + a)^2, x)

### 3.44 $\int (d + ex)^3 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=167

$$\frac{(d + ex)^4 (a + b \csc^{-1}(cx))}{4e} + \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}} (9c^2 d^2 + e^2)}{6c^3} + \frac{bd (2c^2 d^2 + e^2) \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{2c^3} + \frac{bde^2 x^2 \sqrt{1 - \frac{1}{c^2 x^2}}}{2c} + \frac{be^3 x^3}{6c^3}$$

[Out] (b\*e\*(9\*c^2\*d^2 + e^2)\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(6\*c^3) + (b\*d\*e^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(2\*c) + (b\*e^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3)/(12\*c) - (b\*d^4\*ArcCsc[c\*x])/(4\*e) + ((d + e\*x)^4\*(a + b\*ArcCsc[c\*x]))/(4\*e) + (b\*d\*(2\*c^2\*d^2 + e^2)\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(2\*c^3)

**Rubi [A]** time = 0.401839, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {5227, 1568, 1475, 1807, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^4 (a + b \csc^{-1}(cx))}{4e} + \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}} (9c^2 d^2 + e^2)}{6c^3} + \frac{bd (2c^2 d^2 + e^2) \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{2c^3} + \frac{bde^2 x^2 \sqrt{1 - \frac{1}{c^2 x^2}}}{2c} + \frac{be^3 x^3}{6c^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*e\*(9\*c^2\*d^2 + e^2)\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(6\*c^3) + (b\*d\*e^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(2\*c) + (b\*e^3\*Sqrt[1 - 1/(c^2\*x^2)]\*x^3)/(12\*c) - (b\*d^4\*ArcCsc[c\*x])/(4\*e) + ((d + e\*x)^4\*(a + b\*ArcCsc[c\*x]))/(4\*e) + (b\*d\*(2\*c^2\*d^2 + e^2)\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(2\*c^3)

#### Rule 5227

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(e\*(m + 1)), x] + Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 1568

Int[(x\_)^(m\_.)\*((d\_) + (e\_.)\*(x\_)^(mn\_.))^(q\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.))^(p\_.), x\_Symbol] := Int[x^(m + mn\*q)\*(e + d/x^mn)^q\*(a + c\*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2\*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])

#### Rule 1475

Int[(x\_)^(m\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(d + e\*x)^q\*(a + c\*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2\*n] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 1807

Int[(Pq)\*((c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, c\*x, x], R = PolynomialRemainder[Pq, c\*x, x]}, Simp[(R\*(c\*x)^(m + 1)\*(a + b\*x^2)^(p + 1))/(a\*c\*(m + 1)), x] + Dist[1/(a\*c\*(m + 1)), Int[(c\*x)^(m + 1)\*(a + b\*x^2)^p\*ExpandToSum[a\*c\*(m + 1)\*Q - b\*R\*(m

+ 2\*p + 3)\*x, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ[m, -1] && (IntegerQ[2\*p] || NeQ[Expon[Pq, x], 1])

#### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps

$$\begin{aligned}
\int (d+ex)^3 (a+b \csc^{-1}(cx)) dx &= \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} + \frac{b \int \frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{4ce} \\
&= \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} + \frac{b \int \frac{\left(\frac{e+d}{x}\right)^4 x^2}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{4ce} \\
&= \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} - \frac{b \operatorname{Subst}\left(\int \frac{(e+dx)^4}{x^4 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{4ce} \\
&= \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} + \frac{b \operatorname{Subst}\left(\int \frac{-12de^3-2e^2\left(9d^2+\frac{e^2}{c^2}\right)x-12d^3ex^2-}{x^3 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{12ce} \\
&= \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} + \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} - \frac{b \operatorname{Subst}\left(\int \frac{4e^2\left(9d^2+\frac{e^2}{c^2}\right)x-12d^3ex^2-}{x^3 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{12ce} \\
&= \frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} + \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} \\
&= \frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} + \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} + \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} \\
&= \frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} + \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} - \frac{bd^4 \csc^{-1}(cx)}{4e} + \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} \\
&= \frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} + \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} - \frac{bd^4 \csc^{-1}(cx)}{4e} + \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e} \\
&= \frac{be(9c^2d^2+e^2) \sqrt{1-\frac{1}{c^2x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{2c} + \frac{be^3 \sqrt{1-\frac{1}{c^2x^2}} x^3}{12c} - \frac{bd^4 \csc^{-1}(cx)}{4e} + \frac{(d+ex)^4 (a+b \csc^{-1}(cx))}{4e}
\end{aligned}$$

**Mathematica [A]** time = 0.305323, size = 165, normalized size = 0.99

$$\frac{3ac^3x(6d^2ex + 4d^3 + 4de^2x^2 + e^3x^3) + bex\sqrt{1-\frac{1}{c^2x^2}}(c^2(18d^2 + 6dex + e^2x^2) + 2e^2) + 6bd(2c^2d^2 + e^2)\log\left(x\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{12c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(a + b\*ArcCsc[c\*x]),x]

[Out] (3\*a\*c^3\*x\*(4\*d^3 + 6\*d^2\*e\*x + 4\*d\*e^2\*x^2 + e^3\*x^3) + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(2\*e^2 + c^2\*(18\*d^2 + 6\*d\*e\*x + e^2\*x^2)) + 3\*b\*c^3\*x\*(4\*d^3 + 6\*d^2\*e\*x + 4\*d\*e^2\*x^2 + e^3\*x^3)\*ArcCsc[c\*x] + 6\*b\*d\*(2\*c^2\*d^2 + e^2)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x])/(12\*c^3)

**Maple [B]** time = 0.138, size = 485, normalized size = 2.9

$$\frac{ae^3x^4}{4} + ae^2x^3d + \frac{3aex^2d^2}{2} + axd^3 + \frac{ad^4}{4e} + \frac{be^3\operatorname{arccsc}(cx)x^4}{4} + be^2\operatorname{arccsc}(cx)x^3d + \frac{3be\operatorname{arccsc}(cx)x^2d^2}{2} + b\operatorname{arccsc}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(a+b\*arccsc(c\*x)), x)

[Out]  $\frac{1}{4}ae^3x^4 + ae^2x^3d + \frac{3}{2}aex^2d^2 + axd^3 + \frac{1}{4}a/e*d^4 + \frac{1}{4}b*e^3*\operatorname{arccsc}(c*x)*x^4 + b*e^2*\operatorname{arccsc}(c*x)*x^3*d + \frac{3}{2}b*e*\operatorname{arccsc}(c*x)*x^2*d^2 + b*\operatorname{arccsc}(c*x)*x*d^3 + \frac{1}{4}b*d^4*\operatorname{arccsc}(c*x)/e - \frac{1}{4}c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d^4*\operatorname{arctan}(1/(c^2*x^2-1)^{(1/2)}) + 1/c^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d^3*\ln(c*x+(c^2*x^2-1)^{(1/2)}) + 1/12/c*b*e^3/((c^2*x^2-1)/c^2/x^2)^{(1/2)*x^3} + 1/12/c^3*b*e^3/((c^2*x^2-1)/c^2/x^2)^{(1/2)*x} + 1/2/c*b*e^2/((c^2*x^2-1)/c^2/x^2)^{(1/2)*d*x^2} - 1/2/c^3*b*e^2/((c^2*x^2-1)/c^2/x^2)^{(1/2)*d} + 3/2/c*b*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)*x*d^2} - 3/2/c^3*b*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)*x*d} + 1/2/c^4*b*e^2*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d*\ln(c*x+(c^2*x^2-1)^{(1/2)}) - 1/6/c^5*b*e^3/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x$

**Maxima [A]** time = 1.04641, size = 363, normalized size = 2.17

$$\frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2\operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2e + \frac{1}{4}\left(4x^3\operatorname{arccsc}(cx) + \frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(a+b\*arccsc(c\*x)), x, algorithm="maxima")

[Out]  $\frac{1}{4}ae^3x^4 + a*d*e^2*x^3 + \frac{3}{2}a*d^2*e*x^2 + \frac{3}{2}*(x^2*\operatorname{arccsc}(c*x) + x*\operatorname{sqrt}(-1/(c^2*x^2) + 1)/c)*b*d^2*e + \frac{1}{4}*(4*x^3*\operatorname{arccsc}(c*x) + (2*\operatorname{sqrt}(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + \log(\operatorname{sqrt}(-1/(c^2*x^2) + 1) + 1)/c^2 - \log(\operatorname{sqrt}(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + \frac{1}{12}*(3*x^4*\operatorname{arccsc}(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^{(3/2)} + 3*x*\operatorname{sqrt}(-1/(c^2*x^2) + 1))/c^3)*b*e^3 + a*d^3*x + \frac{1}{2}*(2*c*x*\operatorname{arccsc}(c*x) + \log(\operatorname{sqrt}(-1/(c^2*x^2) + 1) + 1) - \log(-\operatorname{sqrt}(-1/(c^2*x^2) + 1) + 1))*b*d^3/c$

**Fricas [A]** time = 3.76748, size = 633, normalized size = 3.79

$$3ac^4e^3x^4 + 12ac^4de^2x^3 + 18ac^4d^2ex^2 + 12ac^4d^3x + 3(bc^4e^3x^4 + 4bc^4de^2x^3 + 6bc^4d^2ex^2 + 4bc^4d^3x - 4bc^4d^3 - 6bc^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(a+b\*arccsc(c\*x)), x, algorithm="fricas")

[Out]  $\frac{1}{12}*(3*a*c^4*e^3*x^4 + 12*a*c^4*d*e^2*x^3 + 18*a*c^4*d^2*e*x^2 + 12*a*c^4*d^3*x + 3*(b*c^4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^4*d^3*x - 4*b*c^4*d^3 - 6*b*c^4))$

$$\frac{d^3 x - 4bc^4 d^3 - 6bc^4 d^2 e - 4bc^4 d e^2 - bc^4 e^3}{c^4} \operatorname{arccsc}(cx) - 6(4bc^4 d^3 + 6bc^4 d^2 e + 4bc^4 d e^2 + bc^4 e^3) \operatorname{arctan}\left(\frac{-cx + \sqrt{c^2 x^2 - 1}}{c^2 x^2 - 1}\right) - 6(2bc^3 d^3 + bc^2 d e^2) \log(-cx + \sqrt{c^2 x^2 - 1}) + (bc^2 e^3 x^2 + 6bc^2 d e^2 x + 18bc^2 d^2 e + 2b e^3) \sqrt{c^2 x^2 - 1}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx)) (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(a+b\*arccsc(c\*x)),x)

[Out] Integral((a + b\*arccsc(c\*x))\*(d + e\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (b \operatorname{arccsc}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*arccsc(c\*x) + a), x)

### 3.45 $\int (d + ex)^2 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=123

$$\frac{(d + ex)^3 (a + b \csc^{-1}(cx))}{3e} + \frac{b(6c^2d^2 + e^2) \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c^3} + \frac{bdex\sqrt{1 - \frac{1}{c^2x^2}}}{c} + \frac{be^2x^2\sqrt{1 - \frac{1}{c^2x^2}}}{6c} - \frac{bd^3 \csc^{-1}(cx)}{3e}$$

[Out] (b\*d\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/c + (b\*e^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(6\*c) - (b\*d^3\*ArcCsc[c\*x])/(3\*e) + ((d + e\*x)^3\*(a + b\*ArcCsc[c\*x]))/(3\*e) + (b\*(6\*c^2\*d^2 + e^2)\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(6\*c^3)

**Rubi [A]** time = 0.276766, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {5227, 1568, 1475, 1807, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^3 (a + b \csc^{-1}(cx))}{3e} + \frac{b(6c^2d^2 + e^2) \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c^3} + \frac{bdex\sqrt{1 - \frac{1}{c^2x^2}}}{c} + \frac{be^2x^2\sqrt{1 - \frac{1}{c^2x^2}}}{6c} - \frac{bd^3 \csc^{-1}(cx)}{3e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*d\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/c + (b\*e^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x^2)/(6\*c) - (b\*d^3\*ArcCsc[c\*x])/(3\*e) + ((d + e\*x)^3\*(a + b\*ArcCsc[c\*x]))/(3\*e) + (b\*(6\*c^2\*d^2 + e^2)\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(6\*c^3)

#### Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] || !IntegerQ[p])
```

#### Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 1807

```
Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps



$$\begin{aligned}
\int (d+ex)^2 (a+b \operatorname{csc}^{-1}(cx)) dx &= \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} + \frac{b \int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{3ce} \\
&= \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} + \frac{b \int \frac{\left(\frac{e+d}{x}\right)^3 x}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{3ce} \\
&= \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} - \frac{b \operatorname{Subst} \left( \int \frac{(e+dx)^3}{x^3 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{3ce} \\
&= \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} + \frac{b \operatorname{Subst} \left( \int \frac{-6de^2 - e \left(6d^2 + \frac{e^2}{c^2}\right) x - 2d^3 x^2}{x^2 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{6ce} \\
&= \frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} - \frac{b \operatorname{Subst} \left( \int \frac{e \left(6d^2 + \frac{e^2}{c^2}\right) x}{x^2 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{6ce} \\
&= \frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} - \frac{(bd^3) \operatorname{Subst} \left( \int \frac{e \left(6d^2 + \frac{e^2}{c^2}\right) x}{x^2 \sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{3ce} \\
&= \frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} - \frac{bd^3 \operatorname{csc}^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} \\
&= \frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} - \frac{bd^3 \operatorname{csc}^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e} \\
&= \frac{bde \sqrt{1-\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}} x^2}{6c} - \frac{bd^3 \operatorname{csc}^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b \operatorname{csc}^{-1}(cx))}{3e}
\end{aligned}$$

**Mathematica [A]** time = 0.185477, size = 122, normalized size = 0.99

$$\frac{c^2 x \left( 2ac(3d^2 + 3dex + e^2 x^2) + be \sqrt{1 - \frac{1}{c^2 x^2}} (6d + ex) \right) + b(6c^2 d^2 + e^2) \log \left( x \left( \sqrt{1 - \frac{1}{c^2 x^2}} + 1 \right) \right) + 2bc^3 x \operatorname{csc}^{-1}(cx) (3d^2 + 3dex + e^2 x^2)}{6c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(a + b\*ArcCsc[c\*x]),x]

[Out] (c^2\*x\*(b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*(6\*d + e\*x) + 2\*a\*c\*(3\*d^2 + 3\*d\*e\*x + e^2\*x^2)) + 2\*b\*c^3\*x\*(3\*d^2 + 3\*d\*e\*x + e^2\*x^2)\*ArcCsc[c\*x] + b\*(6\*c^2\*d^2 + 3\*d\*e\*x + e^2)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x])/(6\*c^3)

**Maple [B]** time = 0.141, size = 361, normalized size = 2.9

$$\frac{ae^2x^3}{3} + ax^2de + axd^2 + \frac{ad^3}{3e} + \frac{be^2 \operatorname{arccsc}(cx) x^3}{3} + b \operatorname{arccsc}(cx) x^2 d + b \operatorname{arccsc}(cx) x d^2 + \frac{bd^3 \operatorname{arccsc}(cx)}{3e} - \frac{bd^3}{3ecx} \sqrt{c^2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/3*a*e^2*x^3+a*x^2*d*e+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arccsc(c*x)*x^3+b*e*a
rccsc(c*x)*x^2*d+b*arccsc(c*x)*x*d^2+1/3*b*d^3*arccsc(c*x)/e-1/3/c*b/e*(c^2
*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))
+1/c^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^
2-1)^(1/2))+1/6/c*b*e^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-1/6/c^3*b*e^2/((c^2
*x^2-1)/c^2/x^2)^(1/2)+1/c*b*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d-1/c^3*b*e/((
c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+1/6/c^4*b*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

**Maxima [A]** time = 1.00296, size = 267, normalized size = 2.17

$$\frac{1}{3}ae^2x^3 + adex^2 + \left( x^2 \operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c} \right) bde + \frac{1}{12} \left( 4x^3 \operatorname{arccsc}(cx) + \frac{2\sqrt{-\frac{1}{c^2x^2} + 1}}{c^2\left(\frac{1}{c^2x^2} - 1\right) + c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} - 1} + 1\right)}{c^2} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*
b*d*e + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^
2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2
) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/
(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c
```

**Fricas [A]** time = 2.87185, size = 462, normalized size = 3.76

$$\frac{2ac^3e^2x^3 + 6ac^3dex^2 + 6ac^3d^2x + 2(bc^3e^2x^3 + 3bc^3dex^2 + 3bc^3d^2x - 3bc^3d^2 - 3bc^3de - bc^3e^2) \operatorname{arccsc}(cx) - 4(3bc^3d^2 - 6bc^3d^2 + 6bc^3d^2e + 6bc^3d^2e^2 + 6bc^3d^2e^3 + 6bc^3d^2e^4 + 6bc^3d^2e^5 + 6bc^3d^2e^6 + 6bc^3d^2e^7 + 6bc^3d^2e^8 + 6bc^3d^2e^9 + 6bc^3d^2e^{10} + 6bc^3d^2e^{11} + 6bc^3d^2e^{12} + 6bc^3d^2e^{13} + 6bc^3d^2e^{14} + 6bc^3d^2e^{15} + 6bc^3d^2e^{16} + 6bc^3d^2e^{17} + 6bc^3d^2e^{18} + 6bc^3d^2e^{19} + 6bc^3d^2e^{20} + 6bc^3d^2e^{21} + 6bc^3d^2e^{22} + 6bc^3d^2e^{23} + 6bc^3d^2e^{24} + 6bc^3d^2e^{25} + 6bc^3d^2e^{26} + 6bc^3d^2e^{27} + 6bc^3d^2e^{28} + 6bc^3d^2e^{29} + 6bc^3d^2e^{30} + 6bc^3d^2e^{31} + 6bc^3d^2e^{32} + 6bc^3d^2e^{33} + 6bc^3d^2e^{34} + 6bc^3d^2e^{35} + 6bc^3d^2e^{36} + 6bc^3d^2e^{37} + 6bc^3d^2e^{38} + 6bc^3d^2e^{39} + 6bc^3d^2e^{40} + 6bc^3d^2e^{41} + 6bc^3d^2e^{42} + 6bc^3d^2e^{43} + 6bc^3d^2e^{44} + 6bc^3d^2e^{45} + 6bc^3d^2e^{46} + 6bc^3d^2e^{47} + 6bc^3d^2e^{48} + 6bc^3d^2e^{49} + 6bc^3d^2e^{50} + 6bc^3d^2e^{51} + 6bc^3d^2e^{52} + 6bc^3d^2e^{53} + 6bc^3d^2e^{54} + 6bc^3d^2e^{55} + 6bc^3d^2e^{56} + 6bc^3d^2e^{57} + 6bc^3d^2e^{58} + 6bc^3d^2e^{59} + 6bc^3d^2e^{60} + 6bc^3d^2e^{61} + 6bc^3d^2e^{62} + 6bc^3d^2e^{63} + 6bc^3d^2e^{64} + 6bc^3d^2e^{65} + 6bc^3d^2e^{66} + 6bc^3d^2e^{67} + 6bc^3d^2e^{68} + 6bc^3d^2e^{69} + 6bc^3d^2e^{70} + 6bc^3d^2e^{71} + 6bc^3d^2e^{72} + 6bc^3d^2e^{73} + 6bc^3d^2e^{74} + 6bc^3d^2e^{75} + 6bc^3d^2e^{76} + 6bc^3d^2e^{77} + 6bc^3d^2e^{78} + 6bc^3d^2e^{79} + 6bc^3d^2e^{80} + 6bc^3d^2e^{81} + 6bc^3d^2e^{82} + 6bc^3d^2e^{83} + 6bc^3d^2e^{84} + 6bc^3d^2e^{85} + 6bc^3d^2e^{86} + 6bc^3d^2e^{87} + 6bc^3d^2e^{88} + 6bc^3d^2e^{89} + 6bc^3d^2e^{90} + 6bc^3d^2e^{91} + 6bc^3d^2e^{92} + 6bc^3d^2e^{93} + 6bc^3d^2e^{94} + 6bc^3d^2e^{95} + 6bc^3d^2e^{96} + 6bc^3d^2e^{97} + 6bc^3d^2e^{98} + 6bc^3d^2e^{99} + 6bc^3d^2e^{100}}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(b*c^3*e^2*x^3 +
3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)*a
rccsc(c*x) - 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqrt(c
^2*x^2 - 1)) - (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*e
^2*x + 6*b*c*d*e)*sqrt(c^2*x^2 - 1)/c^3
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acsc}(cx))(d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x)**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (b \operatorname{arccsc}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arccsc(c*x) + a), x)
```

### 3.46 $\int (d + ex) \left( a + b \csc^{-1}(cx) \right) dx$

**Optimal.** Leaf size=83

$$\frac{(d + ex)^2 (a + b \csc^{-1}(cx))}{2e} + \frac{bd \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}}}{2c} - \frac{bd^2 \csc^{-1}(cx)}{2e}$$

[Out] (b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(2\*c) - (b\*d^2\*ArcCsc[c\*x])/(2\*e) + ((d + e\*x)^2\*(a + b\*ArcCsc[c\*x]))/(2\*e) + (b\*d\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

**Rubi [A]** time = 0.166665, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {5227, 1568, 1396, 1807, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^2 (a + b \csc^{-1}(cx))}{2e} + \frac{bd \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + \frac{bex \sqrt{1 - \frac{1}{c^2 x^2}}}{2c} - \frac{bd^2 \csc^{-1}(cx)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(2\*c) - (b\*d^2\*ArcCsc[c\*x])/(2\*e) + ((d + e\*x)^2\*(a + b\*ArcCsc[c\*x]))/(2\*e) + (b\*d\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

#### Rule 5227

```
Int[((a_) + ArcCsc[(c_)*(x_)])*(b_.)*((d_) + (e_.)*(x_)^(m_.)), x_Symbol]
  := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol]
  := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])
```

#### Rule 1396

```
Int[((d_) + (e_.)*(x_)^(n_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol]
  := -Subst[Int[((d + e/x^n)^q*(a + c/x^(2*n))^p)/x^2, x], x, 1/x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

#### Rule 1807

```
Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, Simp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

#### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rubi steps

$$\begin{aligned}
\int (d+ex)(a+b\csc^{-1}(cx)) dx &= \frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e} + \frac{b \int \frac{(d+ex)^2 dx}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{2ce} \\
&= \frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e} + \frac{b \int \frac{\left(\frac{e+d}{x}\right)^2 dx}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{2ce} \\
&= \frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e} - \frac{b \operatorname{Subst}\left(\int \frac{(e+dx)^2 dx, x, \frac{1}{x}}{x^2 \sqrt{1-\frac{x^2}{c^2}}}\right)}{2ce} \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e} + \frac{b \operatorname{Subst}\left(\int \frac{-2de-d^2x}{x\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2ce} \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e} - \frac{(bd) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{c} - \frac{(bd^2) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2c} \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} - \frac{bd^2 \csc^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e} + (bcd) \operatorname{Subst}\left(\int \frac{1}{c^2-c^2x} dx\right) \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} - \frac{bd^2 \csc^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e} + \frac{bd \tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.195894, size = 113, normalized size = 1.36

$$adx + \frac{1}{2}aex^2 + \frac{bdx\sqrt{1-\frac{1}{c^2x^2}} \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} + \frac{bex\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c} + bdx \csc^{-1}(cx) + \frac{1}{2}bex^2 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(a + b\*ArcCsc[c\*x]), x]

[Out] a\*d\*x + (a\*e\*x^2)/2 + (b\*e\*x\*sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(2\*c) + b\*d\*x\*ArcCsc[c\*x] + (b\*e\*x^2\*ArcCsc[c\*x])/2 + (b\*d\*sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTan[h[(c\*x)/sqrt[-1 + c^2\*x^2]]]/sqrt[-1 + c^2\*x^2])

**Maple [A]** time = 0.18, size = 140, normalized size = 1.7

$$\frac{ax^2e}{2} + adx + \frac{\operatorname{arccsc}(cx)x^2e}{2} + \operatorname{arccsc}(cx)xd + \frac{bd}{c^2x}\sqrt{c^2x^2-1} \ln\left(cx + \sqrt{c^2x^2-1}\right) - \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bex}{2c} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{be}{2c^3x} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(a+b\*arccsc(c\*x)), x)

[Out]  $\frac{1}{2}ax^2e^{ax+d} + \frac{1}{2}b\operatorname{arccsc}(cx)x^2e^{ax+d} + \frac{1}{c^2}b\left(\frac{(c^2x^2-1)^{1/2}}{x} + \frac{d\ln(c^2x^2-1)^{1/2}}{c} + \frac{1}{2c}\right) + \frac{1}{c^2}b\left(\frac{(c^2x^2-1)^{1/2}}{x} + \frac{d\ln(c^2x^2-1)^{1/2}}{c} + \frac{1}{2c}\right)e^{ax+d}$

**Maxima [A]** time = 1.00062, size = 124, normalized size = 1.49

$$\frac{1}{2}aex^2 + \frac{1}{2}\left(x^2 \operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c}\right)be + adx + \frac{\left(2cx \operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{2}aex^2 + \frac{1}{2}(x^2 \operatorname{arccsc}(cx) + x\sqrt{-1/(c^2x^2) + 1}/c)be + adx + \frac{1}{2}(2cx \operatorname{arccsc}(cx) + \log(\sqrt{-1/(c^2x^2) + 1} + 1) - \log(-\sqrt{-1/(c^2x^2) + 1} + 1))b*d/c$

**Fricas [A]** time = 2.19668, size = 302, normalized size = 3.64

$$\frac{ac^2ex^2 + 2ac^2dx - 2bcd \log(-cx + \sqrt{c^2x^2 - 1}) + \sqrt{c^2x^2 - 1}be + (bc^2ex^2 + 2bc^2dx - 2bc^2d - bc^2e) \operatorname{arccsc}(cx) - 2(2bc^2d - bc^2e) \arctan(-cx + \sqrt{c^2x^2 - 1})}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out]  $\frac{1}{2}(ac^2ex^2 + 2ac^2dx - 2bc^2d \log(-cx + \sqrt{c^2x^2 - 1}) + \sqrt{c^2x^2 - 1}be + (bc^2ex^2 + 2bc^2dx - 2bc^2d - bc^2e) \operatorname{arccsc}(cx) - 2(2bc^2d - bc^2e) \arctan(-cx + \sqrt{c^2x^2 - 1}))/c^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx))(d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(a+b\*arccsc(c\*x)),x)

[Out] Integral((a + b\*arccsc(c\*x))\*(d + e\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(b \operatorname{arccsc}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x + d)\*(b\*arccsc(c\*x) + a), x)

### 3.47 $\int (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=31

$$ax + \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \csc^{-1}(cx)$$

[Out] a\*x + b\*x\*ArcCsc[c\*x] + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

**Rubi [A]** time = 0.0229732, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5215, 266, 63, 208}

$$ax + \frac{b \tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCsc[c\*x],x]

[Out] a\*x + b\*x\*ArcCsc[c\*x] + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/c

#### Rule 5215

Int[ArcCsc[(c\_.)\*(x\_)], x\_Symbol] := Simp[x\*ArcCsc[c\*x], x] + Dist[1/c, Int[1/(x\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /; FreeQ[c, x]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rubi steps



$$\begin{aligned}
\int (a + b \csc^{-1}(cx)) dx &= ax + b \int \csc^{-1}(cx) dx \\
&= ax + bx \csc^{-1}(cx) + \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{c} \\
&= ax + bx \csc^{-1}(cx) - \frac{b \operatorname{Subst} \left( \int \frac{1}{x \sqrt{1 - \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{2c} \\
&= ax + bx \csc^{-1}(cx) + (bc) \operatorname{Subst} \left( \int \frac{1}{c^2 - c^2 x^2} dx, x, \sqrt{1 - \frac{1}{c^2 x^2}} \right) \\
&= ax + bx \csc^{-1}(cx) + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.0525964, size = 58, normalized size = 1.87

$$ax + \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}} \tanh^{-1} \left( \frac{cx}{\sqrt{c^2 x^2 - 1}} \right)}{\sqrt{c^2 x^2 - 1}} + bx \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCsc[c\*x], x]

[Out] a\*x + b\*x\*ArcCsc[c\*x] + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/Sqrt[-1 + c^2\*x^2]

**Maple [A]** time = 0.173, size = 37, normalized size = 1.2

$$ax + bx \operatorname{arccsc}(cx) + \frac{b}{c} \ln \left( cx + cx \sqrt{1 - \frac{1}{c^2 x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arccsc(c\*x), x)

[Out] a\*x+b\*x\*arccsc(c\*x)+b/c\*ln(c\*x+c\*x\*(1-1/c^2/x^2)^(1/2))

**Maxima [A]** time = 0.977055, size = 72, normalized size = 2.32

$$ax + \frac{\left( 2cx \operatorname{arccsc}(cx) + \log \left( \sqrt{-\frac{1}{c^2 x^2} + 1} + 1 \right) - \log \left( -\sqrt{-\frac{1}{c^2 x^2} + 1} + 1 \right) \right) b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsc(c\*x), x, algorithm="maxima")

[Out] a\*x + 1/2\*(2\*c\*x\*arccsc(c\*x) + log(sqrt(-1/(c^2\*x^2) + 1) + 1) - log(-sqrt(-1/(c^2\*x^2) + 1) + 1))\*b/c

---

**Fricas [B]** time = 2.07279, size = 154, normalized size = 4.97

$$\frac{acx - 2bc \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bcx - bc) \operatorname{arccsc}(cx) - b \log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsc(c\*x),x, algorithm="fricas")

[Out] (a\*c\*x - 2\*b\*c\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + (b\*c\*x - b\*c)\*arccsc(c\*x) - b\*log(-c\*x + sqrt(c^2\*x^2 - 1)))/c

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*acsc(c\*x),x)

[Out] Integral(a + b\*acsc(c\*x), x)

---

**Giac [A]** time = 1.13005, size = 63, normalized size = 2.03

$$\left( x \arcsin\left(\frac{1}{cx}\right) - \frac{c \log\left(\left|-x|c| + \sqrt{c^2x^2 - 1}\right|\right)}{|c|^2 \operatorname{sgn}(x)} \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsc(c\*x),x, algorithm="giac")

[Out] (x\*arcsin(1/(c\*x)) - c\*log(abs(-x\*abs(c) + sqrt(c^2\*x^2 - 1)))/(abs(c)^2\*sgn(x)))\*b + a\*x

$$3.48 \quad \int \frac{a+b \operatorname{csc}^{-1}(cx)}{d+ex} dx$$

**Optimal.** Leaf size=257

$$\frac{\operatorname{ibPolyLog}\left(2, \frac{i(e-\sqrt{e^2-c^2d^2})e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e} - \frac{\operatorname{ibPolyLog}\left(2, \frac{i(\sqrt{e^2-c^2d^2}+e)e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e} + \frac{\operatorname{ibPolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2e} + \frac{(a+b \operatorname{csc}^{-1}(cx))}{d+ex}$$

[Out] ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*(e - Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*(e + Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e - ((a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/e - (I\*b\*PolyLog[2, (I\*(e - Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e - (I\*b\*PolyLog[2, (I\*(e + Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e + ((I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/e

**Rubi [A]** time = 0.405979, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {5225, 2518}

$$\frac{\operatorname{ibPolyLog}\left(2, \frac{i(e-\sqrt{e^2-c^2d^2})e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e} - \frac{\operatorname{ibPolyLog}\left(2, \frac{i(\sqrt{e^2-c^2d^2}+e)e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e} + \frac{\operatorname{ibPolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2e} + \frac{(a+b \operatorname{csc}^{-1}(cx))}{d+ex}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/(d + e\*x), x]

[Out] ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*(e - Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*(e + Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e - ((a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/e - (I\*b\*PolyLog[2, (I\*(e - Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e - (I\*b\*PolyLog[2, (I\*(e + Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e + ((I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/e

#### Rule 5225

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*(e - Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e, x] + (Dist[b/(c\*e), Int[Log[1 - (I\*(e - Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d)]/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] + Dist[b/(c\*e), Int[Log[1 - (I\*(e + Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d)]/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] - Dist[b/(c\*e), Int[Log[1 - E^(2\*I\*ArcCsc[c\*x])]/(c\*d)]/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] + Simp[((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*(e + Sqrt[-(c^2\*d^2) + e^2]))\*E^(I\*ArcCsc[c\*x])]/(c\*d))/e, x] - Simp[((a + b\*ArcCsc[c\*x])\*Log[1 - E^(2\*I\*ArcCsc[c\*x])])/e, x] /; FreeQ[{a, b, c, d, e}, x]

#### Rule 2518

Int[Log[v\_]\*(u\_), x\_Symbol] :> With[{w = DerivativeDivides[v, u\*(1 - v), x]}, Simp[w\*PolyLog[2, 1 - v], x] /; !FalseQ[w]]

#### Rubi steps

$$\int \frac{a + b \operatorname{csc}^{-1}(cx)}{d + ex} dx = \frac{(a + b \operatorname{csc}^{-1}(cx)) \log\left(1 - \frac{i(e - \sqrt{-c^2 d^2 + e^2}) e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e} + \frac{(a + b \operatorname{csc}^{-1}(cx)) \log\left(1 - \frac{i(e + \sqrt{-c^2 d^2 + e^2}) e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e}$$

$$= \frac{(a + b \operatorname{csc}^{-1}(cx)) \log\left(1 - \frac{i(e - \sqrt{-c^2 d^2 + e^2}) e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e} + \frac{(a + b \operatorname{csc}^{-1}(cx)) \log\left(1 - \frac{i(e + \sqrt{-c^2 d^2 + e^2}) e^{i \operatorname{csc}^{-1}(cx)}}{cd}\right)}{e}$$

**Mathematica [A]** time = 0.658497, size = 411, normalized size = 1.6

$$\frac{a \log(d + ex)}{e} + \frac{b \left( 8i \left( \operatorname{PolyLog}\left(2, \frac{i(\sqrt{e^2 - c^2 d^2} - e) e^{-i \operatorname{csc}^{-1}(cx)}}{cd}\right) + \operatorname{PolyLog}\left(2, -\frac{i(\sqrt{e^2 - c^2 d^2} + e) e^{-i \operatorname{csc}^{-1}(cx)}}{cd}\right) \right) + 4i \left( \operatorname{csc}^{-1}(cx)^2 + \operatorname{Poly}\right)}{e}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x), x]

[Out] (a\*Log[d + e\*x])/e + (b\*(I\*(Pi - 2\*ArcCsc[c\*x])^2 + (32\*I)\*ArcSin[Sqrt[1 + e/(c\*d)]]/Sqrt[2]]\*ArcTan[((c\*d - e)\*Cot[(Pi + 2\*ArcCsc[c\*x])/4])/Sqrt[-(c^2\*d^2) + e^2]] - 4\*(Pi - 2\*ArcCsc[c\*x] + 4\*ArcSin[Sqrt[1 + e/(c\*d)]]/Sqrt[2]])\*Log[1 + (I\*(e - Sqrt[-(c^2\*d^2) + e^2]))/(c\*d\*E^(I\*ArcCsc[c\*x]))] - 4\*(Pi - 2\*ArcCsc[c\*x] - 4\*ArcSin[Sqrt[1 + e/(c\*d)]]/Sqrt[2]])\*Log[1 + (I\*(e + Sqrt[-(c^2\*d^2) + e^2]))/(c\*d\*E^(I\*ArcCsc[c\*x]))] - 8\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + 4\*(Pi - 2\*ArcCsc[c\*x])\*Log[e + d/x] + 8\*ArcCsc[c\*x]\*Log[e + d/x] + (8\*I)\*(PolyLog[2, (I\*(-e + Sqrt[-(c^2\*d^2) + e^2]))/(c\*d\*E^(I\*ArcCsc[c\*x]))] + PolyLog[2, ((-I)\*(e + Sqrt[-(c^2\*d^2) + e^2]))/(c\*d\*E^(I\*ArcCsc[c\*x]))]) + (4\*I)\*(ArcCsc[c\*x]^2 + PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]))/(8\*e)

**Maple [B]** time = 0.417, size = 881, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/(e\*x+d), x)

[Out] a\*ln(c\*e\*x+c\*d)/e-b\*e\*arccsc(c\*x)/(c^2\*d^2-e^2)\*ln((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e-(c^2\*d^2-e^2)^(1/2))/(I\*e-(c^2\*d^2-e^2)^(1/2))-b\*e\*arccsc(c\*x)/(c^2\*d^2-e^2)\*ln((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e+(c^2\*d^2-e^2)^(1/2))/(I\*e+(c^2\*d^2-e^2)^(1/2))+I\*b/e\*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-b/e\*arccsc(c\*x)\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-I\*c^2\*b/e/(c^2\*d^2-e^2)\*dilog((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e-(c^2\*d^2-e^2)^(1/2))/(I\*e-(c^2\*d^2-e^2)^(1/2))) \*d^2-I\*c^2\*b/e/(c^2\*d^2-e^2)\*dilog((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e+(c^2\*d^2-e^2)^(1/2))/(I\*e+(c^2\*d^2-e^2)^(1/2))) \*d^2+c^2\*b\*d^2/e\*arccsc(c\*x)/(c^2\*d^2-e^2)\*ln((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e-(c^2\*d^2-e^2)^(1/2))/(I\*e-(c^2\*d^2-e^2)^(1/2))+c^2\*b\*d^2/e\*arccsc(c\*x)/(c^2\*d^2-e^2)\*ln((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e+(c^2\*d^2-e^2)^(1/2))/(I\*e+(c^2\*d^2-e^2)^(1/2)))-I\*b/e\*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*b\*e/(c^2\*d^2-e^2)\*dilog((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e+(c^2\*d^2-e^2)^(1/2))/(I\*e+(c^2\*d^2-e^2)^(1/2))+I\*b\*e/(c^2\*d^2-e^2)\*dilog((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e-(c^2\*d^2-e^2)^(1/2))/(I\*e-(c^2\*d^2-e^2)^(1/2))+I\*b\*e/(c^2\*d^2-e^2)\*dilog((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e-(c^2\*d^2-e^2)^(1/2))/(I\*e-(c^2\*d^2-e^2)^(1/2))+I\*b\*e/(c^2\*d^2-e^2)\*dilog((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e+(c^2\*d^2-e^2)^(1/2))/(I\*e+(c^2\*d^2-e^2)^(1/2))+I\*b\*e/(c^2\*d^2-e^2)\*dilog((d\*c\*(I/c/x+(1-1/c^2/x^2)^(1/2))+I\*e+(c^2\*d^2-e^2)^(1/2))/(I\*e+(c^2\*d^2-e^2)^(1/2))))

$*d^2 - e^2)^{1/2} / (I * e - (c^2 * d^2 - e^2)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b \int \frac{\arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right)}{ex+d} dx + \frac{a \log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d),x, algorithm="maxima")

[Out] b\*integrate(arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e\*x + d), x) + a\*log(e\*x + d)/e

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d),x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/(e\*x + d), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x+d),x)

[Out] Integral((a + b\*acsc(c\*x))/(d + e\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(e\*x + d), x)

$$3.49 \quad \int \frac{a+b \csc^{-1}(cx)}{(d+ex)^2} dx$$

**Optimal.** Leaf size=102

$$-\frac{a+b \csc^{-1}(cx)}{e(d+ex)} + \frac{b \tanh^{-1}\left(\frac{c^2 d + \frac{e}{x}}{c \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2}}\right)}{d \sqrt{c^2 d^2 - e^2}} + \frac{b \csc^{-1}(cx)}{de}$$

[Out] (b\*ArcCsc[c\*x])/(d\*e) - (a + b\*ArcCsc[c\*x])/(e\*(d + e\*x)) + (b\*ArcTanh[(c^2\*d + e/x)/(c\*Sqrt[c^2\*d^2 - e^2]\*Sqrt[1 - 1/(c^2\*x^2)])])/(d\*Sqrt[c^2\*d^2 - e^2])

**Rubi [A]** time = 0.155473, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5227, 1568, 1475, 844, 216, 725, 206}

$$-\frac{a+b \csc^{-1}(cx)}{e(d+ex)} + \frac{b \tanh^{-1}\left(\frac{c^2 d + \frac{e}{x}}{c \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2}}\right)}{d \sqrt{c^2 d^2 - e^2}} + \frac{b \csc^{-1}(cx)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/(d + e\*x)^2, x]

[Out] (b\*ArcCsc[c\*x])/(d\*e) - (a + b\*ArcCsc[c\*x])/(e\*(d + e\*x)) + (b\*ArcTanh[(c^2\*d + e/x)/(c\*Sqrt[c^2\*d^2 - e^2]\*Sqrt[1 - 1/(c^2\*x^2)])])/(d\*Sqrt[c^2\*d^2 - e^2])

#### Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^(m_.)), x_Symbol]
  := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
  FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol]
  := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])
```

#### Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol]
  := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 844

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
  := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

ist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \csc^{-1}(cx)}{(d + ex)^2} dx &= -\frac{a + b \csc^{-1}(cx)}{e(d + ex)} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 (d + ex)} dx}{ce} \\
 &= -\frac{a + b \csc^{-1}(cx)}{e(d + ex)} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} \left(\frac{e+d}{x}\right) x^3} dx}{ce} \\
 &= -\frac{a + b \csc^{-1}(cx)}{e(d + ex)} + \frac{b \operatorname{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{ce} \\
 &= -\frac{a + b \csc^{-1}(cx)}{e(d + ex)} - \frac{b \operatorname{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{cd} + \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{cde} \\
 &= \frac{b \csc^{-1}(cx)}{de} - \frac{a + b \csc^{-1}(cx)}{e(d + ex)} + \frac{b \operatorname{Subst}\left(\int \frac{1}{d^2 - \frac{e^2}{c^2} x^2} dx, x, \frac{d + \frac{e}{c^2 x}}{\sqrt{1 - \frac{1}{c^2 x^2}}}\right)}{cd} \\
 &= \frac{b \csc^{-1}(cx)}{de} - \frac{a + b \csc^{-1}(cx)}{e(d + ex)} + \frac{b \tanh^{-1}\left(\frac{c^2 d + \frac{e}{x}}{c \sqrt{c^2 d^2 - e^2} \sqrt{1 - \frac{1}{c^2 x^2}}}\right)}{d \sqrt{c^2 d^2 - e^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.220739, size = 141, normalized size = 1.38

$$-\frac{a}{e(d + ex)} - \frac{b \log\left(cx\left(cd - \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2}\right) + e\right)}{d \sqrt{c^2 d^2 - e^2}} + \frac{b \log(d + ex)}{d \sqrt{c^2 d^2 - e^2}} + \frac{b \sin^{-1}\left(\frac{1}{cx}\right)}{de} - \frac{b \csc^{-1}(cx)}{e(d + ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x)^2, x]

[Out]  $-(a/(e*(d + e*x))) - (b*\text{ArcCsc}[c*x])/(e*(d + e*x)) + (b*\text{ArcSin}[1/(c*x)])/(d*e) + (b*\text{Log}[d + e*x])/(d*\text{Sqrt}[c^2*d^2 - e^2]) - (b*\text{Log}[e + c*(c*d - \text{Sqrt}[c^2*d^2 - e^2]*\text{Sqrt}[1 - 1/(c^2*x^2)])*x])/(d*\text{Sqrt}[c^2*d^2 - e^2])$

**Maple [B]** time = 0.263, size = 214, normalized size = 2.1

$$-\frac{ac}{(ecx + dc)e} - \frac{c\text{arccsc}(cx)}{(ecx + dc)e} + \frac{b}{ecx d} \sqrt{c^2 x^2 - 1} \arctan\left(\frac{1}{\sqrt{c^2 x^2 - 1}}\right) \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{b}{ecx d} \sqrt{c^2 x^2 - 1} \ln\left(2 \frac{1}{ecx + dc} \left(\sqrt{\frac{c^2 d^2 - e^2}{e^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsc(c*x))/(e*x+d)^2,x)`

[Out]  $-c*a/(c*e*x+c*d)/e - c*b/(c*e*x+c*d)/e*\text{arccsc}(c*x) + 1/c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d*\text{arctan}(1/(c^2*x^2-1)^{(1/2)}) - 1/c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/((c^2*d^2-e^2)/e^2)^{(1/2)}*\ln(2*((c^2*d^2-e^2)/e^2)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-d*c^2*x-e)/(c*e*x+c*d)$

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [B]** time = 2.36889, size = 960, normalized size = 9.41

$$\frac{ac^2d^3 - ade^2 - \sqrt{c^2d^2 - e^2}(be^2x + bde) \log\left(\frac{c^3d^2x + cde + \sqrt{c^2d^2 - e^2}(c^2dx + e) + (c^2d^2 + \sqrt{c^2d^2 - e^2}cd - e^2)\sqrt{c^2x^2 - 1}}{ex + d}\right) + (bc^2d^3 - bde^2) \text{arccsc}(c*x)}{c^2d^4e - d^2e^3 + (c^2d^3e^2 - de^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="fricas")`

[Out]  $[-(a*c^2*d^3 - a*d*e^2 - \text{sqrt}(c^2*d^2 - e^2)*(b*e^2*x + b*d*e))*\text{log}((c^3*d^2*x + c*d*e + \text{sqrt}(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + \text{sqrt}(c^2*d^2 - e^2)*c*d - e^2)*\text{sqrt}(c^2*x^2 - 1)))/(e*x + d)) + (b*c^2*d^3 - b*d*e^2)*\text{arccsc}(c*x) + 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*\text{arctan}(-c*x + \text{sqrt}(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a*c^2*d^3 - a*d*e^2 + 2*\text{sqrt}(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e))*\text{arctan}(-(\text{sqrt}(-c^2*d^2 + e^2)*\text{sqrt}(c^2*x^2 - 1)*e - \text{sqrt}(-c^2*d^2 + e^2)*(c*e*x + c*d)))/(c^2*d^2 - e^2)) + (b*c^2*d^3 - b*d*e^2)*\text{arccsc}(c*x) + 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*\text{arctan}(-c*x + \text{sqrt}(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x]$



---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x+d)\*\*2,x)

[Out] Integral((a + b\*acsc(c\*x))/(d + e\*x)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(e\*x + d)^2, x)

### 3.50 $\int \frac{a+b \operatorname{csc}^{-1}(cx)}{(d+ex)^3} dx$

**Optimal.** Leaf size=172

$$-\frac{a+b \operatorname{csc}^{-1}(cx)}{2e(d+ex)^2} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d(c^2d^2-e^2)\left(\frac{d}{x}+e\right)} + \frac{b(2c^2d^2-e^2)\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2(c^2d^2-e^2)^{3/2}} + \frac{b \operatorname{csc}^{-1}(cx)}{2d^2e}$$

[Out]  $-(b*c*e*\operatorname{Sqrt}[1-1/(c^2*x^2)])/(2*d*(c^2*d^2-e^2)*(e+d/x))+(b*\operatorname{ArcCsc}[c*x])/(2*d^2*e)-(a+b*\operatorname{ArcCsc}[c*x])/(2*e*(d+e*x)^2)+(b*(2*c^2*d^2-e^2)*\operatorname{ArcTanh}[(c^2*d+e/x)/(c*\operatorname{Sqrt}[c^2*d^2-e^2]*\operatorname{Sqrt}[1-1/(c^2*x^2)])])/(2*d^2*(c^2*d^2-e^2)^{(3/2)})$

**Rubi [A]** time = 0.291905, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5227, 1568, 1475, 1651, 844, 216, 725, 206}

$$-\frac{a+b \operatorname{csc}^{-1}(cx)}{2e(d+ex)^2} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d(c^2d^2-e^2)\left(\frac{d}{x}+e\right)} + \frac{b(2c^2d^2-e^2)\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2(c^2d^2-e^2)^{3/2}} + \frac{b \operatorname{csc}^{-1}(cx)}{2d^2e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+b*\operatorname{ArcCsc}[c*x])/(d+e*x)^3,x]$

[Out]  $-(b*c*e*\operatorname{Sqrt}[1-1/(c^2*x^2)])/(2*d*(c^2*d^2-e^2)*(e+d/x))+(b*\operatorname{ArcCsc}[c*x])/(2*d^2*e)-(a+b*\operatorname{ArcCsc}[c*x])/(2*e*(d+e*x)^2)+(b*(2*c^2*d^2-e^2)*\operatorname{ArcTanh}[(c^2*d+e/x)/(c*\operatorname{Sqrt}[c^2*d^2-e^2]*\operatorname{Sqrt}[1-1/(c^2*x^2)])])/(2*d^2*(c^2*d^2-e^2)^{(3/2)})$

#### Rule 5227

$\operatorname{Int}[(a + \operatorname{ArcCsc}[c*x])*(d + e*x)^m, x\_Symbol] \rightarrow \operatorname{Simp}[(d + e*x)^{m+1}*(a + b*\operatorname{ArcCsc}[c*x])]/(e*(m+1)), x] + \operatorname{Dist}[b/(c*e*(m+1)), \operatorname{Int}[(d + e*x)^{m+1}/(x^2*\operatorname{Sqrt}[1-1/(c^2*x^2)]), x], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \operatorname{NeQ}[m, -1]$

#### Rule 1568

$\operatorname{Int}[x^m*(d + e*x)^{mn}*(a + c*x^n)^p, x\_Symbol] \rightarrow \operatorname{Int}[x^{m+mn*q}*(e + d/x^{mn})^q*(a + c*x^n)^p, x] /;$   
 $\operatorname{FreeQ}\{a, c, d, e, m, mn, p\}, x \ \&\& \operatorname{EqQ}[n2, -2*mn] \ \&\& \operatorname{IntegerQ}[q] \ \&\& (\operatorname{PosQ}[n2] \ \|\ \operatorname{IntegerQ}[p])$

#### Rule 1475

$\operatorname{Int}[x^m*(a + c*x^n)^p*(d + e*x)^q, x\_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(Simplify[(m+1)/n]-1)*(d+e*x)^q*(a+c*x^2)^p}, x], x, x^n], x] /;$   
 $\operatorname{FreeQ}\{a, c, d, e, m, n, p, q\}, x \ \&\& \operatorname{EqQ}[n2, 2*n] \ \&\& \operatorname{IntegerQ}[Simplify[(m+1)/n]]$

#### Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
  With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

#### Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

#### Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

#### Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

#### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{(d + ex)^3} dx &= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex)^2} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2} x^2 (d + ex)^2}} dx}{2ce} \\
&= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex)^2} - \frac{b \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2} \left(e + \frac{d}{x}\right)^2 x^4}} dx}{2ce} \\
&= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex)^2} + \frac{b \operatorname{Subst} \left( \int \frac{x^2}{(e + dx)^2 \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2ce} \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} - \frac{a + b \csc^{-1}(cx)}{2e(d + ex)^2} - \frac{(bc) \operatorname{Subst} \left( \int \frac{e - \left(d - \frac{e^2}{c^2 d}\right) x}{(e + dx) \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2e(c^2 d^2 - e^2)} \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} - \frac{a + b \csc^{-1}(cx)}{2e(d + ex)^2} + \frac{b \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2cd^2 e} - \frac{(bc \left(2 - \frac{e^2}{c^2 d^2}\right)) \operatorname{Subst} \left( \int \frac{1}{d^2 - \frac{e^2}{c^2} - x^2} dx, x, \frac{1}{x} \right)}{2(c^2 d^2 - e^2)} \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} + \frac{b \csc^{-1}(cx)}{2d^2 e} - \frac{a + b \csc^{-1}(cx)}{2e(d + ex)^2} + \frac{(bc \left(2 - \frac{e^2}{c^2 d^2}\right)) \operatorname{Subst} \left( \int \frac{1}{d^2 - \frac{e^2}{c^2} - x^2} dx, x, \frac{1}{x} \right)}{2(c^2 d^2 - e^2)} \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{2d(c^2 d^2 - e^2) \left(e + \frac{d}{x}\right)} + \frac{b \csc^{-1}(cx)}{2d^2 e} - \frac{a + b \csc^{-1}(cx)}{2e(d + ex)^2} + \frac{b(2c^2 d^2 - e^2) \tanh^{-1} \left( \frac{c^2 d + \frac{e}{x}}{c \sqrt{c^2 d^2 - e^2} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2d^2 (c^2 d^2 - e^2)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.515473, size = 250, normalized size = 1.45

$$\frac{1}{2} \left( -\frac{a}{e(d + ex)^2} - \frac{bcex \sqrt{1 - \frac{1}{c^2 x^2}}}{d(c^2 d^2 - e^2)(d + ex)} - \frac{b(2c^2 d^2 - e^2) \log \left( cx \left( cd - \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{c^2 d^2 - e^2} \right) + e \right)}{d^2 (cd - e)(cd + e) \sqrt{c^2 d^2 - e^2}} + \frac{b(2c^2 d^2 - e^2) \log(d + ex)}{d^2 (cd - e)(cd + e) \sqrt{c^2 d^2 - e^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x)^3,x]

[Out]  $(-a/(e*(d + e*x)^2)) - (b*c*e*\sqrt{1 - 1/(c^2*x^2)}*x)/(d*(c^2*d^2 - e^2)*(d + e*x)) - (b*ArcCsc[c*x])/(e*(d + e*x)^2) + (b*ArcSin[1/(c*x)])/(d^2*e) + (b*(2*c^2*d^2 - e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*\sqrt{c^2*d^2 - e^2}) - (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - \sqrt{c^2*d^2 - e^2})*\sqrt{1 - 1/(c^2*x^2)}])/(d^2*(c*d - e)*(c*d + e)*\sqrt{c^2*d^2 - e^2}))/2$

**Maple [B]** time = 0.174, size = 1007, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/(e\*x+d)^3,x)

[Out] 
$$-1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arccsc(c*x)+1/2*c^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^{(1/2)})+1/2*c^2*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^{(1/2)})-c^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/((c^2*d^2-e^2)/e^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*d^2-e^2)/e^2)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-d*c^2*x-e)/(c*e*x+c*d))-c^2*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d/((c^2*d^2-e^2)/e^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*d^2-e^2)/e^2)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-d*c^2*x-e)/(c*e*x+c*d))-1/2*c^2*b*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d^2-e^2)/(c*e*x+c*d)+1/2*b*e/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d^2-e^2)/(c*e*x+c*d)-1/2*b*e^2*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/d^2/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^{(1/2)})-1/2*b*e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^{(1/2)})+1/2*b*e^2*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/d^2/((c^2*d^2-e^2)/e^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*d^2-e^2)/e^2)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-d*c^2*x-e)/(c*e*x+c*d))+1/2*b*e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/((c^2*d^2-e^2)/e^2)^{(1/2)}/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*d^2-e^2)/e^2)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-d*c^2*x-e)/(c*e*x+c*d))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\left( (c^2 e^3 x^2 + 2 c^2 d e^2 x + c^2 d^2 e) \int \frac{x e^{\left( \frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1) \right)}}{c^2 e^3 x^4 + 2 c^2 d e^2 x^3 - 2 d e^2 x + (c^2 e^3 x^4 + 2 c^2 d e^2 x^3 - 2 d e^2 x - d^2 e + (c^2 d^2 e - e^3) x^2)(cx+1)(cx-1) - d^2 e + (c^2 d^2 e - e^3) x^2} dx + \right)}{2 (e^3 x^2 + 2 d e^2 x + d^2 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^3,x, algorithm="maxima")

[Out] 
$$-1/2*(2*(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)*integrate(1/2*x*e^{(1/2)*\log(c*x + 1) + 1/2*\log(c*x - 1)}/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x - d^2*e + (c^2*d^2*e - e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x - d^2*e + (c^2*d^2*e - e^3)*x^2)*e^{(\log(c*x + 1) + \log(c*x - 1))}, x) + arctan(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}))/b/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)$$

**Fricas [B]** time = 6.21897, size = 2223, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^3,x, algorithm="fricas")

[Out] 
$$[-1/2*(a*c^4*d^6 + b*c^3*d^5*e - 2*a*c^2*d^4*e^2 - b*c*d^3*e^3 + a*d^2*e^4 + (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*\sqrt{c^2*d^2 - e^2}*\log((c^3*d^2*x + c*d*e + \sqrt{c^2*d^2 - e^2}*(c^2*d*x + e) + (c^2*d^2 + \sqrt{c^2*d^2 - e^2})*c*d - e^2)*\sqrt{c^2*x^2 - 1})/(e*x + d) + 2*(b*c^3*d^4*e^2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc(c*x)]$$

+ 2\*(b\*c^4\*d^6 - 2\*b\*c^2\*d^4\*e^2 + b\*d^2\*e^4 + (b\*c^4\*d^4\*e^2 - 2\*b\*c^2\*d^2\*e^4 + b\*e^6)\*x^2 + 2\*(b\*c^4\*d^5\*e - 2\*b\*c^2\*d^3\*e^3 + b\*d\*e^5)\*x)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + (b\*c^2\*d^4\*e^2 - b\*d^2\*e^4 + (b\*c^2\*d^3\*e^3 - b\*d\*e^5)\*x)\*sqrt(c^2\*x^2 - 1))/(c^4\*d^8\*e - 2\*c^2\*d^6\*e^3 + d^4\*e^5 + (c^4\*d^6\*e^3 - 2\*c^2\*d^4\*e^5 + d^2\*e^7)\*x^2 + 2\*(c^4\*d^7\*e^2 - 2\*c^2\*d^5\*e^4 + d^3\*e^6)\*x), -1/2\*(a\*c^4\*d^6 + b\*c^3\*d^5\*e - 2\*a\*c^2\*d^4\*e^2 - b\*c\*d^3\*e^3 + a\*d^2\*e^4 + (b\*c^3\*d^3\*e^3 - b\*c\*d\*e^5)\*x^2 + 2\*(2\*b\*c^2\*d^4\*e - b\*d^2\*e^3 + (2\*b\*c^2\*d^2\*e^3 - b\*e^5)\*x^2 + 2\*(2\*b\*c^2\*d^3\*e^2 - b\*d\*e^4)\*x)\*sqrt(-c^2\*d^2 + e^2)\*arctan(-(sqrt(-c^2\*d^2 + e^2)\*sqrt(c^2\*x^2 - 1)\*e - sqrt(-c^2\*d^2 + e^2)\*(c\*e\*x + c\*d))/(c^2\*d^2 - e^2)) + 2\*(b\*c^3\*d^4\*e^2 - b\*c\*d^2\*e^4)\*x + (b\*c^4\*d^6 - 2\*b\*c^2\*d^4\*e^2 + b\*d^2\*e^4)\*arccsc(c\*x) + 2\*(b\*c^4\*d^6 - 2\*b\*c^2\*d^4\*e^2 + b\*d^2\*e^4 + (b\*c^4\*d^4\*e^2 - 2\*b\*c^2\*d^2\*e^4 + b\*e^6)\*x^2 + 2\*(b\*c^4\*d^5\*e - 2\*b\*c^2\*d^3\*e^3 + b\*d\*e^5)\*x)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) + (b\*c^2\*d^4\*e^2 - b\*d^2\*e^4 + (b\*c^2\*d^3\*e^3 - b\*d\*e^5)\*x)\*sqrt(c^2\*x^2 - 1))/(c^4\*d^8\*e - 2\*c^2\*d^6\*e^3 + d^4\*e^5 + (c^4\*d^6\*e^3 - 2\*c^2\*d^4\*e^5 + d^2\*e^7)\*x^2 + 2\*(c^4\*d^7\*e^2 - 2\*c^2\*d^5\*e^4 + d^3\*e^6)\*x)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x+d)\*\*3,x)

[Out] Integral((a + b\*acsc(c\*x))/(d + e\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(e\*x + d)^3, x)

### 3.51 $\int x^2 \sqrt{d+ex} (a + b \operatorname{csc}^{-1}(cx)) dx$

**Optimal.** Leaf size=496

$$\frac{4bd\sqrt{1-c^2x^2}(9c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{2d^2(d+ex)^{3/2}(a+b\operatorname{csc}^{-1}(cx))}{3e^3} - \frac{4d(d+ex)^{5/2}}{3e^3}$$

```
[Out] (4*b*d*Sqrt[d + e*x]*(1 - c^2*x^2))/(105*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) - (
4*b*(d + e*x)^(3/2)*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3)
+ (4*b*(5*c^2*d^2 - 9*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSi
n[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(105*c^4*e^2*Sqrt[1 - 1/(c^2*x^
2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*(9*c^2*d^2 - e^2)*Sqrt[(c*(d
+ e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]
], (2*e)/(c*d + e)]/(105*c^4*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) -
(32*b*d^4*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(105*c*e^3*Sqrt[1 - 1/(c^2*x^
2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 2.8273, antiderivative size = 693, normalized size of antiderivative = 1.4, number of steps used = 31, number of rules used = 16, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$ , Rules used = {43, 5247, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537, 833}

$$\frac{2d^2(d+ex)^{3/2}(a+b\operatorname{csc}^{-1}(cx))}{3e^3} - \frac{4d(d+ex)^{5/2}(a+b\operatorname{csc}^{-1}(cx))}{5e^3} + \frac{2(d+ex)^{7/2}(a+b\operatorname{csc}^{-1}(cx))}{7e^3} - \frac{32bd^3\sqrt{1-c^2x^2}\sqrt{d+ex}}{105c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (4*b*d*Sqrt[d + e*x]*(1 - c^2*x^2))/(105*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) - (
4*b*(d + e*x)^(3/2)*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3)
+ (32*b*d^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)]/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(
d + e*x))/(c*d + e)]) - (4*b*(c^2*d^2 + 3*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x
^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(35*c^4*e^2*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (32*b*d^3*Sqrt[(c*
(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)]/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
- (4*b*d*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^
2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(105*c^4*e^2*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(c*(d + e*x))/(c*d
+ e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)]/(105*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```

$Q[7*m + 4*n + 4, 0] \parallel LtQ[9*m + 5*(n + 1), 0] \parallel GtQ[m + n + 2, 0]$

### Rule 5247

$Int[((a_.) + ArcCsc[(c_.)*(x_.)]*(b_.))*(u_), x\_Symbol] \rightarrow With[\{v = IntHide[u, x]\}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2*sqrt[1 - 1/(c^2*x^2)])], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[\{a, b, c\}, x]$

### Rule 12

$Int[(a_)*(u_), x\_Symbol] \rightarrow Dist[a, Int[u, x], x] /; FreeQ[a, x] \&\& !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]$

### Rule 6721

$Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x\_Symbol] \rightarrow Dist[(b^IntPart[p]*(a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), Int[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[\{a, b, p\}, x] \&\& !IntegerQ[p] \&\& !LtQ[n, 0] \&\& !RationalFunctionQ[u, x] \&\& IntegerQ[p + 1/2]$

### Rule 6742

$Int[u_, x\_Symbol] \rightarrow With[\{v = ExpandIntegrand[u, x]\}, Int[v, x] /; SumQ[v]]$

### Rule 743

$Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x\_Symbol] \rightarrow Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m - 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[\{a, c, d, e, m, p\}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] \&\& NeQ[m + 2*p + 1, 0] \&\& IntQuadraticQ[a, 0, c, d, e, m, p, x]$

### Rule 844

$Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x\_Symbol] \rightarrow Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[\{a, c, d, e, f, g, m, p\}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& !IGtQ[m, 0]$

### Rule 719

$Int[((d_) + (e_.)*(x_))^(m_)/sqrt[(a_) + (c_.)*(x_)^2], x\_Symbol] \rightarrow Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*sqrt[1 + (c*x^2)/a])/(c*sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/sqrt[1 - x^2], x], x, sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[\{a, c, d, e\}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& EqQ[m^2, 1/4]$

### Rule 424

$Int[sqrt[(a_) + (b_.)*(x_)^2]/sqrt[(c_) + (d_.)*(x_)^2], x\_Symbol] \rightarrow Simp[(sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[\{a, b, c, d\}, x] \&\& NegQ[d/c] \&\& GtQ[c, 0] \&\& GtQ[a, 0]$

### Rule 419



```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rule 958

```
Int[((f_) + (g_)*(x_)^(n_)/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^
2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
+ g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

### Rule 932

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

### Rule 168

```
Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_)
]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rule 833

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[(g*(d + e*x)^(m*(a + c*x^2)^(p + 1)))/(c*(m + 2*p + 2
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

### Rubi steps



**Mathematica [C]** time = 13.9094, size = 870, normalized size = 1.75

$$b \frac{c \left( \frac{d}{x} + e \right) x \left( -\frac{16c^3 \operatorname{csc}^{-1}(cx)d^3}{105e^3} - \frac{2}{7}c^3x^3 \operatorname{csc}^{-1}(cx) - \frac{2c^2x^2 \left( 2\sqrt{1-\frac{1}{c^2x^2}}e+cd \operatorname{csc}^{-1}(cx) \right)}{35e} - \frac{8cx \left( cde\sqrt{1-\frac{1}{c^2x^2}}-c^2d^2 \operatorname{csc}^{-1}(cx) \right)}{105e^2} - \frac{4(9e^2-5c^2d^2)\sqrt{1-\frac{1}{c^2x^2}}}{105e^2} \right)}{\sqrt{d+ex}} - \frac{2\sqrt{\frac{d}{x}+e}\sqrt{cx}}{2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]),x]

[Out]  $-\left( (a*d^3*\operatorname{Sqrt}[d + e*x]*\operatorname{Beta}\left[-\left(\frac{e*x}{d}\right), 3, 3/2\right]/\left(e^3*\operatorname{Sqrt}[1 + (e*x)/d]\right) \right) +$   
 $(b*(-((c*(e + d/x)*x*((-4*(-5*c^2*d^2 + 9*e^2)*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]))/(105$   
 $*e^2) - (16*c^3*d^3*\operatorname{ArcCsc}[c*x])/(105*e^3) - (2*c^3*x^3*\operatorname{ArcCsc}[c*x])/7 - (2$   
 $*c^2*x^2*(2*e*\operatorname{Sqrt}[1 - 1/(c^2*x^2)] + c*d*\operatorname{ArcCsc}[c*x]))/(35*e) - (8*c*x*(c*$   
 $d*e*\operatorname{Sqrt}[1 - 1/(c^2*x^2)] - c^2*d^2*\operatorname{ArcCsc}[c*x]))/(105*e^2))/\operatorname{Sqrt}[d + e*x]$   
 $) - (2*\operatorname{Sqrt}[e + d/x]*\operatorname{Sqrt}[c*x]*((2*(9*c^3*d^3*e - c*d*e^3)*\operatorname{Sqrt}[(c*d + c*e*$   
 $x)/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2$   
 $*e)/(c*d + e)])/(\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*\operatorname{Sqrt}[e + d/x]*(c*x)^(3/2)) + (2*(8*c$   
 $^4*d^4 + 5*c^2*d^2*e^2 - 9*e^4)*\operatorname{Sqrt}[(c*d + c*e*x)/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*$   
 $x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(\operatorname{Sqrt}[1$   
 $- 1/(c^2*x^2)]*\operatorname{Sqrt}[e + d/x]*(c*x)^(3/2)) + (2*(-5*c^3*d^3*e + 9*c*d*e^3)*$   
 $\operatorname{Cos}[2*\operatorname{ArcCsc}[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*\operatorname{Sqrt}[(c*d + c*e*$   
 $x)/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2$   
 $*e)/(c*d + e) - (c*x*(1 + c*x)*\operatorname{Sqrt}[(e - c*e*x)/(c*d + e)]*\operatorname{Sqrt}[(c*d + c*e$   
 $*x)/(c*d - e)]*((c*d + e)*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[(c*d + c*e*x)/(c*d - e)]],$   
 $(c*d - e)/(c*d + e) - e*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[(c*d + c*e*x)/(c*d - e)]], ($   
 $c*d - e)/(c*d + e)))/\operatorname{Sqrt}[(e*(1 + c*x))/(-c*d + e) + c*e*x*\operatorname{Sqrt}[(c*d +$   
 $c*e*x)/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}$   
 $[2]], (2*e)/(c*d + e)))/(c*d*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*\operatorname{Sqrt}[e + d/x]*\operatorname{Sqrt}[c*x]$   
 $*(-2 + c^2*x^2)))/(105*e^3*\operatorname{Sqrt}[d + e*x]))/c^4$

**Maple [B]** time = 0.285, size = 1222, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2),x)

[Out]  $2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*(e*x+d)^(5/2)*d+1/3*(e*x+d)^(3/2)*d^2)+b*(1$   
 $/7*\operatorname{arccsc}(c*x)*(e*x+d)^(7/2)-2/5*\operatorname{arccsc}(c*x)*(e*x+d)^(5/2)*d+1/3*\operatorname{arccsc}(c*x$   
 $)*(e*x+d)^(3/2)*d^2+2/105/c^4*(3*(c/(c*d-e))^(1/2)*(e*x+d)^(7/2)*c^3-7*(c/($   
 $c*d-e))^(1/2)*(e*x+d)^(5/2)*c^3*d+5*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^3*d^2$   
 $+4*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*\operatorname{El$   
 $lipticF}((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+5*$   
 $(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*\operatorname{Ellip$   
 $ticE}((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-8*d^3$

```
*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^3*d^3-5*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+5*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e-3*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c*e^2+8*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-9*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c*d*e^2+9*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3-9*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 \operatorname{arccsc}(cx) + ax^2\right)\sqrt{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x + d), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + d}(b \operatorname{arccsc}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x^2, x)
```

### 3.52 $\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)dx$

**Optimal.** Leaf size=404

$$\frac{4b\sqrt{1-c^2x^2}(3c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^2) - (8*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*(3*c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 2.19491, antiderivative size = 502, normalized size of antiderivative = 1.24, number of steps used = 24, number of rules used = 15, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.79$ , Rules used = {43, 5247, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537}

$$-\frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{8bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^2) - (8*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHide[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
```

$(x^2 \sqrt{1 - 1/(c^2 x^2)})$ , x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 6721

Int[(u\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 743

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[1/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 2)\*Simp[c\*d^2\*(m + 2\*p + 1) - a\*e^2\*(m - 1) + 2\*c\*d\*e\*(m + p)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2\*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 719

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

### Rule 958

```
Int[((f_.) + (g_.)*(x_)^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

### Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

### Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rubi steps



$$\begin{aligned}
\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))dx &= -\frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{b\int\frac{2(d+ex)^{3/2}(-2d+ex)}{15e^2\sqrt{1-\frac{c^2x^2}{d+ex}}}dx}{c} \\
&= -\frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{(2b)\int\frac{(d+ex)^{3/2}(-2d+ex)}{\sqrt{1-\frac{c^2x^2}{d+ex}}}dx}{15ce} \\
&= -\frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{(2b\sqrt{1-c^2x^2})}{15ce^2} \\
&= -\frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{(2b\sqrt{1-c^2x^2})}{15ce^2} \\
&= -\frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} - \frac{(4bd\sqrt{1-c^2x^2})}{15ce^2\sqrt{1-\frac{c^2x^2}{d+ex}}} \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}
\end{aligned}$$

**Mathematica [C]** time = 1.61413, size = 368, normalized size = 0.91

$$\frac{1}{15} \left( \frac{4ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}} \left( (-c^2d^2 - 2cde + e^2) \text{EllipticF} \left( i \sinh^{-1} \left( \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right), \frac{cd+e}{cd-e} \right) + 2c^2d^2 \Pi \left( \frac{e}{cd} + 1; i \sinh^{-1} \left( \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right) \right) \right)}{c^3e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{c}{cd+e}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]),x]

```
[Out] ((4*b*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2)*ArcCsc[c*x])/e^2 - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-c*d) + e])*Sqrt[(e - c*e*x)/(c*d + e)]*(-2*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e))]]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (-(c^2*d^2) - 2*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)))/(c^3*e^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x))/15
```

**Maple [B]** time = 0.281, size = 836, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e^2*(a*(1/5*(e*x+d)^(5/2)-1/3*(e*x+d)^(3/2)*d)+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)-1/3*arccsc(c*x)*(e*x+d)^(3/2)*d+2/15/c^3*((c/(c*d-e))^(1/2)*(e*x+d)^(5/2)*c^2-2*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^2*d-d^2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2+2*d^2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^2*d^2+2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*e^2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + d}(b \operatorname{arccsc}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x, x)
```

### 3.53 $\int \sqrt{d+ex} \left( a + b \csc^{-1}(cx) \right) dx$

**Optimal.** Leaf size=315

$$\frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{4bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

[Out] (2\*(d + e\*x)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*e) - (4\*b\*Sqrt[d + e\*x]\*Sqrt[1 - c^2\*x^2]\*EllipticE[ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)))/(3\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]) - (4\*b\*d\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)))/(3\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x]) - (4\*b\*d^2\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)))/(3\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])

**Rubi [A]** time = 0.48491, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5227, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424}

$$\frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{4bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]),x]

[Out] (2\*(d + e\*x)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*e) - (4\*b\*Sqrt[d + e\*x]\*Sqrt[1 - c^2\*x^2]\*EllipticE[ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)))/(3\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]) - (4\*b\*d\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)))/(3\*c^2\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x]) - (4\*b\*d^2\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)))/(3\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])

#### Rule 5227

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCsc[c\*x]))/(e\*(m + 1)), x] + Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 - 1/(c^2\*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 1574

Int[(x\_)^(m\_.)\*((a\_.) + (c\_.)\*(x\_)^(mn2\_.))^(p\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[(x^(2\*n\*FracPart[p]))\*(a + c/x^(2\*n))^FracPart[p]]/(c + a\*x^(2\*n))^FracPart[p], Int[x^(m - 2\*n\*p)\*(d + e\*x^n)^q\*(c + a\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2\*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

#### Rule 958

```
Int[((f_.) + (g_.)*(x_)^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

### Rule 719

```
Int[((d_) + (e_.)*(x_)^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]
```

### Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rule 844

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c)
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{d+ex} (a+b \csc^{-1}(cx)) dx &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} + \frac{(2b) \int \frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{3ce} \\
 &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{(d+ex)^{3/2}}{x\sqrt{-\frac{1}{c^2}+x^2}} dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}} \\
 &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \left(\frac{2de}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \dots\right) dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}} \\
 &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} + \frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{3c\sqrt{1-\frac{1}{c^2x^2}}} + \frac{\left(2bd^2\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}} \\
 &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}} dx}{3c\sqrt{1-\frac{1}{c^2x^2}}} - \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{3c\sqrt{1-\frac{1}{c^2x^2}}} \\
 &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} - \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(4bd^2\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}} \\
 &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}}{3c\sqrt{1-\frac{1}{c^2x^2}}} \\
 &= \frac{2(d+ex)^{3/2} (a+b \csc^{-1}(cx))}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}}{3c\sqrt{1-\frac{1}{c^2x^2}}}
 \end{aligned}$$

**Mathematica [B]** time = 13.2616, size = 657, normalized size = 2.09

$$\frac{2a(d+ex)^{3/2}}{3e} - \frac{b(cd+ce x) \left( \frac{4d\sqrt{-c^2x^2\left(1-\frac{1}{c^2x^2}\right)}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}(cd+e)\sqrt{\frac{cd+ce x}{cd+e}}} + \frac{\sin(4 \csc^{-1}(cx)) \sec(2 \csc^{-1}(cx)) \left( c^2 dx \sqrt{-c^2x^2\left(1-\frac{1}{c^2x^2}\right)} \sqrt{\frac{cd+ce x}{cd+e}} \text{EllipticE}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right) \right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{cd+ce x}{cd+e}}} \right)}{3c\sqrt{1-\frac{1}{c^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (2*a*(d + e*x)^(3/2))/(3*e) - (b*(c*d + c*e*x)*((-2*(2*e*Sqrt[1 - 1/(c^2*x^2)] + c*d*ArcCsc[c*x] + c*e*x*ArcCsc[c*x]))/e + (4*d*Sqrt[-(c^2*(1 - 1/(c^2*x^2))]*x^2)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x)/(c*d + e)] - (4*(-(c*d + e)*Sqrt[-(c^2*(1 - 1/(c^2*x^2))]*x^2)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x)/(c*d + e)] + ((c^2*(1 - 1/(c^2*x^2)))*x^2*(c*d + c*e*x) + c^2*d*x*Sqrt[-(c^2*(1 - 1/(c^2*x^2))]*x^2)]*Sqrt[(c*d + c*e*x)/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d + e)] + c*e*x*Sqrt[-(c^2*(1 - 1/(c^2*x^2))]*x^2)]*Sqrt[(c*d + c*e*x)/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]*Sec[2*ArcCsc[c*x]]*Sin[4*ArcCsc[c*x]])/(c^2*(1 - 1/(c^2*x^2))*(e + d/x)*x^2)))/(3*c^2*Sqrt[d + e*x])
```

---

**Maple [A]** time = 0.273, size = 388, normalized size = 1.2

$$2 \frac{1}{e} \left( \frac{1}{3} (ex + d)^{3/2} a + b \left( \frac{1}{3} (ex + d)^{3/2} \operatorname{arccsc}(cx) + 2/3 \frac{1}{c^2 x} \left( 2 d \operatorname{EllipticF} \left( \sqrt{ex + d} \sqrt{\frac{c}{dc - e}}, \sqrt{\frac{dc - e}{dc + e}} \right) c - \operatorname{EllipticE} \left( \sqrt{ex + d} \sqrt{\frac{c}{dc - e}}, \sqrt{\frac{dc - e}{dc + e}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arccsc(c*x)+2/3/c^2*(2*d*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*(-(e*x+d)*c-d*c+e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x+d)\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + d}(b \operatorname{arccsc}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a), x)



$$3.54 \quad \int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x}, x\right)$$

[Out] Unintegrable[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x, x]

**Rubi [A]** time = 0.0708815, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x,x]

[Out] Defer[Int] [(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x, x]

Rubi steps

$$\int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x} dx = \int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

**Mathematica [A]** time = 17.5124, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x,x]

[Out] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x, x]

**Maple [A]** time = 5.796, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x} \sqrt{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x,x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x+d)\*\*(1/2)/x,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/x, x)

$$3.55 \quad \int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x^2}, x\right)$$

[Out] Unintegrable[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

**Rubi [A]** time = 0.0757837, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

[Out] Defer[Int] [(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

**Mathematica [A]** time = 7.99731, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

[Out] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

**Maple [A]** time = 4.339, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2} \sqrt{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x^2, x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x^2, x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/x^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x+d)\*\*(1/2)/x\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x+d)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/x^2, x)

### 3.56 $\int (d + ex)^{3/2} (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=372

$$\frac{4b\sqrt{1-c^2x^2}(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e} - \frac{4bd^3\sqrt{1-c^2x^2}}{5c}$$

```
[Out] (-4*b*e*Sqrt[d + e*x]*(1 - c^2*x^2))/((15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (28*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2 + e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 0.754832, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {5227, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e} - \frac{4b\sqrt{1-c^2x^2}(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{15c^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\middle|\frac{2e}{cd+e}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*e*Sqrt[d + e*x]*(1 - c^2*x^2))/((15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (28*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2 + e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_.) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^(FracPart[p]))/(c + a*x^(2*n))^(FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

$e, f, g, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

#### Rule 424

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/\text{Sqrt}[(c_) + (d_)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /;$   $\text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

#### Rule 931

$\text{Int}[(d_) + (e_)*(x_)^{(m_)}]/(\text{Sqrt}[(f_) + (g_)*(x_)]*\text{Sqrt}[(a_) + (c_)*(x_)^2]), x\_Symbol] \rightarrow \text{Simp}[(2*e^2*(d + e*x)^{(m-2)}*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(c*g*(2*m - 1)), x] - \text{Dist}[1/(c*g*(2*m - 1)), \text{Int}[(d + e*x)^{(m-3)}]*\text{Simp}[a*e^2*(d*g + 2*e*f*(m-2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m-1)*x^2, x]/(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]), x], x] /;$   $\text{FreeQ}[\{a, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{IntegerQ}[2*m] \&\& \text{GeQ}[m, 2]$

#### Rule 1584

$\text{Int}[(u_)*(x_)^{(m_)}*((a_)*(x_)^{(p_)} + (b_)*(x_)^{(q_)})^{(n_)}, x\_Symbol] \rightarrow \text{Int}[u*x^{(m+n*p)}*(a + b*x^{(q-p)})^n, x] /;$   $\text{FreeQ}[\{a, b, m, p, q\}, x] \&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p]$

#### Rubi steps

$$\begin{aligned}
\int (d+ex)^{3/2} (a+b \csc^{-1}(cx)) dx &= \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} + \frac{(2b) \int \frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{5ce} \\
&= \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{(d+ex)^{5/2}}{x\sqrt{-\frac{1}{c^2}+x^2}} dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right) \int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{d^3}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \dots\right) dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}} \\
&= \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} + \frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{5c\sqrt{1-\frac{1}{c^2x^2}}} + \frac{\left(2bd^3\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} dx}{5c\sqrt{1-\frac{1}{c^2x^2}}} \\
&= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}} + \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} + \frac{\left(6bd\sqrt{-\frac{1}{c^2}+x^2}\right) \int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}} dx}{5c\sqrt{1-\frac{1}{c^2x^2}}} \\
&= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}} + \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} - \frac{12bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}E}{5c^2\sqrt{1-\frac{1}{c^2x^2}}} \\
&= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}} + \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} - \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}E}{5c^2\sqrt{1-\frac{1}{c^2x^2}}} \\
&= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}} + \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} - \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}E}{5c^2\sqrt{1-\frac{1}{c^2x^2}}} \\
&= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}} + \frac{2(d+ex)^{5/2} (a+b \csc^{-1}(cx))}{5e} - \frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E}{15c^2\sqrt{1-\frac{1}{c^2x^2}}}
\end{aligned}$$

**Mathematica [C]** time = 1.42274, size = 333, normalized size = 0.9

$$\frac{1}{15} \left( \frac{4ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}} \left( (9c^2d^2 - 7cde + e^2) \text{EllipticF} \left( i \sinh^{-1} \left( \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right), \frac{cd+e}{cd-e} \right) - 3c^2d^2 \Pi \left( \frac{e}{cd} + 1; i \sinh^{-1} \left( \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right) \right) \right)}{c^3ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{c}{cd+e}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]



```
[Out] ((4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/e
+ (6*b*(d + e*x)^(5/2)*ArcCsc[c*x])/e - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-c*d
+ e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sq
rt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 - 7*c*
d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d +
e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)))]/(c^3*e*Sqrt[-(c/(c*d + e))]*Sq
rt[1 - 1/(c^2*x^2)]*x))/15
```

**Maple [B]** time = 0.278, size = 810, normalized size = 2.2

$$2 \frac{1}{e} \left( \frac{1}{5} (ex + d)^{5/2} a + b \left( \frac{1}{5} \operatorname{arccsc}(cx) (ex + d)^{5/2} + \frac{2}{15} \frac{1}{c^3 x} \left( \sqrt{\frac{c}{dc - e}} (ex + d)^{5/2} c^2 - 2 \sqrt{\frac{c}{dc - e}} (ex + d)^{3/2} c^2 d + 9 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a+b*arccsc(c*x)), x)
```

```
[Out] 2/e*(1/5*(e*x+d)^(5/2)*a+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)+2/15/c^3*((c/(c*d
-e))^(1/2)*(e*x+d)^(5/2)*c^2-2*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^2*d+9*d^2*
(-(e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ellip
ticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-7*(-((e*x
+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e
*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-3*d^2*(-((e*
x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))
^(1/2))*c^2+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^2*d^2+7*(-((e*x+d)*c-d*c+e)/(
c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-7*(-((e*x+d)*c-d*c+e)/(c*d-e
))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+(-(e*x+d)*c-d*c+e)/(c*d-e))^(1/2
)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*e^2)/(c/(c
*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1
/2)))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{3}{2}}(b \operatorname{arccsc}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

$$3.57 \quad \int \frac{x^3 (a + b \csc^{-1}(cx))}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=714

$$\frac{64bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{32bd\sqrt{1-c^2x^2}(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

[Out]  $(-4*b*\text{Sqrt}[d + e*x]*(1 - c^2*x^2))/(35*c^3*e*\text{Sqrt}[1 - 1/(c^2*x^2)]) + (4*b*d*\text{Sqrt}[d + e*x]*(1 - c^2*x^2))/(21*c^3*e^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x) - (2*d^3*\text{Sqrt}[d + e*x]*(a + b*\text{ArcCsc}[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*\text{ArcCsc}[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*\text{ArcCsc}[c*x]))/(5*e^4) + (2*(d + e*x)^(7/2)*(a + b*\text{ArcCsc}[c*x]))/(7*e^4) - (24*b*d^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(35*c^2*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]) + (4*b*(2*c^2*d^2 - 9*e^2)*\text{Sqrt}[d + e*x]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(105*c^4*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]) + (64*b*d^3*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(35*c^2*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x]) - (32*b*d*(c*d - e)*(c*d + e)*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(105*c^4*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x]) + (64*b*d^4*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(35*c*e^4*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 2.58945, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 17, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.81$ , Rules used = {43, 5247, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844, 942, 1654}

$$-\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x], x]

[Out]  $(-4*b*\text{Sqrt}[d + e*x]*(1 - c^2*x^2))/(35*c^3*e*\text{Sqrt}[1 - 1/(c^2*x^2)]) + (4*b*d*\text{Sqrt}[d + e*x]*(1 - c^2*x^2))/(21*c^3*e^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x) - (2*d^3*\text{Sqrt}[d + e*x]*(a + b*\text{ArcCsc}[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*\text{ArcCsc}[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*\text{ArcCsc}[c*x]))/(5*e^4) + (2*(d + e*x)^(7/2)*(a + b*\text{ArcCsc}[c*x]))/(7*e^4) - (24*b*d^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(35*c^2*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]) + (4*b*(2*c^2*d^2 - 9*e^2)*\text{Sqrt}[d + e*x]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(105*c^4*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]) + (64*b*d^3*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(35*c^2*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x]) - (32*b*d*(c*d - e)*(c*d + e)*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(105*c^4*e^3*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x]) + (64*b*d^4*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(35*c*e^4*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x])$

$1/(c^2*x^2)]*x*Sqrt[d + e*x])$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\text{!IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

#### Rule 5247

$\text{Int}[(a_.) + \text{ArcCsc}[(c_.)*(x_.)]*(b_.)]*(u_.), x\_Symbol] \rightarrow \text{With}\{v = \text{IntHide}[u, x]\}, \text{Dist}[a + b*\text{ArcCsc}[c*x], v, x] + \text{Dist}[b/c, \text{Int}[\text{SimplifyIntegrand}[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; \text{InverseFunctionFreeQ}[v, x] /; \text{FreeQ}\{a, b, c\}, x]$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 6721

$\text{Int}[(u_.)*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(x^{(n*\text{FracPart}[p])}*(1 + a/(x^n*b))^{\text{FracPart}[p]}), \text{Int}[u*x^{(n*p)}*(1 + a/(x^n*b))^p, x], x] /; \text{FreeQ}\{a, b, p\}, x \ \&\& \ \text{!IntegerQ}[p] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{!RationalFunctionQ}[u, x] \ \&\& \ \text{IntegerQ}[p + 1/2]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

#### Rule 719

$\text{Int}[(d_.) + (e_.)*(x_.))^{(m_.)}/\text{Sqrt}[(a_.) + (c_.)*(x_.)^2], x\_Symbol] \rightarrow \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/(c*\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] /; \text{FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{EqQ}[m^2, 1/4]$

#### Rule 424

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*(x_.)^2]/\text{Sqrt}[(c_.) + (d_.)*(x_.)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0]$

#### Rule 944

$\text{Int}[\text{Sqrt}[(f_.) + (g_.)*(x_.)]/(((d_.) + (e_.)*(x_.))*\text{Sqrt}[(a_.) + (c_.)*(x_.)^2]), x\_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[1/(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]), x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[1/((d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]), x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rule 932

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

### Rule 168

```
Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_)
]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rule 833

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

### Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 942

```
Int[((d_) + (e_)*(x_))^(m_)*Sqrt[(f_) + (g_)*(x_)]/Sqrt[(a_) + (c_)*
(x_)^2], x_Symbol] := Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*Sqrt[a + c
*x^2))/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)*Simp
[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(
4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d*g*(4*m - 1))*x^2, x)]/(Sqrt[f +
```

```
g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

#### Rule 1654

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \csc^{-1}(cx))}{\sqrt{d + ex}} dx &= -\frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} - \frac{6d (d + ex)^{5/2} (a + b \csc^{-1}(cx))}{5e^4} \\
&= -\frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} - \frac{6d (d + ex)^{5/2} (a + b \csc^{-1}(cx))}{5e^4} \\
&= -\frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} - \frac{6d (d + ex)^{5/2} (a + b \csc^{-1}(cx))}{5e^4} \\
&= -\frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} - \frac{6d (d + ex)^{5/2} (a + b \csc^{-1}(cx))}{5e^4} \\
&= -\frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} - \frac{6d (d + ex)^{5/2} (a + b \csc^{-1}(cx))}{5e^4} \\
&= -\frac{4b \sqrt{d + ex} (1 - c^2 x^2)}{35c^3 e \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{8bd \sqrt{d + ex} (1 - c^2 x^2)}{35c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4b \sqrt{d + ex} (1 - c^2 x^2)}{35c^3 e \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{4bd \sqrt{d + ex} (1 - c^2 x^2)}{21c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4b \sqrt{d + ex} (1 - c^2 x^2)}{35c^3 e \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{4bd \sqrt{d + ex} (1 - c^2 x^2)}{21c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4b \sqrt{d + ex} (1 - c^2 x^2)}{35c^3 e \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{4bd \sqrt{d + ex} (1 - c^2 x^2)}{21c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4b \sqrt{d + ex} (1 - c^2 x^2)}{35c^3 e \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{4bd \sqrt{d + ex} (1 - c^2 x^2)}{21c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{2d^3 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2d^2 (d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4}
\end{aligned}$$

**Mathematica [C]** time = 13.7471, size = 873, normalized size = 1.22

$$\frac{a\sqrt{\frac{ex}{d}} + 1B_{-\frac{ex}{d}}\left(4, \frac{1}{2}\right)d^4}{e^4\sqrt{d+ex}} + \frac{b \left( \frac{2\sqrt{\frac{d}{x}+e}\sqrt{cx} \left( \frac{2(40c^3ed^3+8ce^3d)\sqrt{\frac{cd+ce}{cd+e}}\sqrt{1-c^2x^2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}} \right) + \frac{2(48c^4d^4+16c^2e^2d^2+9e^4)\sqrt{\frac{cd+ce}{cd+e}}\sqrt{1-c^2x^2}\Gamma\left(2,\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}} \right)}{e^4\sqrt{d+ex}} \right)}{e^4\sqrt{d+ex}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x], x]
```

```
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*(-((c*(e + d/x))*x*((-4*(16*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105*e^
3) + (32*c^3*d^3*ArcCsc[c*x]))/(35*e^4) - (2*c^3*x^3*ArcCsc[c*x])/(7*e) - (4
*c^2*x^2*(e*Sqrt[1 - 1/(c^2*x^2)] - 3*c*d*ArcCsc[c*x]))/(35*e^2) + (4*c*x*(
5*c*d*e*Sqrt[1 - 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsc[c*x]))/(105*e^3))/Sqrt[d
+ e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(40*c^3*d^3*e + 8*c*d*e^3)*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2))
+ (2*(48*c^4*d^4 + 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*S
qrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-16*c^3*d^3*e
- 9*c*d*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sq
rt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-c*d) + e] + c*e*x
*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[
1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d
/x]*Sqrt[c*x]*(-2 + c^2*x^2)))/(105*e^4*Sqrt[d + e*x]))/c^4
```

**Maple [A]** time = 0.286, size = 1248, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2), x)
```

```
[Out] 2/e^4*(a*(1/7*(e*x+d)^(7/2)-3/5*(e*x+d)^(5/2)*d+(e*x+d)^(3/2)*d^2-d^3*(e*x+
d)^(1/2))+b*(1/7*arccsc(c*x)*(e*x+d)^(7/2)-3/5*arccsc(c*x)*(e*x+d)^(5/2)*d+
arccsc(c*x)*(e*x+d)^(3/2)*d^2-arccsc(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^4*(3*(c
/(c*d-e))^(1/2)*(e*x+d)^(7/2)*c^3-14*(c/(c*d-e))^(1/2)*(e*x+d)^(5/2)*c^3*d+
19*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^3*d^2-24*(-((e*x+d)*c-d*c+e)/(c*d-e))^(
1/2)*(-((e*x+d)*c-d*c+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e)
)^(1/2), ((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-16*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1
/2)*(-((e*x+d)*c-d*c+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(
1/2), ((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+48*d^3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(
```



$$\begin{aligned} & (1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticPi((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)}, 1/c*(c*d-e)/d, (c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)})*c^3-8*(c/(c*d-e))^{(1/2)}*(e*x+d)^{(1/2)}*c^3*d^3+16*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)})*c^2*d^2*e-16*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)})*c^2*d^2*e-3*(c/(c*d-e))^{(1/2)}*(e*x+d)^{(3/2)}*c*e^2+(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)})*c*d*e^2-9*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)})*c*d*e^2+8*(c/(c*d-e))^{(1/2)}*(e*x+d)^{(1/2)}*c*d*e^2+9*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)})*e^3-9*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)})*e^3)/(c/(c*d-e))^{(1/2)}/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^{(1/2)}) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))/(e\*x+d)\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x + d), x)
```

$$3.58 \quad \int \frac{x^2 (a + b \csc^{-1}(cx))}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=530

$$\frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{4b\sqrt{1-c^2x^2}(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/((15*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*d^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3) + (4*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(5*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e]]) - (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*c^4*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 1.90737, antiderivative size = 530, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {43, 5247, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844}

$$\frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} - \frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}}{15c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/((15*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*d^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3) + (4*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(5*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e]]) - (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*c^4*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rule 43**

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```

$Q[7*m + 4*n + 4, 0] \ || \ LtQ[9*m + 5*(n + 1), 0] \ || \ GtQ[m + n + 2, 0]$

#### Rule 5247

$Int[(a_.) + ArcCsc[(c_.)*(x_.)]*(b_.)]*(u_.), x\_Symbol] \ :> \ With[\{v = IntHide[u, x]\}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2*sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[\{a, b, c\}, x]$

#### Rule 12

$Int[(a_)*(u_), x\_Symbol] \ :> \ Dist[a, Int[u, x], x] /; FreeQ[a, x] \ \&\& \ !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]$

#### Rule 6721

$Int[(u_.)*((a_.) + (b_.)*(x_.)^n)]^p, x\_Symbol] \ :> \ Dist[(b^IntPart[p]*(a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), Int[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[\{a, b, p\}, x] \ \&\& \ !IntegerQ[p] \ \&\& \ !LtQ[n, 0] \ \&\& \ !RationalFunctionQ[u, x] \ \&\& \ IntegerQ[p + 1/2]$

#### Rule 6742

$Int[u_, x\_Symbol] \ :> \ With[\{v = ExpandIntegrand[u, x]\}, Int[v, x] /; SumQ[v]]$

#### Rule 719

$Int[((d_) + (e_.)*(x_.))^m]/sqrt[(a_) + (c_.)*(x_.)^2], x\_Symbol] \ :> \ Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*sqrt[1 + (c*x^2)/a])/(c*sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m]/sqrt[1 - x^2], x], x, sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[\{a, c, d, e\}, x] \ \&\& \ NeQ[c*d^2 + a*e^2, 0] \ \&\& \ EqQ[m^2, 1/4]$

#### Rule 424

$Int[sqrt[(a_) + (b_.)*(x_.)^2]/sqrt[(c_) + (d_.)*(x_.)^2], x\_Symbol] \ :> \ Simp[(sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[\{a, b, c, d\}, x] \ \&\& \ NegQ[d/c] \ \&\& \ GtQ[c, 0] \ \&\& \ GtQ[a, 0]$

#### Rule 944

$Int[sqrt[(f_.) + (g_.)*(x_.)]/(((d_.) + (e_.)*(x_.))*sqrt[(a_) + (c_.)*(x_.)^2]), x\_Symbol] \ :> \ Dist[g/e, Int[1/(sqrt[f + g*x]*sqrt[a + c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*sqrt[f + g*x]*sqrt[a + c*x^2]), x], x] /; FreeQ[\{a, c, d, e, f, g\}, x] \ \&\& \ NeQ[e*f - d*g, 0] \ \&\& \ NeQ[c*d^2 + a*e^2, 0]$

#### Rule 419

$Int[1/(sqrt[(a_) + (b_.)*(x_.)^2]*sqrt[(c_) + (d_.)*(x_.)^2]), x\_Symbol] \ :> \ Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(sqrt[a]*sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[\{a, b, c, d\}, x] \ \&\& \ NegQ[d/c] \ \&\& \ GtQ[c, 0] \ \&\& \ GtQ[a, 0] \ \&\& \ !(NegQ[b/a] \ \&\& \ SimplerSqrtQ[-(b/a), -(d/c)])$

#### Rule 932

```
Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(f_.) + (g_.)*(x_.)]*Sqrt[(a_.) + (c_.)*(x_.)^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

### Rule 168

```
Int[1/(((a_.) + (b_.)*(x_.))*Sqrt[(c_.) + (d_.)*(x_.)]*Sqrt[(e_.) + (f_.)*(x_.)]*Sqrt[(g_.) + (h_.)*(x_.)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 538

```
Int[1/(((a_.) + (b_.)*(x_)^2)*Sqrt[(c_.) + (d_.)*(x_)^2]*Sqrt[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_.) + (b_.)*(x_)^2)*Sqrt[(c_.) + (d_.)*(x_)^2]*Sqrt[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rule 833

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^(m*(a + c*x^2)^(p + 1)))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

### Rule 844

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rubi steps



**Mathematica [C]** time = 13.5729, size = 784, normalized size = 1.48

$$b \left[ \frac{2\sqrt{cx}\sqrt{\frac{d}{x}+e} \left( \frac{2\sqrt{1-c^2x^2}(7c^2d^2e+e^3)\sqrt{\frac{cd+ce}{cd+e}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}(cx)^3/2}\sqrt{\frac{d}{x}+e}} \right) - \frac{6cde \cos(2 \operatorname{csc}^{-1}(cx)) \left( c^2 dx \sqrt{1-c^2x^2} \sqrt{\frac{cd+ce}{cd+e}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right) - \frac{cx(cx+1)\sqrt{\frac{e}{c}}}{cd+e} \right)}{\sqrt{1-\frac{1}{c^2x^2}(cx)^3/2}\sqrt{\frac{d}{x}+e}}}{\sqrt{1-\frac{1}{c^2x^2}(cx)^3/2}\sqrt{\frac{d}{x}+e}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x], x]

[Out]  $-\left(\frac{a d^3 \sqrt{1 + (e x)/d} \operatorname{Beta}\left[-\frac{(e x)/d}{d}, 3, 1/2\right]}{e^3 \sqrt{d + e x}}\right) + \left(\frac{b \left(-\frac{c(e + d/x) x \left(4 c d \sqrt{1 - 1/(c^2 x^2)}\right)}{5 e^2} - \frac{16 c^2 d^2 \operatorname{ArcCsc}[c x]}{15 e^3} - \frac{2 c^2 x^2 \operatorname{ArcCsc}[c x]}{5 e} - \frac{4 c x (e \sqrt{1 - 1/(c^2 x^2)} - 2 c d \operatorname{ArcCsc}[c x])}{15 e^2}\right)}{\sqrt{d + e x}} - \frac{2 \sqrt{e + d/x} \sqrt{c x} \left(2 (7 c^2 d^2 e + e^3) \sqrt{(c d + c e x)/(c d + e)} \sqrt{1 - c^2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{1 - c x}/\sqrt{2}\right], (2 e)/(c d + e)\right]}{\sqrt{1 - 1/(c^2 x^2)} \sqrt{e + d/x} (c x)^{3/2}} + \frac{2 (8 c^3 d^3 + 3 c d e^2) \sqrt{(c d + c e x)/(c d + e)} \sqrt{1 - c^2 x^2} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\sqrt{1 - c x}/\sqrt{2}\right], (2 e)/(c d + e)\right]}{\sqrt{1 - 1/(c^2 x^2)} \sqrt{e + d/x} (c x)^{3/2}} - \frac{6 c d e \cos\left[2 \operatorname{ArcCsc}[c x]\right] \left((c d + c e x)(-1 + c^2 x^2) + c^2 d x \sqrt{(c d + c e x)/(c d + e)} \sqrt{1 - c^2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{1 - c x}/\sqrt{2}\right], (2 e)/(c d + e)\right] - (c x (1 + c x) \sqrt{(e - c e x)/(c d + e)} \sqrt{(c d + c e x)/(c d - e)} + (c d + e) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{(c d + c e x)/(c d - e)}\right], (c d - e)/(c d + e)\right] - e \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{(c d + c e x)/(c d - e)}\right], (c d - e)/(c d + e)\right]\right)}{\sqrt{(e(1 + c x))/(-c d + e)}} + \frac{c e x \sqrt{(c d + c e x)/(c d + e)} \sqrt{1 - c^2 x^2} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\sqrt{1 - c x}/\sqrt{2}\right], (2 e)/(c d + e)\right]}{\sqrt{1 - 1/(c^2 x^2)} \sqrt{e + d/x} \sqrt{c x} (-2 + c^2 x^2)}\right)}{15 e^3 \sqrt{d + e x}}\right)/c^3$

**Maple [A]** time = 0.276, size = 862, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x+d)^(1/2), x)

[Out]  $\frac{2}{e^3} \left( a \left( \frac{1}{5} (e x + d)^{5/2} - \frac{2}{3} (e x + d)^{3/2} d + d^2 (e x + d)^{1/2} \right) + b \left( \frac{1}{5} a \operatorname{arccsc}(c x) (e x + d)^{5/2} - \frac{2}{3} \operatorname{arccsc}(c x) (e x + d)^{3/2} d + \operatorname{arccsc}(c x) d^2 (e x + d)^{1/2} + \frac{2}{15} c^3 \left( \frac{c}{c d - e} \right)^{1/2} (e x + d)^{5/2} c^2 - 2 \left( \frac{c}{c d - e} \right)^{1/2} (e x + d)^{3/2} c^2 d + 4 d^2 \left( -\frac{(e x + d) c - d c + e}{c d - e} \right)^{1/2} \left( -\frac{(e x + d) c - d c - e}{c d + e} \right)^{1/2} \operatorname{EllipticF}\left(\frac{(e x + d)^{1/2} c}{c d - e}, \frac{(c d - e)}{c d + e}\right)^{1/2} c^2 + 3 \left( -\frac{(e x + d) c - d c + e}{c d - e} \right)^{1/2} \left( -\frac{(e x + d) c - d c - e}{c d + e} \right)^{1/2} \operatorname{EllipticE}\left(\frac{(e x + d)^{1/2} c}{c d - e}, \frac{(c d - e)}{c d + e}\right)^{1/2} c^2 d^2 - 8 d^2 \left( -\frac{(e x + d) c - d c + e}{c d - e} \right)^{1/2} \left( -\frac{(e x + d) c - d c - e}{c d + e} \right)^{1/2} \operatorname{EllipticPi}\left(\frac{(e x + d)^{1/2} c}{c d - e}, \frac{1}{c} \frac{c d - e}{d}, \frac{c}{c d + e}\right)^{1/2} \left( \frac{c}{c d - e} \right)^{1/2} c^2 + \left( \frac{c}{c d - e} \right)^{1/2} (e x + d)^{1/2} c^2 d^2 - 3 \left( -\frac{(e x + d) c - d c + e}{c d - e} \right)^{1/2} \left( -\frac{(e x + d) c - d c - e}{c d + e} \right)^{1/2} \right)$

```
2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e
+3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*E
llipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+(-((
e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(
1/2)*(e*x+d)^(1/2)*e^2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)
+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x + d), x)
```



$$3.59 \quad \int \frac{x(a+b \csc^{-1}(cx))}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=344

$$\frac{8bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{2d\sqrt{d+ex}(a+b \csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b \csc^{-1}(cx))}{3e^2}$$

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 1.60028, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$ , Rules used = {43, 5247, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537}

$$-\frac{2d\sqrt{d+ex}(a+b \csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b \csc^{-1}(cx))}{3e^2} + \frac{8bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHide[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]
```

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 6721

Int[(u\_)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 719

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

#### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 944

Int[Sqrt[(f\_.) + (g\_.)\*(x\_)]/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[g/e, Int[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] + Dist[(e\*f - d\*g)/e, Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0]

#### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

#### Rule 932

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(f\_.) + (g\_.)\*(x\_)]\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

#### Rule 168

Int[1/(((a\_.) + (b\_.)\*(x\_))\*Sqrt[(c\_.) + (d\_.)\*(x\_)]\*Sqrt[(e\_.) + (f\_.)\*(x\_)]\*Sqrt[(g\_.) + (h\_.)\*(x\_)]), x\_Symbol] := Dist[-2, Subst[Int[1/(Simp[b\*c - a\*d - b\*x^2, x]\*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]]), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e

, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \csc^{-1}(cx))}{\sqrt{d + ex}} dx &= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} + \frac{b \int \frac{2(-2d+ex)\sqrt{d+ex}}{3e^2\sqrt{1-\frac{1}{c^2x^2}}x^2} dx}{c} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} + \frac{(2b) \int \frac{(-2d+ex)\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2} dx}{3ce^2} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} + \frac{(2b\sqrt{1 - c^2x^2}) \int \frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1 - \frac{1}{c^2x^2}}x} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} + \frac{(2b\sqrt{1 - c^2x^2}) \int \left(\frac{e\sqrt{d+ex}}{\sqrt{1-c^2x^2}} - \frac{2d}{x\sqrt{1-c^2x^2}}\right) dx}{3ce^2\sqrt{1 - \frac{1}{c^2x^2}}x} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} - \frac{(4bd\sqrt{1 - c^2x^2}) \int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1 - \frac{1}{c^2x^2}}x} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} - \frac{(4bd^2\sqrt{1 - c^2x^2}) \int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1 - \frac{1}{c^2x^2}}x} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d + ex}\sqrt{1 - c^2x^2}E\left(\sin^{-1}\left(\frac{c\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\right)\right)}{3c^2e\sqrt{1 - \frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d + ex}\sqrt{1 - c^2x^2}E\left(\sin^{-1}\left(\frac{c\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\right)\right)}{3c^2e\sqrt{1 - \frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d + ex}\sqrt{1 - c^2x^2}E\left(\sin^{-1}\left(\frac{c\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\right)\right)}{3c^2e\sqrt{1 - \frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= -\frac{2d\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{2(d + ex)^{3/2}(a + b \csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d + ex}\sqrt{1 - c^2x^2}E\left(\sin^{-1}\left(\frac{c\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\right)\right)}{3c^2e\sqrt{1 - \frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

**Mathematica [C]** time = 1.42907, size = 289, normalized size = 0.84

$$2 \left( \frac{2ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cx}{cd+e}}\left((cd+e)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)+(cd-e)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)\right)-2cd\Pi\left(\frac{e}{cd}+1;i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{c}{cd+e}}}\right)$$


---


$$3e^2$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x], x]

[Out] (2\*(a\*(-2\*d + e\*x)\*Sqrt[d + e\*x] + b\*(-2\*d + e\*x)\*Sqrt[d + e\*x]\*ArcCsc[c\*x] + ((2\*I)\*b\*Sqrt[(e\*(1 + c\*x))/(-c\*d) + e])\*Sqrt[(e - c\*e\*x)/(c\*d + e)]\*((

$$\frac{c*d - e}{c*d - e} * \text{EllipticE}\left[\text{I} * \text{ArcSinh}\left[\text{Sqrt}\left[-\frac{c}{c*d + e}\right]\right] * \text{Sqrt}[d + e*x], \frac{c*d + e}{c*d - e}\right] + \frac{c*d + e}{c*d - e} * \text{EllipticF}\left[\text{I} * \text{ArcSinh}\left[\text{Sqrt}\left[-\frac{c}{c*d + e}\right]\right] * \text{Sqrt}[d + e*x], \frac{c*d + e}{c*d - e}\right] - 2*c*d * \text{EllipticPi}\left[1 + \frac{e}{c*d}, \text{I} * \text{ArcSinh}\left[\text{Sqrt}\left[-\frac{c}{c*d + e}\right]\right] * \text{Sqrt}[d + e*x], \frac{c*d + e}{c*d - e}\right] \Big/ \left(\frac{c^2 * \text{Sqrt}\left[-\frac{c}{c*d + e}\right] * \text{Sqrt}[1 - 1/(c^2*x^2)] * x\right) \Big/ (3*e^2)$$

**Maple [A]** time = 0.278, size = 412, normalized size = 1.2

$$2 \frac{1}{e^2} \left( a \left( \frac{1}{3} (ex + d)^{3/2} - d \sqrt{ex + d} \right) + b \left( \frac{1}{3} (ex + d)^{3/2} \operatorname{arccsc}(cx) - \operatorname{arccsc}(cx) d \sqrt{ex + d} - 2/3 \frac{1}{c^2 x} \left( d \operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{c^2 x^2}} \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsc(c\*x))/(e\*x+d)^(1/2),x)

[Out]  $2/e^2 * (a * (1/3 * (e*x+d)^{(3/2)} - d * (e*x+d)^{(1/2)}) + b * (1/3 * (e*x+d)^{(3/2)} * \operatorname{arccsc}(c*x) - \operatorname{arccsc}(c*x) * d * (e*x+d)^{(1/2)} - 2/3/c^2 * (d * \operatorname{EllipticF}((e*x+d)^{(1/2)} * (c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)}) * c + \operatorname{EllipticE}((e*x+d)^{(1/2)} * (c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)}) * c * d - 2 * d * \operatorname{EllipticPi}((e*x+d)^{(1/2)} * (c/(c*d-e))^{(1/2)}, 1/c * (c*d-e)/d, (c/(c*d+e))^{(1/2)} / (c/(c*d-e))^{(1/2)}) * c - \operatorname{EllipticF}((e*x+d)^{(1/2)} * (c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)}) * e + \operatorname{EllipticE}((e*x+d)^{(1/2)} * (c/(c*d-e))^{(1/2)}, ((c*d-e)/(c*d+e))^{(1/2)}) * e) * (-((e*x+d) * c - d * c - e) / (c*d+e))^{(1/2)} * (-((e*x+d) * c - d * c + e) / (c*d-e))^{(1/2)} / (c/(c*d-e))^{(1/2)} / x / ((c^2 * (e*x+d)^2 - 2 * d * c^2 * (e*x+d) + c^2 * d^2 - e^2) / c^2 / e^2 / x^2)^{(1/2)}))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(a + b \operatorname{arccsc}(cx))}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)`

[Out] `Integral(x*(a + b*acsc(c*x))/sqrt(d + e*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x/sqrt(e*x + d), x)`

$$3.60 \quad \int \frac{a+b \csc^{-1}(cx)}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=212

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b \csc^{-1}(cx))}{e} - \frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/ (c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 0.318515, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5227, 1574, 944, 719, 419, 933, 168, 538, 537}

$$\frac{2\sqrt{d+ex}(a+b \csc^{-1}(cx))}{e} - \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x], x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/ (c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x])/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_.) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

#### Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_
^2)]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] && !GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps



$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{\sqrt{d + ex}} dx &= \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e} + \frac{(2b) \int \frac{\sqrt{d+ex}}{\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{ce} \\
&= \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e} + \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{\sqrt{d+ex}}{x \sqrt{-\frac{1}{c^2} + x^2}} dx}{ce \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e} + \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{\sqrt{d+ex} \sqrt{-\frac{1}{c^2} + x^2}} dx}{c \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{\left(2bd \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x \sqrt{d+ex} \sqrt{-\frac{1}{c^2} + x^2}} dx}{ce \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e} + \frac{\left(2bd \sqrt{1 - c^2 x^2}\right) \int \frac{1}{x \sqrt{1-cx} \sqrt{1+cx} \sqrt{d+ex}} dx}{ce \sqrt{1 - \frac{1}{c^2 x^2}}} - \frac{\left(4b \sqrt{\frac{d+ex}{d+\frac{e}{c}}} \sqrt{1 - c^2 x^2}\right)}{c^2} \\
&= \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e} - \frac{4b \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2 \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{\left(4bd \sqrt{1 - c^2 x^2}\right) S}{c^2} \\
&= \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e} - \frac{4b \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2 \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{\left(4bd \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2}\right) S}{c^2} \\
&= \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e} - \frac{4b \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c^2 \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{4bd \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2}}{ce \sqrt{d + ex}}
\end{aligned}$$

**Mathematica [A]** time = 3.50003, size = 243, normalized size = 1.15

$$2 \left[ \frac{2bcx^2 \sqrt{cx+1} \sqrt{\frac{c(d+ex)}{cd+e}} \left( \cos\left(\frac{1}{2} \csc^{-1}(cx)\right) + \sin\left(\frac{1}{2} \csc^{-1}(cx)\right) \right) \left( \cos\left(\frac{1}{2} \csc^{-1}(cx)\right) - \sin\left(\frac{1}{2} \csc^{-1}(cx)\right) \right)^3 \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{\sqrt{1-cx}(c^2 x^2 - 1)} + \frac{a(d+ex)}{e} + \frac{b \left( \frac{2cdx}{\sqrt{d+ex}} \right)}{\sqrt{d+ex}} \right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/Sqrt[d + e\*x], x]

[Out] (2\*((a\*(d + e\*x))/e + (b\*((d + e\*x)\*ArcCsc[c\*x] + (2\*c\*d\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)])/Sqrt[1 - c^2\*x^2]))/e + (2\*b\*c\*x^2\*Sqrt[1 + c\*x]\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)]\*(Cos[ArcCsc[c\*x]/2] - Sin[ArcCsc[c\*x]/2])^3\*(Cos[ArcCsc[c\*x]/2] + Sin[ArcCsc[c\*x]/2]))/(Sqrt[1 - c\*x]\*(-1 + c^2\*x^2)))/Sqrt[d + e\*x]

**Maple [A]** time = 0.272, size = 254, normalized size = 1.2

$$2 \frac{1}{e} \left( a \sqrt{ex+d} + b \left( \sqrt{ex+d} \operatorname{arccsc}(cx) + 2 \frac{1}{cx} \sqrt{-\frac{(ex+d)c-dc+e}{dc-e}} \sqrt{-\frac{(ex+d)c-dc-e}{dc+e}} \right) \operatorname{EllipticF} \left( \sqrt{ex+d} \sqrt{\frac{c}{dc-e}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsc(c*x))/(e*x+d)^(1/2),x)`

[Out]  $2/e*(a*(e*x+d)^{(1/2)}+b*((e*x+d)^{(1/2)}*arccsc(c*x)+2/c*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*(EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})-EllipticPi((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)}))/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^{(1/2)}/x/(c/(c*d-e))^{(1/2)})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/(e*x+d)**(1/2),x)`

[Out] `Integral((a + b*acsc(c*x))/sqrt(d + e*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/sqrt(e*x + d), x)
```

$$3.61 \quad \int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x]), x]

**Rubi [A]** time = 0.0722318, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x]), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x]), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex}} dx = \int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex}} dx$$

**Mathematica [A]** time = 6.28854, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x]), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x]), x]

**Maple [A]** time = 3.53, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x} \frac{1}{\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x+d)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))/x/(e\*x+d)^(1/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{ex^2 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/(e\*x^2 + d\*x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{x\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x/(e\*x+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsc(c\*x))/(x\*sqrt(d + e\*x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(sqrt(e\*x + d)\*x), x)

$$3.62 \quad \int \frac{a+b \csc^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x^2 \sqrt{d+ex}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

**Rubi [A]** time = 0.079183, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x^2 \sqrt{d+ex}} dx = \int \frac{a+b \csc^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

**Mathematica [A]** time = 8.75705, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

**Maple [A]** time = 4.408, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x^2} \frac{1}{\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(1/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/(e\*x^3 + d\*x^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*2/(e\*x+d)\*\*(1/2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arccsc}(cx)+a}{\sqrt{ex+dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(sqrt(e\*x + d)\*x^2), x)

$$3.63 \quad \int \frac{x^3 (a + b \operatorname{csc}^{-1}(cx))}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=551

$$\frac{8bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

[Out]  $(-4*b*\operatorname{Sqrt}[d + e*x]*(1 - c^2*x^2))/(15*c^3*e^2*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x) + (2*d^3*(a + b*\operatorname{ArcCsc}[c*x]))/(e^4*\operatorname{Sqrt}[d + e*x]) + (6*d^2*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcCsc}[c*x]))/e^4 - (2*d*(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcCsc}[c*x]))/e^4 + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcCsc}[c*x]))/(5*e^4) + (32*b*d*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(15*c^2*e^3*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]) - (8*b*d^2*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(c^2*e^3*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (4*b*(2*c^2*d^2 + e^2)*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(15*c^4*e^3*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (64*b*d^3*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(5*c*e^4*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

**Rubi [A]** time = 2.47404, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {43, 5247, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2d^3(a + b \operatorname{csc}^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a + b \operatorname{csc}^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{e^4} + \frac{2(d+ex)^{5/2}(a + b \operatorname{csc}^{-1}(cx))}{5e^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3*(a + b*\operatorname{ArcCsc}[c*x]))/(d + e*x)^{(3/2)}, x]$

[Out]  $(-4*b*\operatorname{Sqrt}[d + e*x]*(1 - c^2*x^2))/(15*c^3*e^2*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x) + (2*d^3*(a + b*\operatorname{ArcCsc}[c*x]))/(e^4*\operatorname{Sqrt}[d + e*x]) + (6*d^2*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcCsc}[c*x]))/e^4 - (2*d*(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcCsc}[c*x]))/e^4 + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcCsc}[c*x]))/(5*e^4) + (32*b*d*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(15*c^2*e^3*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]) - (8*b*d^2*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(c^2*e^3*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (4*b*(2*c^2*d^2 + e^2)*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(15*c^4*e^3*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (64*b*d^3*\operatorname{Sqrt}[(c*(d + e*x))/(c*d + e)]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[2]], (2*e)/(c*d + e)])/(5*c*e^4*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

**Rule 43**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (!\operatorname{IntegerQ}[n] || (\operatorname{EqQ}[c, 0] \&\& \operatorname{Le}$



$Q[7*m + 4*n + 4, 0] \parallel LtQ[9*m + 5*(n + 1), 0] \parallel GtQ[m + n + 2, 0]$

#### Rule 5247

$Int[(a_.) + ArcCsc[(c_.)*(x_.)]*(b_.)]*(u_.), x\_Symbol] \rightarrow With[\{v = IntHide[u, x]\}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2*sqrt[1 - 1/(c^2*x^2)])], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[\{a, b, c\}, x]$

#### Rule 12

$Int[(a_.)*(u_.), x\_Symbol] \rightarrow Dist[a, Int[u, x], x] /; FreeQ[a, x] \&\& !MatchQ[u, (b_.)*(v_.) /; FreeQ[b, x]]$

#### Rule 6721

$Int[(u_.)*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow Dist[(b^{IntPart[p]}*(a + b*x^n)^{FracPart[p]})/(x^{(n*FracPart[p])}*(1 + a/(x^n*b))^{FracPart[p]}), Int[u*x^{(n*p)}*(1 + a/(x^n*b))^p, x], x] /; FreeQ[\{a, b, p\}, x] \&\& !IntegerQ[p] \&\& !LtQ[n, 0] \&\& !RationalFunctionQ[u, x] \&\& IntegerQ[p + 1/2]$

#### Rule 6742

$Int[u_, x\_Symbol] \rightarrow With[\{v = ExpandIntegrand[u, x]\}, Int[v, x] /; SumQ[v]]$

#### Rule 719

$Int[((d_.) + (e_.)*(x_.))^{(m_.)}/sqrt[(a_.) + (c_.)*(x_.)^2], x\_Symbol] \rightarrow Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*sqrt[1 + (c*x^2)/a])/(c*sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/sqrt[1 - x^2], x], x, sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[\{a, c, d, e\}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& EqQ[m^2, 1/4]$

#### Rule 419

$Int[1/(sqrt[(a_.) + (b_.)*(x_.)^2]*sqrt[(c_.) + (d_.)*(x_.)^2]), x\_Symbol] \rightarrow Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(sqrt[a]*sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[\{a, b, c, d\}, x] \&\& NegQ[d/c] \&\& GtQ[c, 0] \&\& GtQ[a, 0] \&\& !(NegQ[b/a] \&\& SimplerSqrtQ[-(b/a), -(d/c)])$

#### Rule 932

$Int[1/(((d_.) + (e_.)*(x_.))*sqrt[(f_.) + (g_.)*(x_.)]*sqrt[(a_.) + (c_.)*(x_.)^2]), x\_Symbol] \rightarrow With[\{q = Rt[-(c/a), 2]\}, Dist[1/sqrt[a], Int[1/((d + e*x)*sqrt[f + g*x]*sqrt[1 - q*x]*sqrt[1 + q*x]), x], x] /; FreeQ[\{a, c, d, e, f, g\}, x] \&\& NeQ[e*f - d*g, 0] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& GtQ[a, 0]$

#### Rule 168

$Int[1/(((a_.) + (b_.)*(x_.))*sqrt[(c_.) + (d_.)*(x_.)]*sqrt[(e_.) + (f_.)*(x_.)]*sqrt[(g_.) + (h_.)*(x_.)]), x\_Symbol] \rightarrow Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, sqrt[c + d*x]], x] /; FreeQ[\{a, b, c, d, e, f, g, h\}, x] \&\& GtQ[(d*e - c*f)/d, 0]$

#### Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

#### Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplersqrtQ[-(f/e), -(d/c)])]
```

#### Rule 844

```
Int[((d_) + (e_)*(x_)^m)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^p, x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

#### Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 931

```
Int[((d_) + (e_)*(x_)^m)/(Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a + c*x^2]/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x]/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[m, 2]
```

#### Rule 1584

```
Int[(u_)*(x_)^m*((a_)*(x_)^p + (b_)*(x_)^q)^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex)^{3/2}} dx &= \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= -\frac{4b\sqrt{d + ex} (1 - c^2 x^2)}{15c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2} x}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= -\frac{4b\sqrt{d + ex} (1 - c^2 x^2)}{15c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2} x}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= -\frac{4b\sqrt{d + ex} (1 - c^2 x^2)}{15c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2} x}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= -\frac{4b\sqrt{d + ex} (1 - c^2 x^2)}{15c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2} x}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots \\
&= -\frac{4b\sqrt{d + ex} (1 - c^2 x^2)}{15c^3 e^2 \sqrt{1 - \frac{1}{c^2 x^2} x}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{e^4} + \dots
\end{aligned}$$

**Mathematica [C]** time = 13.8299, size = 814, normalized size = 1.48

$$\frac{a \left(\frac{ex}{d} + 1\right)^{3/2} B_{-\frac{ex}{d}} \left(4, -\frac{1}{2}\right) d^4}{e^4 (d + ex)^{3/2}} + \frac{b \left( c^2 \left(\frac{d}{x} + e\right)^2 \left( \frac{2c^2 \csc^{-1}(cx) d^2}{e^3 \left(\frac{d}{x} + e\right)} - \frac{32c^2 \csc^{-1}(cx) d^2}{5e^4} + \frac{32c \sqrt{1 - \frac{1}{c^2 x^2} d}}{15e^3} - \frac{2c^2 x^2 \csc^{-1}(cx)}{5e^2} - \frac{2cx \left(2e \sqrt{1 - \frac{1}{c^2 x^2}} - 9cd \csc^{-1}(cx)\right)}{15e^3} \right) x^2}{(d + ex)^{3/2}} \right)}{e^4 (d + ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2), x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2))
+ (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 - 1/(c^2*x^2)])/(15*e^3) - (3
2*c^2*d^2*ArcCsc[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsc[c*x])/(e^3*(e + d/x)) -
(2*c^2*x^2*ArcCsc[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 - 1/(c^2*x^2)] - 9*c*d
*ArcCsc[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^(3/2)
*((2*(32*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(Sqrt[1 - 1/(c^2*
x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(48*c^3*d^3 + 8*c*d*e^2)*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)]/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) -
(16*c*d*e*cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[
(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt
[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c
*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*
d - e)]], (c*d - e)/(c*d + e)])))/Sqrt[(e*(1 + c*x))/(-c*d + e)] + c*e*x*S
qrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- c*x]/Sqrt[2]], (2*e)/(c*d + e)])))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sq
rt[c*x]*(-2 + c^2*x^2)))/(15*e^4*(d + e*x)^(3/2)))/c^4
```

**Maple [A]** time = 0.281, size = 890, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2), x)
```

```
[Out] 2/e^4*(a*(1/5*(e*x+d)^(5/2)-(e*x+d)^(3/2)*d+3*d^2*(e*x+d)^(1/2)+d^3/(e*x+d)
^(1/2))+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)-arccsc(c*x)*(e*x+d)^(3/2)*d+3*arcc
sc(c*x)*d^2*(e*x+d)^(1/2)+arccsc(c*x)*d^3/(e*x+d)^(1/2)+2/15/c^3*((c/(c*d-e
))^(1/2)*(e*x+d)^(5/2)*c^2-2*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^2*d+24*d^2*(
-(e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2), ((c*d-e)/(c*d+e))^(1/2))*c^2+8*(-((e*x+
d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2), ((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-48*d^2*(-((e*
x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2), 1/c*(c*d-e)/d, (c/(c*d+e))^(1/2)/(c/(c*d-e)
)^(1/2))*c^2+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^2*d^2-8*(-((e*x+d)*c-d*c+e)/(
c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2), ((c*d-e)/(c*d+e))^(1/2))*c*d*e+8*(-((e*x+d)*c-d*c+e)/(c*d-e
))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2), ((c*d-e)/(c*d+e))^(1/2))*c*d*e+(-(e*x+d)*c-d*c+e)/(c*d-e)^(1/2
))*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2), ((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*e^2)/(c/(c
*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1
/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

$$3.64 \quad \int \frac{x^2(a+b \operatorname{csc}^{-1}(cx))}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=369

$$\frac{20bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{2d^2(a+b \operatorname{csc}^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{2(d+ex)}{e^3}$$

```
[Out] (-2*d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*
b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*
d + e)]) + (20*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) + (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(
3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 1.81351, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {43, 5247, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424}

$$-\frac{2d^2(a+b \operatorname{csc}^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{3e^3} + \frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2; \operatorname{si}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2), x]
```

```
[Out] (-2*d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*
b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*
d + e)]) + (20*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) + (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(
3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; F
reeQ[{a, b, c}, x]
```

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 6721

Int[(u\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 719

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

#### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

#### Rule 932

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(f\_.) + (g\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

#### Rule 168

Int[1/(((a\_.) + (b\_.)\*(x\_))\*Sqrt[(c\_.) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_.) + (f\_.)\*(x\_)^2]\*Sqrt[(g\_.) + (h\_.)\*(x\_)]), x\_Symbol] := Dist[-2, Subst[Int[1/(Simp[b\*c - a\*d - b\*x^2, x]\*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]]), x], x, Sqrt[c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

#### Rule 538

Int[1/(((a\_) + (b\_.)\*(x\_)^2)\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/((a + b\*x^2)\*Sqrt[1 + (d\*x^2)/c]\*Sqrt[e + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

#### Rule 537

Int[1/(((a\_) + (b\_.)\*(x\_)^2)\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e)])/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0])

&& SimplerSqrtQ[-(f/e), -(d/c)]

### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] :> Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2 (a + b \csc^{-1}(cx))}{(d + ex)^{3/2}} dx &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} - \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} - \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} - \dots \\
 &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} - \frac{4d\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{2(d + ex)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} - \dots
 \end{aligned}$$



**Mathematica [C]** time = 13.6764, size = 750, normalized size = 2.03

$$b \left( \frac{2(cx)^{3/2} \left( \frac{d}{x} + e \right)^{3/2} \frac{10cde\sqrt{1-c^2x^2} \sqrt{\frac{cd+cx}{cd+e}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}(cx)^{3/2} \sqrt{\frac{d}{x}+e}} - \frac{2e \cos(2 \operatorname{csc}^{-1}(cx)) \left( c^2 dx \sqrt{1-c^2x^2} \sqrt{\frac{cd+cx}{cd+e}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right) - \frac{cx(cx+1) \sqrt{\frac{e-cex}{cd+e}} \sqrt{\frac{d}{x}+e}}{\sqrt{1-\frac{1}{c^2x^2}}} \right)}{\sqrt{1-\frac{1}{c^2x^2}}(cx)^{3/2} \sqrt{\frac{d}{x}+e}} \right)}{\sqrt{1-\frac{1}{c^2x^2}}(cx)^{3/2} \sqrt{\frac{d}{x}+e}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x)^(3/2), x]

[Out] -((a\*d^3\*(1 + (e\*x)/d)^(3/2)\*Beta[-((e\*x)/d), 3, -1/2])/(e^3\*(d + e\*x)^(3/2))) + (b\*(-((c^2\*(e + d/x)^2\*x^2\*((-4\*sqrt[1 - 1/(c^2\*x^2)]))/(3\*e^2) + (16\*c\*d\*ArcCsc[c\*x])/(3\*e^3) - (2\*c\*d\*ArcCsc[c\*x])/(e^2\*(e + d/x)) - (2\*c\*x\*ArcCsc[c\*x])/(3\*e^2)))/(d + e\*x)^(3/2)) + (2\*(e + d/x)^(3/2)\*(c\*x)^(3/2)\*((10\*c\*d\*e\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)]/(sqrt[1 - 1/(c^2\*x^2)]\*sqrt[e + d/x]\*(c\*x)^(3/2)) + (2\*(8\*c^2\*d^2 + e^2)\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)]/(sqrt[1 - 1/(c^2\*x^2)]\*sqrt[e + d/x]\*(c\*x)^(3/2)) - (2\*e\*cos[2\*ArcCsc[c\*x]]\*((c\*d + c\*e\*x)\*(-1 + c^2\*x^2) + c^2\*d\*x\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)] - (c\*x\*(1 + c\*x)\*sqrt[(e - c\*e\*x)/(c\*d + e)]\*sqrt[(c\*d + c\*e\*x)/(c\*d - e)]\*((c\*d + e)\*EllipticE[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - e)]], (c\*d - e)/(c\*d + e)] - e\*EllipticF[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - e)]], (c\*d - e)/(c\*d + e)])))/sqrt[(e\*(1 + c\*x))/(-c\*d + e)] + c\*e\*x\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)])))/(sqrt[1 - 1/(c^2\*x^2)]\*sqrt[e + d/x]\*sqrt[c\*x]\*(-2 + c^2\*x^2))))/(3\*e^3\*(d + e\*x)^(3/2)))/c^3

**Maple [A]** time = 0.278, size = 441, normalized size = 1.2

$$2 \frac{1}{e^3} \left( a \left( \frac{1}{3} (ex + d)^{3/2} - 2d\sqrt{ex + d} - \frac{d^2}{\sqrt{ex + d}} \right) + b \left( \frac{1}{3} (ex + d)^{3/2} \operatorname{arccsc}(cx) - 2 \operatorname{arccsc}(cx) d\sqrt{ex + d} - \frac{\operatorname{arccsc}(cx)}{\sqrt{ex + d}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x+d)^(3/2), x)

[Out] 2/e^3\*(a\*(1/3\*(e\*x+d)^(3/2)-2\*d\*(e\*x+d)^(1/2)-d^2/(e\*x+d)^(1/2))+b\*(1/3\*(e\*x+d)^(3/2)\*arccsc(c\*x)-2\*arccsc(c\*x)\*d\*(e\*x+d)^(1/2)-arccsc(c\*x)\*d^2/(e\*x+d)^(1/2))-2/3/c^2\*(4\*d\*EllipticF((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2), ((c\*d-e)/(c\*d+e))^(1/2))\*c+EllipticE((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2), ((c\*d-e)/(c\*d+e))^(1/2))\*c\*d-8\*d\*EllipticPi((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2), 1/c\*(c\*d-e)/d, (c/(c\*d+e))^(1/2)/(c/(c\*d-e))^(1/2))\*c-EllipticF((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2), ((c\*d-e)/(c\*d+e))^(1/2))\*e+EllipticE((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2), ((c\*d-e)/(c\*d+e))^(1/2))\*e)\*(-(e\*x+d)\*c-d\*c-e)/(c\*d+e)^(1/2)\*(-(e\*x+d)\*c-d\*

$$\frac{c+e}{(c*d-e)^{1/2}} / \left( \frac{c}{(c*d-e)^{1/2}} / x / \left( \frac{c^2*(e*x+d)^2 - 2*d*c^2*(e*x+d) + c^2*d^2 - e^2}{c^2/e^2/x^2} \right)^{1/2} \right)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsc(c\*x))/(e\*x+d)\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^2/(e\*x + d)^(3/2), x)

$$3.65 \quad \int \frac{x(a+b \csc^{-1}(cx))}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=238

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b \csc^{-1}(cx))}{e^2} + \frac{2d(a+b \csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{8bd\sqrt{1-c^2x^2}}{c^2e}$$

```
[Out] (2*d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 1.57528, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$ , Rules used = {43, 5247, 12, 6721, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{2\sqrt{d+ex}(a+b \csc^{-1}(cx))}{e^2} + \frac{2d(a+b \csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{8bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}}{c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHide[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

#### Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_
^2)]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

#### Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

#### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_
)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

#### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_
)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \csc^{-1}(cx))}{(d + ex)^{3/2}} dx &= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{b \int \frac{2(2d+ex)}{e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x^2 \sqrt{d+ex}} dx}{c} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{(2b) \int \frac{2d+ex}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 \sqrt{d+ex}} dx}{ce^2} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{(2b\sqrt{1 - c^2 x^2}) \int \frac{2d+ex}{x\sqrt{d+ex}\sqrt{1 - c^2 x^2}} dx}{ce^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{(2b\sqrt{1 - c^2 x^2}) \int \left( \frac{e}{\sqrt{d+ex}\sqrt{1 - c^2 x^2}} + \frac{1}{x\sqrt{d+ex}\sqrt{1 - c^2 x^2}} \right) dx}{ce^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{(4bd\sqrt{1 - c^2 x^2}) \int \frac{1}{x\sqrt{d+ex}\sqrt{1 - c^2 x^2}} dx}{ce^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} + \frac{(4bd\sqrt{1 - c^2 x^2}) \int \frac{1}{x\sqrt{1 - cx}\sqrt{1 + cx}\sqrt{d+ex}} dx}{ce^2 \sqrt{1 - \frac{1}{c^2 x^2}} x} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1 - cx}}{\sqrt{2}}\right), \frac{cd+e}{cd-e}\right)}{c^2 e \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1 - cx}}{\sqrt{2}}\right), \frac{cd+e}{cd-e}\right)}{c^2 e \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1 - c^2 x^2} F\left(\sin^{-1}\left(\frac{\sqrt{1 - cx}}{\sqrt{2}}\right), \frac{cd+e}{cd-e}\right)}{c^2 e \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + ex}}
\end{aligned}$$

**Mathematica [C]** time = 1.73969, size = 226, normalized size = 0.95

$$\frac{2 \left( -\frac{2ib\sqrt{\frac{e(cx+1)}{e-cd}} \sqrt{\frac{e-cex}{cd+e}} \left( \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right), \frac{cd+e}{cd-e}\right) - 2\text{Pi}\left(\frac{e}{cd} + 1; i \sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right), \frac{cd+e}{cd-e}\right)\right)}{cx\sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{-\frac{c}{cd+e}}} + \frac{a(2d+ex)}{\sqrt{d+ex}} + \frac{b \csc^{-1}(cx)(2d+ex)}{\sqrt{d+ex}} \right)}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x)^(3/2), x]

[Out] (2\*((a\*(2\*d + e\*x))/Sqrt[d + e\*x] + (b\*(2\*d + e\*x)\*ArcCsc[c\*x])/Sqrt[d + e\*x] - ((2\*I)\*b\*Sqrt[(e\*(1 + c\*x))/(-c\*d) + e])\*Sqrt[(e - c\*e\*x)/(c\*d + e)]\*(EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d + e))]]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)] - 2\*EllipticPi[1 + e/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d + e))]]\*Sqrt[d + e\*x]], (c\*d + e)/(c\*d - e)))/(c\*Sqrt[-(c/(c\*d + e))]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/e^2

---

**Maple [A]** time = 0.276, size = 277, normalized size = 1.2

$$2 \frac{1}{e^2} \left( a \left( \sqrt{ex+d} + \frac{d}{\sqrt{ex+d}} \right) + b \left( \sqrt{ex+d} \operatorname{arccsc}(cx) + \frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}} + 2 \frac{1}{cx} \sqrt{\frac{(ex+d)c-dc+e}{dc-e}} \sqrt{\frac{(ex+d)c-dc-e}{dc+e}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x)`

[Out]  $2/e^2*(a*((e*x+d)^{(1/2)}+d/(e*x+d)^{(1/2)})+b*((e*x+d)^{(1/2)}*\operatorname{arccsc}(c*x)+\operatorname{arccsc}(c*x)*d/(e*x+d)^{(1/2)}+2/c*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*(\operatorname{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})-2*\operatorname{EllipticPi}((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)}/(c/(c*d-e))^{(1/2)}))/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-2-e^2)/c^2/e^2/x^2)^{(1/2)}/x/(c/(c*d-e))^{(1/2)})$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(3/2), x)
```

### 3.66 $\int \frac{a+b \csc^{-1}(cx)}{(d+ex)^{3/2}} dx$

**Optimal.** Leaf size=119

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{2(a+b \csc^{-1}(cx))}{e\sqrt{d+ex}}$$

[Out]  $(-2*(a + b*\text{ArcCsc}[c*x]))/(e*\text{Sqrt}[d + e*x]) + (4*b*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(c*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 0.22537, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5227, 1574, 933, 168, 538, 537}

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{2(a+b \csc^{-1}(cx))}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCsc}[c*x])/(d + e*x)^{(3/2)}, x]$

[Out]  $(-2*(a + b*\text{ArcCsc}[c*x]))/(e*\text{Sqrt}[d + e*x]) + (4*b*\text{Sqrt}[(c*(d + e*x))/(c*d + e)]*\text{Sqrt}[1 - c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]], (2*e)/(c*d + e)))/(c*e*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x])$

#### Rule 5227

$\text{Int}[(a_.) + \text{ArcCsc}[(c_.)*(x_.)]*(b_.)]*((d_.) + (e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{ArcCsc}[c*x])]/(e*(m + 1)), x] + \text{Dist}[b/(c*e*(m + 1)), \text{Int}[(d + e*x)^{(m + 1)}/(x^2*\text{Sqrt}[1 - 1/(c^2*x^2)]), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 1574

$\text{Int}[(x_)^{(m_.)}*((a_.) + (c_.)*(x_)^{(mn2_.)})^{(p_.)}*((d_.) + (e_.)*(x_)^{(n_.)})^{(q_.)}, x\_Symbol] \rightarrow \text{Dist}[(x^{(2*n*FracPart[p])}*(a + c/x^{(2*n)})^{FracPart[p]})/(c + a*x^{(2*n)})^{FracPart[p]}, \text{Int}[x^{(m - 2*n*p)}*(d + e*x^n)^q*(c + a*x^{(2*n)})^p, x], x] /;$   
 $\text{FreeQ}\{a, c, d, e, m, n, p, q\}, x] \ \&\& \ \text{EqQ}[mn2, -2*n] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !\text{IntegerQ}[q] \ \&\& \ \text{PosQ}[n]$

#### Rule 933

$\text{Int}[1/(((d_.) + (e_.)*(x_.))*\text{Sqrt}[(f_.) + (g_.)*(x_.)]*\text{Sqrt}[(a_.) + (c_.)*(x_)^2]), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 2]\}, \text{Dist}[\text{Sqrt}[1 + (c*x^2)/a]/\text{Sqrt}[a + c*x^2], \text{Int}[1/((d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 - q*x]*\text{Sqrt}[1 + q*x]), x], x] /;$   
 $\text{FreeQ}\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ !\text{GtQ}[a, 0]$

#### Rule 168

$\text{Int}[1/(((a_.) + (b_.)*(x_.))*\text{Sqrt}[(c_.) + (d_.)*(x_.)]*\text{Sqrt}[(e_.) + (f_.)*(x_.)]*\text{Sqrt}[(g_.) + (h_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(\text{Simp}[b*c -$



$a*d - b*x^2, x]$ \*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]], x, Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

### Rule 538

Int[1/(((a\_) + (b\_.)\*(x\_)^2)\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/((a + b\*x^2)\*Sqrt[1 + (d\*x^2)/c]\*Sqrt[e + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

### Rule 537

Int[1/(((a\_) + (b\_.)\*(x\_)^2)\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e)])/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \csc^{-1}(cx)}{(d + ex)^{3/2}} dx &= -\frac{2(a + b \csc^{-1}(cx))}{e\sqrt{d + ex}} - \frac{(2b) \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2} x^2 \sqrt{d + ex}}} dx}{ce} \\
 &= -\frac{2(a + b \csc^{-1}(cx))}{e\sqrt{d + ex}} - \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x \sqrt{d + ex} \sqrt{-\frac{1}{c^2} + x^2}} dx}{ce \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
 &= -\frac{2(a + b \csc^{-1}(cx))}{e\sqrt{d + ex}} - \frac{\left(2b \sqrt{1 - c^2 x^2}\right) \int \frac{1}{x \sqrt{1 - cx} \sqrt{1 + cx} \sqrt{d + ex}} dx}{ce \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
 &= -\frac{2(a + b \csc^{-1}(cx))}{e\sqrt{d + ex}} + \frac{\left(4b \sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{1}{(1 - x^2) \sqrt{2 - x^2} \sqrt{d + \frac{e}{c} - \frac{ex^2}{c}} dx, x, \sqrt{1 - cx}\right)}{ce \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
 &= -\frac{2(a + b \csc^{-1}(cx))}{e\sqrt{d + ex}} + \frac{\left(4b \sqrt{\frac{c(d + ex)}{cd + e}} \sqrt{1 - c^2 x^2}\right) \text{Subst}\left(\int \frac{1}{(1 - x^2) \sqrt{2 - x^2} \sqrt{1 - \frac{ex^2}{c(d + \frac{e}{c})}} dx, x, \sqrt{1 - cx}\right)}{ce \sqrt{1 - \frac{1}{c^2 x^2} x} \sqrt{d + ex}} \\
 &= -\frac{2(a + b \csc^{-1}(cx))}{e\sqrt{d + ex}} + \frac{4b \sqrt{\frac{c(d + ex)}{cd + e}} \sqrt{1 - c^2 x^2} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1 - cx}}{\sqrt{2}}\right) \Big| \frac{2e}{cd + e}\right)}{ce \sqrt{1 - \frac{1}{c^2 x^2} x} \sqrt{d + ex}}
 \end{aligned}$$

**Mathematica [A]** time = 0.20256, size = 124, normalized size = 1.04

$$\frac{4bcx \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{1 - c^2 x^2} \sqrt{\frac{c(d + ex)}{cd + e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1 - cx}}{\sqrt{2}}\right) \Big| \frac{2e}{cd + e}\right) - 2(c^2 x^2 - 1)(a + b \csc^{-1}(cx))}{e(c^2 x^2 - 1) \sqrt{d + ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x)^(3/2),x]

[Out] (-2\*(-1 + c^2\*x^2)\*(a + b\*ArcCsc[c\*x]) + 4\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)]/(e\*Sqrt[d + e\*x]\*(-1 + c^2\*x^2))

**Maple [A]** time = 0.27, size = 217, normalized size = 1.8

$$2\frac{1}{e}\left(-\frac{a}{\sqrt{ex+d}}+b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+2\frac{1}{cdx}\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{dc-e}},\frac{dc}{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/(e\*x+d)^(3/2),x)

[Out] 2/e\*(-1/(e\*x+d)^(1/2)\*a+b\*(-1/(e\*x+d)^(1/2)\*arccsc(c\*x)+2/c/((c^2\*(e\*x+d)^2-2\*d\*c^2\*(e\*x+d)+c^2\*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c\*d-e))^(1/2)\*(-((e\*x+d)\*c-d\*c+e)/(c\*d-e))^(1/2)\*(-((e\*x+d)\*c-d\*c-e)/(c\*d+e))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2),1/c\*(c\*d-e)/d,(c/(c\*d+e))^(1/2)/(c/(c\*d-e))^(1/2))))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)}{e^2x^2+2dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(3/2), x)
```

$$3.67 \quad \int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x(d+ex)^{3/2}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(3/2)), x]

**Rubi [A]** time = 0.0889813, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(3/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(3/2)), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{3/2}} dx = \int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

**Mathematica [A]** time = 11.0698, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(3/2)), x]

**Maple [A]** time = 4.336, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x} (ex+d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x+d)^(3/2), x)

[Out] int((a+b\*arccsc(c\*x))/x/(e\*x+d)^(3/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{e^2x^3 + 2dex^2 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/(e^2\*x^3 + 2\*d\*e\*x^2 + d^2\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x/(e\*x+d)\*\*(3/2),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex+d)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/((e\*x + d)^(3/2)\*x), x)

$$3.68 \quad \int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{3/2}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

**Rubi [A]** time = 0.0938657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{3/2}} dx = \int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

**Mathematica [A]** time = 13.9046, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

**Maple [A]** time = 3.789, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x^2} (ex+d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(3/2), x)

[Out] int((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(3/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b\text{arccsc}(cx)+a)}{e^2x^4+2dex^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/(e^2\*x^4 + 2\*d\*e\*x^3 + d^2\*x^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*2/(e\*x+d)\*\*(3/2),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b\text{arccsc}(cx)+a}{(ex+d)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/((e\*x + d)^(3/2)\*x^2), x)

$$3.69 \quad \int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex)^{5/2}} dx$$

**Optimal.** Leaf size=602

$$\frac{32bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}$$

```
[Out] (-4*b*d^2*(1 - c^2*x^2))/(3*c*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (2*d^3*(a + b*ArcCsc[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^2*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^4) + (8*b*d^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*e^3*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2 - e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*c^2*e^3*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (32*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (64*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 2.89572, antiderivative size = 602, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 18, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$ , Rules used = {43, 5247, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419, 1651}

$$\frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^4} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2), x]
```

```
[Out] (-4*b*d^2*(1 - c^2*x^2))/(3*c*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (2*d^3*(a + b*ArcCsc[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^2*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^4) + (8*b*d^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*e^3*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2 - e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*c^2*e^3*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (32*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (64*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x]
```



$x]$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{IGtQ}[m, 0]$  && ( $!\text{IntegerQ}[n]$  || ( $\text{EqQ}[c, 0]$  &&  $\text{LeQ}[7*m + 4*n + 4, 0]$ ) ||  $\text{LtQ}[9*m + 5*(n + 1), 0]$  ||  $\text{GtQ}[m + n + 2, 0]$ )

#### Rule 5247

$\text{Int}[(a_.) + \text{ArcCsc}[(c_.)*(x_.)]*(b_.)]*(u_.), x\_Symbol] :> \text{With}[\{v = \text{IntHide}[u, x]\}, \text{Dist}[a + b*\text{ArcCsc}[c*x], v, x] + \text{Dist}[b/c, \text{Int}[\text{SimplifyIntegrand}[v/(x^2*\text{Sqrt}[1 - 1/(c^2*x^2)]), x], x], x] /; \text{InverseFunctionFreeQ}[v, x]] /; \text{FreeQ}[\{a, b, c\}, x]$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x]$  &&  $!\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 6721

$\text{Int}[(u_.)*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] :> \text{Dist}[(b^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(x^{(n*\text{FracPart}[p])}*(1 + a/(x^n*b))^{\text{FracPart}[p]}), \text{Int}[u*x^{(n*p)}*(1 + a/(x^n*b))^p, x], x] /; \text{FreeQ}[\{a, b, p\}, x]$  &&  $!\text{IntegerQ}[p]$  &&  $\text{ILtQ}[n, 0]$  &&  $!\text{RationalFunctionQ}[u, x]$  &&  $\text{IntegerQ}[p + 1/2]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] :> \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 745

$\text{Int}[(d_.) + (e_.)*(x_.)]^{(m_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] :> \text{Simp}[(e*(d + e*x)^{(m + 1)}*(a + c*x^2)^{(p + 1)})/((m + 1)*(c*d^2 + a*e^2)), x] + \text{Dist}[c/((m + 1)*(c*d^2 + a*e^2)), \text{Int}[(d + e*x)^{(m + 1)}*\text{Simp}[d*(m + 1) - e*(m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, c, d, e, m, p\}, x]$  &&  $\text{NeQ}[c*d^2 + a*e^2, 0]$  &&  $\text{NeQ}[m, -1]$  && ( $(\text{LtQ}[m, -1]$  &&  $\text{IntQuadraticQ}[a, 0, c, d, e, m, p, x])$  ||  $(\text{SumSimplerQ}[m, 1]$  &&  $\text{IntegerQ}[p])$  ||  $\text{ILtQ}[\text{Simplify}[m + 2*p + 3], 0]$ )

#### Rule 21

$\text{Int}[(u_.)*((a_.) + (b_.)*(v_.))^{(m_.)}*((c_.) + (d_.)*(v_.))^{(n_.)}, x\_Symbol] :> \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m + n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x]$  &&  $\text{EqQ}[b*c - a*d, 0]$  &&  $\text{IntegerQ}[m]$  && ( $!\text{IntegerQ}[n]$  ||  $\text{SimplerQ}[c + d*x, a + b*x]$ )

#### Rule 719

$\text{Int}[(d_.) + (e_.)*(x_.)]^{(m_.)}/\text{Sqrt}[(a_.) + (c_.)*(x_.)^2], x\_Symbol] :> \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/(c*\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] /; \text{FreeQ}[\{a, c, d, e\}, x]$  &&  $\text{NeQ}[c*d^2 + a*e^2, 0]$  &&  $\text{EqQ}[m^2, 1/4]$

#### Rule 424

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*(x_.)^2]/\text{Sqrt}[(c_.) + (d_.)*(x_.)^2], x\_Symbol] :> \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x]$  &&  $\text{NegQ}[d/c]$  &&  $\text{GtQ}[c, 0]$  &&  $\text{GtQ}[a, 0]$

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

`[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])`

### Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :>
  With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex)^{5/2}} dx &= \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2(d + ex)^{3/2}}{e^4} \\
&= \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2(d + ex)^{3/2}}{e^4} \\
&= \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2(d + ex)^{3/2}}{e^4} \\
&= \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} + \frac{2(d + ex)^{3/2}}{e^4} \\
&= -\frac{68bd^2 (1 - c^2 x^2)}{3ce^2 (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{68bd^2 (1 - c^2 x^2)}{3ce^2 (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4bd^2 (1 - c^2 x^2)}{3ce^2 (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4bd^2 (1 - c^2 x^2)}{3ce^2 (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4bd^2 (1 - c^2 x^2)}{3ce^2 (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4} \\
&= -\frac{4bd^2 (1 - c^2 x^2)}{3ce^2 (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} + \frac{2d^3 (a + b \csc^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \csc^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^4}
\end{aligned}$$

**Mathematica [C]** time = 13.935, size = 887, normalized size = 1.47

$$\frac{a \left(\frac{ex}{d} + 1\right)^{5/2} B_{-\frac{ex}{d}}\left(4, -\frac{3}{2}\right) d^4}{e^4 (d + ex)^{5/2}} + \frac{b \left( 2 \cos(2 \operatorname{csc}^{-1}(cx)) \left( dx \sqrt{\frac{cd+cx}{cd+e}} \sqrt{1-c^2x^2} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right) c^2 - \frac{x(cx+1) \sqrt{\frac{e-cex}{cd+e}} \sqrt{\frac{cd+cx}{cd-e}} \left( cd+e \right) \right)}{\right)}{\right)}{e^4 (d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsc[c\*x]))/(d + e\*x)^(5/2), x]

[Out] (a\*d^4\*(1 + (e\*x)/d)^(5/2)\*Beta[-((e\*x)/d), 4, -3/2])/(e^4\*(d + e\*x)^(5/2)) + (b\*(-((c^3\*(e + d/x)^3\*x^3\*(-4\*sqrt[1 - 1/(c^2\*x^2)]))/(3\*e\*(-(c^2\*d^2 + e^2)) + (32\*c\*d\*ArcCsc[c\*x])/(3\*e^4) - (2\*c\*d\*ArcCsc[c\*x])/(3\*e^2\*(e + d/x)^2) - (2\*c\*x\*ArcCsc[c\*x])/(3\*e^3) - (2\*(-2\*c^2\*d^2\*e\*sqrt[1 - 1/(c^2\*x^2)]) - 7\*c^3\*d^3\*ArcCsc[c\*x] + 7\*c\*d\*e^2\*ArcCsc[c\*x]))/(3\*e^3\*(-(c^2\*d^2 + e^2)\*(e + d/x)))))/(d + e\*x)^(5/2)) + (2\*(e + d/x)^(5/2)\*(c\*x)^(5/2)\*((2\*(8\*c^3\*d^3\*e - 8\*c\*d\*e^3)\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)]/(sqrt[1 - 1/(c^2\*x^2)]\*sqrt[e + d/x]\*(c\*x)^(3/2)) + (2\*(16\*c^4\*d^4 - 16\*c^2\*d^2\*e^2 - e^4)\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)]/(sqrt[1 - 1/(c^2\*x^2)]\*sqrt[e + d/x]\*(c\*x)^(3/2)) + (2\*e^3\*cos[2\*ArcCsc[c\*x]]\*((c\*d + c\*e\*x)\*(-1 + c^2\*x^2) + c^2\*d\*x\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)] - (c\*x\*(1 + c\*x)\*sqrt[(e - c\*e\*x)/(c\*d + e)]\*sqrt[(c\*d + c\*e\*x)/(c\*d - e)]\*((c\*d + e)\*EllipticE[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - e)]], (c\*d - e)/(c\*d + e)] - e\*EllipticF[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - e)]], (c\*d - e)/(c\*d + e)]))/sqrt[(e\*(1 + c\*x))/(-c\*d + e)] + c\*e\*x\*sqrt[(c\*d + c\*e\*x)/(c\*d + e)]\*sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/sqrt[2]], (2\*e)/(c\*d + e)]))/(sqrt[1 - 1/(c^2\*x^2)]\*sqrt[e + d/x]\*sqrt[c\*x]\*(-2 + c^2\*x^2))))/(3\*(c\*d - e)\*e^4\*(c\*d + e)\*(d + e\*x)^(5/2))))/c^4

**Maple [A]** time = 0.288, size = 1077, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x+d)^(5/2), x)

[Out] 2/e^4\*(a\*(1/3\*(e\*x+d)^(3/2)-3\*d\*(e\*x+d)^(1/2)-3\*d^2/(e\*x+d)^(1/2)+1/3\*d^3/(e\*x+d)^(3/2))+b\*(1/3\*(e\*x+d)^(3/2)\*arccsc(c\*x)-3\*arccsc(c\*x)\*d\*(e\*x+d)^(1/2)-3\*arccsc(c\*x)\*d^2/(e\*x+d)^(1/2)+1/3\*arccsc(c\*x)\*d^3/(e\*x+d)^(3/2)+2/3/c^2\*((c/(c\*d-e))^(1/2)\*(e\*x+d)^2\*c^3\*d^2-8\*(-((e\*x+d)\*c-d\*c+e)/(c\*d-e))^(1/2)\*(-((e\*x+d)\*c-d\*c+e)/(c\*d+e))^(1/2)\*EllipticF((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2)), ((c\*d-e)/(c\*d+e))^(1/2))\*(e\*x+d)^(1/2)\*c^3\*d^3+16\*(-((e\*x+d)\*c-d\*c+e)/(c\*d-e))^(1/2)\*(-((e\*x+d)\*c-d\*c+e)/(c\*d+e))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*(c/(c\*d-e))^(1/2), 1/c\*(c\*d-e)/d, (c/(c\*d+e))^(1/2)/(c/(c\*d-e))^(1/2))\*(e\*x+d)^(1/2)))/e^4\*(c\*d+e)\*(d+e\*x)^(5/2))

$$\begin{aligned} & 1/2)*c^3*d^3-2*(c/(c*d-e))^{1/2}*(e*x+d)*c^3*d^3+(c/(c*d-e))^{1/2}*c^3*d^4+ \\ & 7*(-((e*x+d)*c-d*c+e)/(c*d-e))^{1/2}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{1/2}*EllipticF((e*x+d)^{1/2}*(c/(c*d-e))^{1/2},((c*d-e)/(c*d+e))^{1/2})*(e*x+d)^{1/2} \\ & )^{1/2}*c*d*e^2+(-((e*x+d)*c-d*c+e)/(c*d-e))^{1/2}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{1/2}*EllipticE((e*x+d)^{1/2}*(c/(c*d-e))^{1/2},((c*d-e)/(c*d+e))^{1/2})*(e \\ & *x+d)^{1/2}*c*d*e^2-16*(-((e*x+d)*c-d*c+e)/(c*d-e))^{1/2}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{1/2}*EllipticPi((e*x+d)^{1/2}*(c/(c*d-e))^{1/2},1/c*(c*d-e)/d, \\ & (c/(c*d+e))^{1/2}/(c/(c*d-e))^{1/2})*(e*x+d)^{1/2}*c*d*e^2-(-((e*x+d)*c-d*c+e)/(c*d-e))^{1/2}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{1/2}*EllipticF((e*x+d)^{1/2} \\ & )^{1/2}*(c/(c*d-e))^{1/2},((c*d-e)/(c*d+e))^{1/2})*(e*x+d)^{1/2}*e^3+(-((e*x+d)*c-d*c+e)/(c*d-e))^{1/2}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{1/2}*EllipticE((e*x+d) \\ & )^{1/2}*(c/(c*d-e))^{1/2},((c*d-e)/(c*d+e))^{1/2})*(e*x+d)^{1/2}*e^3-(c/(c*d-e))^{1/2}*c*d^2*e^2)/(c*d-e)/(c/(c*d-e))^{1/2}/(e*x+d)^{1/2}/(c*d+e)/x/(( \\ & c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^{1/2} \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^3 \operatorname{arccsc}(cx) + ax^3)\sqrt{ex+d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x+d)^(5/2),x, algorithm="fricas")

[Out] integral((b\*x^3\*arccsc(c\*x) + a\*x^3)\*sqrt(e\*x + d)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))/(e\*x+d)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

$$3.70 \quad \int \frac{x^2 (a + b \csc^{-1}(cx))}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=440

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$

```
[Out] (4*b*d*(1 - c^2*x^2))/(3*c*e*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 2.18477, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 17, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.81$ , Rules used = {43, 5247, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419}

$$-\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{4bd(1-c^2x^2)}{3cex\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} - \frac{4bd\sqrt{1-c^2x^2}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2), x]
```

```
[Out] (4*b*d*(1 - c^2*x^2))/(3*c*e*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHide[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; F
```



reeQ[{a, b, c}, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 6721

Int[(u\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 745

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*Simp[d\*(m + 1) - e\*(m + 2\*p + 3)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2\*p + 3], 0])

### Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

### Rule 719

Int[((d\_) + (e\_)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

### Rule 424

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

### Rule 958

Int[((f\_) + (g\_)\*(x\_))^(n\_)/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), (f + g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

### Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

#### Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

#### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

#### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplrSqrtQ[-(f/e), -(d/c)])
```

#### Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

#### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplrSqrtQ[-(b/a), -(d/c)])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \csc^{-1}(cx))}{(d + ex)^{5/2}} dx &= -\frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{b \int \frac{2(8d^2)}{3e^3 \sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} \\
&= -\frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{(2b) \int \frac{8d}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} \\
&= -\frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{(2b\sqrt{1 - c^2 x^2})}{\sqrt{1 - c^2 x^2}} \\
&= -\frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{(2b\sqrt{1 - c^2 x^2})}{\sqrt{1 - c^2 x^2}} \\
&= -\frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} + \frac{(16bd^2 \sqrt{1 - c^2 x^2})}{\sqrt{1 - c^2 x^2}} \\
&= \frac{12bd (1 - c^2 x^2)}{ce (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} - \frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} \\
&= \frac{12bd (1 - c^2 x^2)}{ce (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} - \frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} \\
&= \frac{4bd (1 - c^2 x^2)}{3ce (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} - \frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} \\
&= \frac{4bd (1 - c^2 x^2)}{3ce (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} - \frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} \\
&= \frac{4bd (1 - c^2 x^2)}{3ce (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} - \frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3} \\
&= \frac{4bd (1 - c^2 x^2)}{3ce (c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x \sqrt{d + ex}}} - \frac{2d^2 (a + b \csc^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \csc^{-1}(cx))}{e^3}
\end{aligned}$$

**Mathematica [C]** time = 13.7893, size = 856, normalized size = 1.95

$$b \frac{c^3 \left(\frac{d}{x}+e\right)^3 \left( \frac{4c\sqrt{1-\frac{1}{c^2x^2}}d}{3e^2(e^2-c^2d^2)} + \frac{2\operatorname{csc}^{-1}(cx)}{3e\left(\frac{d}{x}+e\right)^2} - \frac{16\operatorname{csc}^{-1}(cx)}{3e^3} + \frac{4\left(-2c^2\operatorname{csc}^{-1}(cx)d^2-ce\sqrt{1-\frac{1}{c^2x^2}}d+2e^2\operatorname{csc}^{-1}(cx)\right)}{3e^2(e^2-c^2d^2)\left(\frac{d}{x}+e\right)} \right) x^3}{(d+ex)^{5/2}} - \frac{2\left(\frac{d}{x}+e\right)^{5/2}(cx)^{5/2}}{\frac{2(3c^2d^2e-3e^3)\sqrt{\frac{cd+cx}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticE}\left(\operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}}{\sqrt{1-\frac{1}{c^2x^2}}}\right]\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2), x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/2))) + (b*(-((c^3*(e + d/x)^3*x^3*((4*c*d*sqrt[1 - 1/(c^2*x^2)]))/(3*e^2*(-(c^2*d^2) + e^2)) - (16*ArcCsc[c*x])/(3*e^3) + (2*ArcCsc[c*x])/(3*e*(e + d/x)^2) + (4*(-(c*d*e*sqrt[1 - 1/(c^2*x^2)])) - 2*c^2*d^2*ArcCsc[c*x] + 2*e^2*ArcCsc[c*x]))/(3*e^2*(-(c^2*d^2) + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2*(3*c^2*d^2*e - 3*e^3)*sqrt[(c*d + c*e*x)/(c*d + e)]*sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(sqrt[1 - 1/(c^2*x^2)]*sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3 - 9*c*d*e^2)*sqrt[(c*d + c*e*x)/(c*d + e)]*sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(sqrt[1 - 1/(c^2*x^2)]*sqrt[e + d/x]*(c*x)^(3/2)) + (2*c*d*e*cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*sqrt[(c*d + c*e*x)/(c*d + e)]*sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*sqrt[(e - c*e*x)/(c*d + e)]*sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)])))/sqrt[(e*(1 + c*x))/(- (c*d) + e)] + c*e*x*sqrt[(c*d + c*e*x)/(c*d + e)]*sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])))/(sqrt[1 - 1/(c^2*x^2)]*sqrt[e + d/x]*sqrt[c*x]*(-2 + c^2*x^2)))/(3*(c*d - e)*e^3*(c*d + e)*(d + e*x)^(5/2)))/c^3
```

**Maple [B]** time = 0.289, size = 1038, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2), x)
```

```
[Out] 2/e^3*(a*((e*x+d)^(1/2)+2*d/(e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2))+b*((e*x+d)^(1/2)*arccsc(c*x)+2*arccsc(c*x)*d/(e*x+d)^(1/2)-1/3*arccsc(c*x)*d^2/(e*x+d)^(3/2)-2/3/c*((c/(c*d-e))^(1/2)*(e*x+d)^2*c^2*d-4*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2+8*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))))/c^3
```

$$\begin{aligned} & \left. \right)^{(1/2)} \cdot (e \cdot x + d)^{(1/2)} \cdot c^2 \cdot d^2 - 2 \cdot \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \cdot (e \cdot x + d) \cdot c^2 \cdot d^2 - \left( - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c - d \cdot c + e}{c \cdot d - e} \right)^{(1/2)} \cdot \left( - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c - d \cdot c - e}{c \cdot d + e} \right)^{(1/2)} \cdot \text{EllipticF} \left( \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \right), \left( \frac{c \cdot d - e}{c \cdot d + e} \right)^{(1/2)} \right) \cdot (e \cdot x + d)^{(1/2)} \cdot c \cdot d \cdot e + \right. \\ & \left. - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c - d \cdot c - e}{c \cdot d + e} \right)^{(1/2)} \cdot \text{EllipticE} \left( \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \right), \left( \frac{c \cdot d - e}{c \cdot d + e} \right)^{(1/2)} \right) \cdot (e \cdot x + d)^{(1/2)} \cdot c \cdot d \cdot e + \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \cdot c^2 \cdot d^3 + 3 \cdot \left( - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c - d \cdot c - e}{c \cdot d + e} \right)^{(1/2)} \cdot \text{EllipticF} \left( \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \right), \left( \frac{c \cdot d - e}{c \cdot d + e} \right)^{(1/2)} \right) \cdot (e \cdot x + d)^{(1/2)} \cdot e^2 - 8 \cdot \left( - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( - \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c - d \cdot c - e}{c \cdot d + e} \right)^{(1/2)} \cdot \text{EllipticPi} \left( \left( \frac{e \cdot x + d}{c \cdot d - e} \right)^{(1/2)} \cdot \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \right), 1/c \cdot (c \cdot d - e)/d, \left( \frac{c}{c \cdot d + e} \right)^{(1/2)} / \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \right) \cdot (e \cdot x + d)^{(1/2)} \cdot e^2 - \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} \cdot d \cdot e^2 / (c \cdot d - e) / \left( \frac{c}{c \cdot d - e} \right)^{(1/2)} / (e \cdot x + d)^{(1/2)} / (c \cdot d + e) / x / \left( \frac{c^2 \cdot (e \cdot x + d)^2 - 2 \cdot d \cdot c^2 \cdot (e \cdot x + d) + c^2 \cdot d^2 - e^2}{c^2 / e^2 / x^2} \right)^{(1/2)} \right) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx^2 \arccsc(cx) + ax^2) \sqrt{ex + d}}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x+d)^(5/2),x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsc(c\*x) + a\*x^2)\*sqrt(e\*x + d)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsc(c\*x))/(e\*x+d)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arccsc(cx) + a)x^2}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

$$3.71 \quad \int \frac{x(a+b \csc^{-1}(cx))}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=314

$$\frac{2(a+b \csc^{-1}(cx))}{e^2 \sqrt{d+ex}} + \frac{2d(a+b \csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{4b(1-c^2x^2)}{3cx \sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2) \sqrt{d+ex}} + \frac{4b \sqrt{1-c^2x^2} \sqrt{d+ex} E\left(\sin^{-1}\left(\frac{\sqrt{1-c^2x^2}}{\sqrt{2}}\right)\right)}{3ex \sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2) \sqrt{\frac{cd}{ca}}}$$

```
[Out] (-4*b*(1 - c^2*x^2))/(3*c*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d +
e*x]) + (2*d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
c[c*x]))/(e^2*Sqrt[d + e*x]) + (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*e*(c^2*d^2 - e^2)*Sq
rt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*Sqrt[(c*(d + e
x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]
], (2*e)/(c*d + e)))/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 2.00082, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$ , Rules used = {43, 5247, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537}

$$\frac{2(a+b \csc^{-1}(cx))}{e^2 \sqrt{d+ex}} + \frac{2d(a+b \csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{4b(1-c^2x^2)}{3cx \sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2) \sqrt{d+ex}} + \frac{4b \sqrt{1-c^2x^2} \sqrt{d+ex} E\left(\sin^{-1}\left(\frac{\sqrt{1-c^2x^2}}{\sqrt{2}}\right)\right)}{3ex \sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2) \sqrt{\frac{cd}{ca}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2), x]
```

```
[Out] (-4*b*(1 - c^2*x^2))/(3*c*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d +
e*x]) + (2*d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
c[c*x]))/(e^2*Sqrt[d + e*x]) + (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*e*(c^2*d^2 - e^2)*Sq
rt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*Sqrt[(c*(d + e
x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]
], (2*e)/(c*d + e)))/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; F
reeQ[{a, b, c}, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*
a + b*x^n)^FracPart[p]]/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d))]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
+ g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168



```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

#### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

#### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \csc^{-1}(cx))}{(d + ex)^{5/2}} dx &= \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{b \int \frac{2(-2d-3ex)}{3e^2\sqrt{1-\frac{1}{c^2x^2}x^2(d+ex)^{3/2}}} dx}{c} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{(2b) \int \frac{-2d-3ex}{\sqrt{1-\frac{1}{c^2x^2}x^2(d+ex)^{3/2}}} dx}{3ce^2} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{(2b\sqrt{1-c^2x^2}) \int \frac{-2d-3ex}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}x}} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{(2b\sqrt{1-c^2x^2}) \int \left(-\frac{3e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}} - \frac{2d}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\right) dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}x}} \\
&= \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{(4bd\sqrt{1-c^2x^2}) \int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}x}} - \frac{(2b\sqrt{1-c^2x^2}) \int \frac{2d}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}x}} \\
&= -\frac{4b(1-c^2x^2)}{c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}x}\sqrt{d+ex}} + \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{(4bd\sqrt{1-c^2x^2}) \int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}x}} \\
&= -\frac{4b(1-c^2x^2)}{c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}x}\sqrt{d+ex}} + \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{(4b\sqrt{1-c^2x^2}) \int \frac{2d}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}x}} \\
&= -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}x}\sqrt{d+ex}} + \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{(4b\sqrt{1-c^2x^2}) \int \frac{2d}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}x}} \\
&= -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}x}\sqrt{d+ex}} + \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{4b\sqrt{d+ex}}{e(c^2d^2-e^2)} \\
&= -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}x}\sqrt{d+ex}} + \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{4b\sqrt{d+ex}}{e(c^2d^2-e^2)} \\
&= -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}x}\sqrt{d+ex}} + \frac{2d(a + b \csc^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \csc^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{4b\sqrt{d+ex}}{3e(c^2d^2-e^2)}
\end{aligned}$$

**Mathematica [C]** time = 1.64907, size = 345, normalized size = 1.1

$$\frac{4ib\sqrt{-\frac{c}{cd+e}}\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}}\left(-cd\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)+cdE\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)\middle|\frac{cd+e}{cd-e}\right)+2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3c^2de^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x)^(5/2), x]

```
[Out] (4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*a*(2
*d + 3*e*x))/(3*e^2*(d + e*x)^(3/2)) - (2*b*(2*d + 3*e*x)*ArcCsc[c*x])/(3*e
^2*(d + e*x)^(3/2)) + (((4*I)/3)*b*Sqrt[-(c/(c*d + e))]*Sqrt[(e*(1 + c*x))/
(-c*d + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*EllipticE[I*ArcSinh[Sqrt[-(c
/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*EllipticF[I*ArcSinh
[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*(c*d + e)*El
lipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
e)/(c*d - e)))/(c^2*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)
```

**Maple [B]** time = 0.286, size = 913, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e^2*(a*(-1/(e*x+d)^(1/2)+1/3*d/(e*x+d)^(3/2))+b*(-1/(e*x+d)^(1/2)*arccsc(
c*x)+1/3*arccsc(c*x)*d/(e*x+d)^(3/2)+2/3*c*((c/(c*d-e))^(1/2)*(e*x+d)^2*c^2
*d+2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*
EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/
(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)
*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/
2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2+(-((e*x+d)*c-d*c+e)/(c*d-
e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*
d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-2*(c/(c*d-e))^(1
/2)*(e*x+d)*c^2*d^2-(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/
(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(
1/2))*(e*x+d)^(1/2)*c*d*e+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-
d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c
*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^3-2*(-((e*x+d)*c-
d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)
^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))
*(e*x+d)^(1/2)*e^2-(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(e*x+d)^(1/2)/(c*d+e)/(
c/(c*d-e))^(1/2)/d/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x
^2)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx \operatorname{arccsc}(cx) + ax)\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(5/2), x)
```

### 3.72 $\int \frac{a+b \csc^{-1}(cx)}{(d+ex)^{5/2}} dx$

**Optimal.** Leaf size=298

$$\frac{2(a+b \csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{4be(1-c^2x^2)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{1-c^2x^2}}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}}$$

```
[Out] (4*b*e*(1 - c^2*x^2))/(3*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x] - (2*(a + b*ArcCsc[c*x]))/(3*e*(d + e*x)^(3/2)) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 0.408926, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5227, 1574, 958, 745, 21, 719, 424, 933, 168, 538, 537}

$$\frac{2(a+b \csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{4be(1-c^2x^2)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{1-c^2x^2}}{3cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2), x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(3*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x] - (2*(a + b*ArcCsc[c*x]))/(3*e*(d + e*x)^(3/2)) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)))/(3*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_.) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol]
:> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

#### Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol]
:> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
```

+ g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

#### Rule 745

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*Simp[d\*(m + 1) - e\*(m + 2\*p + 3)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2\*p + 3], 0])

#### Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

#### Rule 719

Int[((d\_) + (e\_)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

#### Rule 424

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 933

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(f\_) + (g\_)\*(x\_)]\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c\*x^2)/a]/Sqrt[a + c\*x^2], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && !GtQ[a, 0]

#### Rule 168

Int[1/(((a\_) + (b\_)\*(x\_))\*Sqrt[(c\_) + (d\_)\*(x\_)]\*Sqrt[(e\_) + (f\_)\*(x\_)]\*Sqrt[(g\_) + (h\_)\*(x\_)]), x\_Symbol] := Dist[-2, Subst[Int[1/(Simp[b\*c - a\*d - b\*x^2, x]\*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]]), x], x, Sqrt[c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

#### Rule 538

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/((a + b\*x^2)\*Sqrt[1 + (d\*x^2)/c]\*Sqrt[e + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

#### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{(d + ex)^{5/2}} dx &= \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{(2b) \int \frac{1}{\sqrt{1 - \frac{1}{c^2}x^2} x^2 (d+ex)^{3/2}} dx}{3ce} \\
&= \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x(d+ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{3ce\sqrt{1 - \frac{1}{c^2}x^2}} \\
&= \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \left( \frac{e}{d(d+ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} + \frac{1}{dx\sqrt{d+ex} \sqrt{-\frac{1}{c^2} + x^2}} \right) dx}{3ce\sqrt{1 - \frac{1}{c^2}x^2}} \\
&= \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d+ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{3cd\sqrt{1 - \frac{1}{c^2}x^2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x\sqrt{d+ex} \sqrt{-\frac{1}{c^2} + x^2}} dx}{3cde\sqrt{1 - \frac{1}{c^2}x^2}} \\
&= \frac{4be(1 - c^2x^2)}{3cd(c^2d^2 - e^2)\sqrt{1 - \frac{1}{c^2}x^2}x\sqrt{d + ex}} - \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(4b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{-\frac{d}{2} - \frac{ex}{2}}{\sqrt{d+ex} \sqrt{-\frac{1}{c^2} + x^2}} dx}{3cd\left(d^2 - \frac{e^2}{c^2}\right)\sqrt{1 - \frac{1}{c^2}x^2}} \\
&= \frac{4be(1 - c^2x^2)}{3cd(c^2d^2 - e^2)\sqrt{1 - \frac{1}{c^2}x^2}x\sqrt{d + ex}} - \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2} + x^2}} dx}{3cd\left(d^2 - \frac{e^2}{c^2}\right)\sqrt{1 - \frac{1}{c^2}x^2}} + \dots \\
&= \frac{4be(1 - c^2x^2)}{3cd(c^2d^2 - e^2)\sqrt{1 - \frac{1}{c^2}x^2}x\sqrt{d + ex}} - \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1 - c^2x^2}\right) \text{Subst} \left( \int \dots \right)}{3cde\sqrt{1 - \frac{1}{c^2}x^2}} \\
&= \frac{4be(1 - c^2x^2)}{3cd(c^2d^2 - e^2)\sqrt{1 - \frac{1}{c^2}x^2}x\sqrt{d + ex}} - \frac{2(a + b \csc^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{4b\sqrt{d + ex}\sqrt{1 - c^2x^2}E\left(\sin^{-1}\left(\frac{\sqrt{1 - \frac{1}{c^2}x^2}}{\sqrt{2}}\right)\right)}{3d(c^2d^2 - e^2)\sqrt{1 - \frac{1}{c^2}x^2}x\sqrt{\frac{c(d+ex)}{cd+e}}}
\end{aligned}$$

**Mathematica [B]** time = 13.5131, size = 725, normalized size = 2.43

$$\frac{2(cx)^{5/2} \left(\frac{d}{x} + e\right)^{5/2} \frac{2cd\sqrt{1-c^2x^2} \sqrt{\frac{cd+cx}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right) - 2e \cos(2 \operatorname{csc}^{-1}(cx)) \left(c^2 dx \sqrt{1-c^2x^2} \sqrt{\frac{cd+cx}{cd+e}} \operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right) - \frac{cx(cx+1) \sqrt{\frac{e-cex}{cd+e}} \sqrt{\frac{cd+cx}{cd-e}} \left(\frac{cd+e}{cd+e}\right)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}} (cx)^{3/2} \sqrt{\frac{d}{x} + e}} \right)}{3e(cd-e)(cd+e)(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2), x]
```

```
[Out] (-2*a)/(3*e*(d + e*x)^(3/2)) + (b*(-((c^3*(e + d/x)^3*x^3*(-4*Sqrt[1 - 1/(c^2*x^2)]))/(3*c*d*(c^2*d^2 - e^2)) + (2*ArcCsc[c*x])/(3*c^2*d^2*e) + (2*e*ArcCsc[c*x])/(3*c^2*d^2*(e + d/x)^2) - (4*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) + c^2*d^2*ArcCsc[c*x] - e^2*ArcCsc[c*x]))/(3*c^2*d^2*(c^2*d^2 - e^2)*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2*c*d*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-c*d + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2)))/(3*(c*d - e)*e*(c*d + e)*(d + e*x)^(5/2)))/c
```

**Maple [B]** time = 0.279, size = 886, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(5/2), x)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsc(c*x)-2/3*c*((c/(c*d-e))^(1/2)*(e*x+d)^2*c^2*d-(((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-(((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2+(((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-2*(c/(c*d-e))^(1/2)*(e*x+d)*c^2*d^2-(((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(-(e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(c/(c*d-e))^(1/2)*c^
```



$$2*d^3+(-((e*x+d)*c-d*c+e)/(c*d-e))^{1/2}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{1/2}$$

$$*EllipticPi((e*x+d)^{1/2}*(c/(c*d-e))^{1/2},1/c*(c*d-e)/d,(c/(c*d+e))^{1/2}$$

$$/(c/(c*d-e))^{1/2})*(e*x+d)^{1/2}*e^2-(c/(c*d-e))^{1/2}*d*e^2)/(c*d-e)/(e*x$$

$$+d)^{1/2}/(c*d+e)/(c/(c*d-e))^{1/2}/d^2/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+$$

$$c^2*d^2-e^2)/c^2/e^2/x^2)^{1/2}))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x+d)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(e\*x + d)^(5/2), x)

$$3.73 \quad \int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x(d+ex)^{5/2}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(5/2)), x]

**Rubi [A]** time = 0.0881453, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(5/2)), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{5/2}} dx = \int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

**Mathematica [A]** time = 25.1508, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x)^(5/2)), x]

**Maple [A]** time = 3.867, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x} (ex+d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x+d)^(5/2), x)

[Out] int((a+b\*arccsc(c\*x))/x/(e\*x+d)^(5/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{e^3x^4 + 3de^2x^3 + 3d^2ex^2 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/(e^3\*x^4 + 3\*d\*e^2\*x^3 + 3\*d^2\*e\*x^2 + d^3\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x/(e\*x+d)\*\*(5/2),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex+d)^{\frac{5}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/((e\*x + d)^(5/2)\*x), x)

$$3.74 \quad \int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{5/2}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

**Rubi [A]** time = 0.101364, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{5/2}} dx = \int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

**Mathematica [A]** time = 25.371, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

**Maple [A]** time = 4.704, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2} (ex + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(5/2), x)

[Out] int((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(5/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arccsc}(cx) + a)}{e^3x^5 + 3de^2x^4 + 3d^2ex^3 + d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsc(c\*x) + a)/(e^3\*x^5 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^3 + d^3\*x^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*2/(e\*x+d)\*\*(5/2),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex+d)^{\frac{5}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/((e\*x + d)^(5/2)\*x^2), x)

### 3.75 $\int \frac{a+b \csc^{-1}(cx)}{(d+ex)^{7/2}} dx$

**Optimal.** Leaf size=540

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15dx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} - \frac{2(a+b \csc^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{4be(1-c^2x^2)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{15cx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}}$$

```
[Out] (4*b*e*(1 - c^2*x^2))/(15*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x)^(3/2)) + (16*b*c*e*(1 - c^2*x^2))/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) + (4*b*e*(1 - c^2*x^2))/(5*c*d^2*(c^2*d^2 - e^2)*S
qrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsc[c*x]))/(5*e*(d + e
*x)^(5/2)) - (4*b*(7*c^2*d^2 - 3*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*(c^2*d^3 - d*e^2)
^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*Sqrt[(c*(d
+ e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)]/(15*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
+ e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2
, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(5*c*d^2*e*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[d + e*x])
```

**Rubi [A]** time = 0.751693, antiderivative size = 637, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 14, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {5227, 1574, 958, 745, 835, 844, 719, 424, 419, 21, 933, 168, 538, 537}

$$-\frac{2(a+b \csc^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{16bce(1-c^2x^2)}{15x\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)^2\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{15cdx\sqrt{1-\frac{1}{c^2x^2}}(c^2d^2-e^2)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2), x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(15*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x)^(3/2)) + (16*b*c*e*(1 - c^2*x^2))/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) + (4*b*e*(1 - c^2*x^2))/(5*c*d^2*(c^2*d^2 - e^2)*S
qrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsc[c*x]))/(5*e*(d + e
*x)^(5/2)) - (16*b*c^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(5*d^2*(c^2
*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*S
qrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)]/(15*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[d + e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]/(5*c*d^2*e*Sqrt
[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

#### Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol
] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

Int[(x\_)^(m\_)\*((a\_) + (c\_)\*(x\_)^(mn2\_))^(p\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[(x^(2\*n\*FracPart[p])\*(a + c/x^(2\*n))^FracPart[p])/(c + a\*x^(2\*n))^FracPart[p], Int[x^(m - 2\*n\*p)\*(d + e\*x^n)^q\*(c + a\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2\*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

Rule 958

Int[((f\_) + (g\_)\*(x\_)^(n\_))/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), (f + g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

Rule 745

Int[((d\_) + (e\_)\*(x\_)^(m\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*Simp[d\*(m + 1) - e\*(m + 2\*p + 3)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2\*p + 3], 0])

Rule 835

Int[((d\_) + (e\_)\*(x\_)^(m\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p\*Simp[(c\*d\*f + a\*e\*g)\*(m + 1) - c\*(e\*f - d\*g)\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

Rule 844

Int[((d\_) + (e\_)\*(x\_)^(m\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

Rule 719

Int[((d\_) + (e\_)\*(x\_)^(m\_))/Sqrt[(a\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

Rule 424

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

### Rule 933

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)
^2]), x_Symbol] :=> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] && !GtQ[a, 0]
```

### Rule 168

```
Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_)
]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] :=> Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

### Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] :=> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

### Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] :=> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rubi steps



$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{(d + ex)^{7/2}} dx &= \frac{2(a + b \csc^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{(2b) \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2} x^2 (d + ex)^{5/2}}} dx}{5ce} \\
&= \frac{2(a + b \csc^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{x(d + ex)^{5/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{5ce \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= \frac{2(a + b \csc^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \left( -\frac{e}{d(d + ex)^{5/2} \sqrt{-\frac{1}{c^2} + x^2}} - \frac{e}{d^2(d + ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} + \frac{1}{d^2 x \sqrt{d + ex} \sqrt{-\frac{1}{c^2} + x^2}} \right) dx}{5ce \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= \frac{2(a + b \csc^{-1}(cx))}{5e(d + ex)^{5/2}} + \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{3/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{5cd^2 \sqrt{1 - \frac{1}{c^2 x^2} x}} + \frac{\left(2b \sqrt{-\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{5/2} \sqrt{-\frac{1}{c^2} + x^2}} dx}{5cd \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= \frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x} (d + ex)^{3/2}} + \frac{4be(1 - c^2 x^2)}{5cd^2(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x} \sqrt{d + ex}} - \frac{2(a + b \csc^{-1}(cx))}{5e(d + ex)^{5/2}} \\
&= \frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x} (d + ex)^{3/2}} + \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2 \sqrt{1 - \frac{1}{c^2 x^2} x} \sqrt{d + ex}} + \frac{4be}{5cd^2(c^2 d^2 - e^2)} \\
&= \frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x} (d + ex)^{3/2}} + \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2 \sqrt{1 - \frac{1}{c^2 x^2} x} \sqrt{d + ex}} + \frac{4be}{5cd^2(c^2 d^2 - e^2)} \\
&= \frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x} (d + ex)^{3/2}} + \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2 \sqrt{1 - \frac{1}{c^2 x^2} x} \sqrt{d + ex}} + \frac{4be}{5cd^2(c^2 d^2 - e^2)} \\
&= \frac{4be(1 - c^2 x^2)}{15cd(c^2 d^2 - e^2) \sqrt{1 - \frac{1}{c^2 x^2} x} (d + ex)^{3/2}} + \frac{16bce(1 - c^2 x^2)}{15(c^2 d^2 - e^2)^2 \sqrt{1 - \frac{1}{c^2 x^2} x} \sqrt{d + ex}} + \frac{4be}{5cd^2(c^2 d^2 - e^2)}
\end{aligned}$$

**Mathematica [A]** time = 14.053, size = 1002, normalized size = 1.86

$$b \left( 2 \left( \frac{d}{x} + e \right)^{7/2} (cx)^{7/2} \left[ \frac{2(c^2 d^2 e - c^3) \sqrt{\frac{cd+cx}{cd+e}} \sqrt{1-c^2 x^2} \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{\frac{d}{x} + e(cx)^{3/2}}} + \frac{2(3c^3 d^3 + ce^2 d) \sqrt{\frac{cd+cx}{cd+e}} \sqrt{1-c^2 x^2} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{\frac{d}{x} + e(cx)^{3/2}}} + \frac{2(3e^3 - 7c^2 d^2 e) \cos(2 \operatorname{csc}^{-1}(cx))}{\dots} \right] \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2), x]
```

```
[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((4*(-7*c^2*d^2 + 3*e^2)*Sqrt[1 - 1/(c^2*x^2)]))/(15*c^2*d^2*(-(c^2*d^2) + e^2)^2) + (2*ArcCs
c[c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsc[c*x])/(5*c^3*d^3*(e + d/x)^3) - (2*(2
*c*d*e^2*Sqrt[1 - 1/(c^2*x^2)] - 9*c^2*d^2*e*ArcCsc[c*x] + 9*e^3*ArcCsc[c*x
]))/(15*c^3*d^3*(c^2*d^2 - e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 - 1
/(c^2*x^2)] + 8*c*d*e^3*Sqrt[1 - 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsc[c*x] - 18*
c^2*d^2*e^2*ArcCsc[c*x] + 9*e^4*ArcCsc[c*x]))/(15*c^3*d^3*(c^2*d^2 - e^2)^2
*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*((2*(c^2*d^
2*e - e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e +
d/x]*(c*x)^(3/2)) + (2*(3*c^3*d^3 + c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*
Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-7*c^2*d^2*e
+ 3*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c
*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(
c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
- e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
- e)]], (c*d - e)/(c*d + e)])))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)])))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*
Sqrt[c*x]*(-2 + c^2*x^2))))/(15*c*d*(c*d - e)^2*e*(c*d + e)^2*(d + e*x)^(7/
2))))/c
```

**Maple [B]** time = 0.296, size = 1640, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(7/2), x)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsc(c*x)-2/15/c*(7*(c/(c
*d-e))^(1/2)*(e*x+d)^3*c^4*d^3-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x
+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2), 1/c*
(c*d-e)/d, (c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c^4*d^4-6*(-((
e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d-e))^(1/2), ((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^4
```

$$\begin{aligned}
& *d^4+7*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)} \\
& )*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(3/2)} \\
& *c^4*d^4-13*(c/(c*d-e))^{(1/2)}*(e*x+d)^2*c^4*d^4-7*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)} \\
& *(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)}) \\
& *(e*x+d)^{(3/2)}*c^3*d^3*e+7*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)} \\
& *EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(3/2)}*c^3*d^3*e-3 \\
& *(c/(c*d-e))^{(1/2)}*(e*x+d)^3*c^2*d*e^2+5*(c/(c*d-e))^{(1/2)}*(e*x+d)*c^4*d^5+6*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)} \\
& *(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticPi((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)}) \\
& /((c/(c*d-e))^{(1/2)}*(e*x+d)^{(3/2)}*c^2*d^2*e^2+2*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)} \\
& *EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(3/2)}*c^2*d^2*e^2-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)} \\
& *(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(3/2)}*c^2*d^2*e^2+5 \\
& *(c/(c*d-e))^{(1/2)}*(e*x+d)^2*c^2*d^2*e^2+(c/(c*d-e))^{(1/2)}*c^4*d^6+3*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)} \\
& *EllipticF((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(3/2)}*c*d*e^3-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)} \\
& *(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},((c*d-e)/(c*d+e))^{(1/2)})*(e*x+d)^{(3/2)}*c*d*e^3-8 \\
& *(c/(c*d-e))^{(1/2)}*(e*x+d)*c^2*d^3*e^2-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^{(1/2)}*(-((e*x+d)*c-d*c-e)/(c*d+e))^{(1/2)} \\
& *EllipticPi((e*x+d)^{(1/2)}*(c/(c*d-e))^{(1/2)},1/c*(c*d-e)/d,(c/(c*d+e))^{(1/2)})/(c/(c*d-e))^{(1/2)}*(e*x+d)^{(3/2)}*e^4-2*(c/(c*d-e))^{(1/2)}*c^2*d^4*e^2+3*(c/(c*d-e))^{(1/2)}*(e*x+d)*d*e^4+(c/(c*d-e))^{(1/2)}*d^2*e^4)/(c*d-e)/(e*x+d)^{(3/2)}/(c*d+e)/(c^2*d^2-e^2)/(c/(c*d-e))^{(1/2)}/d^3/x/((c^2*(e*x+d)^2-2*d*c^2*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^{(1/2)})
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x+d)^(7/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex + d)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(7/2), x)
```

### 3.76 $\int x^4 (d + ex^2) (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=206

$$\frac{1}{5}dx^5 (a + b \csc^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \csc^{-1}(cx)) + \frac{bx^4\sqrt{c^2x^2-1}(42c^2d+25e)}{840c^3\sqrt{c^2x^2}} + \frac{bx^2\sqrt{c^2x^2-1}(42c^2d+25e)}{560c^5\sqrt{c^2x^2}} + \frac{bx(42c^2d+25e)\sqrt{c^2x^2-1}}{560c^5\sqrt{c^2x^2}}$$

[Out] (b\*(42\*c^2\*d + 25\*e)\*x^2\*Sqrt[-1 + c^2\*x^2])/(560\*c^5\*Sqrt[c^2\*x^2]) + (b\*(42\*c^2\*d + 25\*e)\*x^4\*Sqrt[-1 + c^2\*x^2])/(840\*c^3\*Sqrt[c^2\*x^2]) + (b\*e\*x^6\*Sqrt[-1 + c^2\*x^2])/(42\*c\*Sqrt[c^2\*x^2]) + (d\*x^5\*(a + b\*ArcCsc[c\*x]))/5 + (e\*x^7\*(a + b\*ArcCsc[c\*x]))/7 + (b\*(42\*c^2\*d + 25\*e)\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(560\*c^6\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.125028, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {14, 5239, 12, 459, 321, 217, 206}

$$\frac{1}{5}dx^5 (a + b \csc^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \csc^{-1}(cx)) + \frac{bx^4\sqrt{c^2x^2-1}(42c^2d+25e)}{840c^3\sqrt{c^2x^2}} + \frac{bx^2\sqrt{c^2x^2-1}(42c^2d+25e)}{560c^5\sqrt{c^2x^2}} + \frac{bx(42c^2d+25e)\sqrt{c^2x^2-1}}{560c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*(42\*c^2\*d + 25\*e)\*x^2\*Sqrt[-1 + c^2\*x^2])/(560\*c^5\*Sqrt[c^2\*x^2]) + (b\*(42\*c^2\*d + 25\*e)\*x^4\*Sqrt[-1 + c^2\*x^2])/(840\*c^3\*Sqrt[c^2\*x^2]) + (b\*e\*x^6\*Sqrt[-1 + c^2\*x^2])/(42\*c\*Sqrt[c^2\*x^2]) + (d\*x^5\*(a + b\*ArcCsc[c\*x]))/5 + (e\*x^7\*(a + b\*ArcCsc[c\*x]))/7 + (b\*(42\*c^2\*d + 25\*e)\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(560\*c^6\*Sqrt[c^2\*x^2])

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_)\*(x\_)]\*(b\_))\*((f\_)\*(x\_))^(m\_)\*((d\_.) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 459

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n)\*(p

+ 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int x^4 (d + ex^2) (a + b \csc^{-1}(cx)) dx &= \frac{1}{5} dx^5 (a + b \csc^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^4(7d+5ex^2)}{35\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= \frac{1}{5} dx^5 (a + b \csc^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^4(7d+5ex^2)}{\sqrt{-1+c^2x^2}} dx}{35\sqrt{c^2x^2}} \\ &= \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{5} dx^5 (a + b \csc^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \csc^{-1}(cx)) - \frac{(bc(-42d - 2ex^2))}{2\sqrt{c^2x^2}} \\ &= \frac{b(42c^2d + 25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{5} dx^5 (a + b \csc^{-1}(cx)) + \frac{1}{7} ex^7 \\ &= \frac{b(42c^2d + 25e)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} + \frac{b(42c^2d + 25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \\ &= \frac{b(42c^2d + 25e)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} + \frac{b(42c^2d + 25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \\ &= \frac{b(42c^2d + 25e)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} + \frac{b(42c^2d + 25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.231158, size = 140, normalized size = 0.68

$$\frac{48ac^7x^5(7d+5ex^2) + bc^2x^2\sqrt{1-\frac{1}{c^2x^2}}(c^4(84dx^2+40ex^4) + 2c^2(63d+25ex^2) + 75e) + 3b(42c^2d+25e)\log\left(x\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{1680c^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]), x]

[Out]  $(48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e*x^2)*\text{ArcCsc}[c*x] + 3*b*(42*c^2*d + 25*e)*\text{Log}[(1 + \text{Sqrt}[1 - 1/(c^2*x^2)])*x])/(1680*c^7)$

**Maple [A]** time = 0.182, size = 338, normalized size = 1.6

$$\frac{aex^7}{7} + \frac{ax^5d}{5} + \frac{\text{barccsc}(cx)ex^7}{7} + \frac{\text{barccsc}(cx)x^5d}{5} + \frac{bx^6e}{42c} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bx^4e}{168c^3} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bx^4d}{20c} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bx^2d}{40c^3} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x)`

[Out]  $\frac{1}{7}aex^7 + \frac{1}{5}ax^5d + \frac{1}{7}b\text{arccsc}(cx)ex^7 + \frac{1}{5}b\text{arccsc}(cx)x^5d + \frac{1}{4} \frac{2/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^6*e + 1/168/c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^4*e + 1/20/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^4*d + 1/40/c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^2*d + 5/336/c^5*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*x^2*e - 3/40/c^5*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*d + 3/40/c^6*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*d*\ln(c*x+(c^2*x^2-1)^{(1/2)}) - 5/112/c^7*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*e + 5/112/c^8*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*e*\ln(c*x+(c^2*x^2-1)^{(1/2)})$

**Maxima [A]** time = 0.986806, size = 400, normalized size = 1.94

$$\frac{1}{7}aex^7 + \frac{1}{5}adx^5 + \frac{1}{80} \left( 16x^5 \text{arccsc}(cx) - \frac{2 \left( 3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 5 \sqrt{-\frac{1}{c^2x^2} + 1} \right)}{c^4 \left( \frac{1}{c^2x^2} - 1 \right)^2 + 2c^4 \left( \frac{1}{c^2x^2} - 1 \right) + c^4} - \frac{3 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} + 1 \right)}{c^4} + \frac{3 \log \left( \sqrt{-\frac{1}{c^2x^2} + 1} - 1 \right)}{c^4} \right) bd + \frac{1}{672}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{7}aex^7 + \frac{1}{5}ax^5d + \frac{1}{80} \left( 16x^5 \text{arccsc}(cx) - (2*(3*(-1/(c^2*x^2) + 1)^{(3/2)} - 5*\text{sqrt}(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*\log(\text{sqrt}(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*\log(\text{sqrt}(-1/(c^2*x^2) + 1) - 1)/c^4)/c \right) *b*d + \frac{1}{672} \left( 96*x^7*\text{arccsc}(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^{(5/2)} - 40*(-1/(c^2*x^2) + 1)^{(3/2)} + 33*\text{sqrt}(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*\log(\text{sqrt}(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*\log(\text{sqrt}(-1/(c^2*x^2) + 1) - 1)/c^6)/c \right) *b*e$

**Fricas [A]** time = 4.59651, size = 459, normalized size = 2.23

$$240ac^7ex^7 + 336ac^7dx^5 + 48(5bc^7ex^7 + 7bc^7dx^5 - 7bc^7d - 5bc^7e)\text{arccsc}(cx) - 96(7bc^7d + 5bc^7e)\text{arctan}\left(-cx + \frac{1}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] 1/1680\*(240\*a\*c^7\*e\*x^7 + 336\*a\*c^7\*d\*x^5 + 48\*(5\*b\*c^7\*e\*x^7 + 7\*b\*c^7\*d\*x^5 - 7\*b\*c^7\*d - 5\*b\*c^7\*e)\*arccsc(c\*x) - 96\*(7\*b\*c^7\*d + 5\*b\*c^7\*e)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) - 3\*(42\*b\*c^2\*d + 25\*b\*e)\*log(-c\*x + sqrt(c^2\*x^2 - 1)) + (40\*b\*c^5\*e\*x^5 + 2\*(42\*b\*c^5\*d + 25\*b\*c^3\*e)\*x^3 + 3\*(42\*b\*c^3\*d + 25\*b\*c\*e)\*x)\*sqrt(c^2\*x^2 - 1))/c^7

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{arccsc}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x\*\*2+d)\*(a+b\*acsc(c\*x)),x)

[Out] Integral(x\*\*4\*(a + b\*acsc(c\*x))\*(d + e\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arccsc}(cx) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*x^4, x)



### 3.77 $\int x^2 (d + ex^2) (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=161

$$\frac{1}{3}dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5}ex^5 (a + b \csc^{-1}(cx)) + \frac{bx^2\sqrt{c^2x^2-1}(20c^2d+9e)}{120c^3\sqrt{c^2x^2}} + \frac{bx(20c^2d+9e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}} + \dots$$

```
[Out] (b*(20*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*e*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + (d*x^3*(a + b*ArcCsc[c*x]))/3 + (e*x^5*(a + b*ArcCsc[c*x]))/5 + (b*(20*c^2*d + 9*e)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(120*c^4*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 0.107796, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {14, 5239, 12, 459, 321, 217, 206}

$$\frac{1}{3}dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5}ex^5 (a + b \csc^{-1}(cx)) + \frac{bx^2\sqrt{c^2x^2-1}(20c^2d+9e)}{120c^3\sqrt{c^2x^2}} + \frac{bx(20c^2d+9e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(20*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*e*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + (d*x^3*(a + b*ArcCsc[c*x]))/3 + (e*x^5*(a + b*ArcCsc[c*x]))/5 + (b*(20*c^2*d + 9*e)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(120*c^4*Sqrt[c^2*x^2])
```

#### Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 5239

```
Int[((a_) + ArcCsc[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m},
```

$n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m + n*(p + 1) + 1, 0]$

### Rule 321

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}, x\_Symbol] := \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n-1] \&\& \text{NeQ}[m+n*p+1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_*) + (b_*)*(x_)^2], x\_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /;$   $\text{FreeQ}[\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

### Rule 206

$\text{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$   $\text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int x^2 (d + ex^2) (a + b \csc^{-1}(cx)) dx &= \frac{1}{3} dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^2(5d+3ex^2) dx}{15\sqrt{-1+c^2x^2}}}{\sqrt{c^2x^2}} \\ &= \frac{1}{3} dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^2(5d+3ex^2) dx}{\sqrt{-1+c^2x^2}}}{15\sqrt{c^2x^2}} \\ &= \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \csc^{-1}(cx)) - \frac{(bc(-20d - 6ex^2))}{6\sqrt{c^2x^2}} \\ &= \frac{b(20c^2d + 9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \csc^{-1}(cx)) \\ &= \frac{b(20c^2d + 9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \csc^{-1}(cx)) \\ &= \frac{b(20c^2d + 9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + \frac{1}{3} dx^3 (a + b \csc^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \csc^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.166245, size = 121, normalized size = 0.75

$$\frac{c^2x^2 \left( 8ac^3x(5d + 3ex^2) + b\sqrt{1 - \frac{1}{c^2x^2}} (c^2(20d + 6ex^2) + 9e) \right) + b(20c^2d + 9e) \log \left( x \left( \sqrt{1 - \frac{1}{c^2x^2}} + 1 \right) \right) + 8bc^5x^3 \csc^{-1}(cx)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (c^2\*x^2\*(8\*a\*c^3\*x\*(5\*d + 3\*e\*x^2) + b\*Sqrt[1 - 1/(c^2\*x^2)]\*(9\*e + c^2\*(20\*d + 6\*e\*x^2))) + 8\*b\*c^5\*x^3\*(5\*d + 3\*e\*x^2)\*ArcCsc[c\*x] + b\*(20\*c^2\*d + 9\*e)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x])/(120\*c^5)

**Maple [B]** time = 0.184, size = 282, normalized size = 1.8

$$\frac{aex^5}{5} + \frac{adx^3}{3} + \frac{\operatorname{arccsc}(cx)ex^5}{5} + \frac{\operatorname{arccsc}(cx)dx^3}{3} + \frac{bx^4e}{20c} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bex^2}{40c^3} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bdx^2}{6c} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{bd}{6c^3} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x)`

[Out]  $\frac{1}{5}aex^5 + \frac{1}{3}adx^3 + \frac{1}{5}b\operatorname{arccsc}(cx)ex^5 + \frac{1}{3}b\operatorname{arccsc}(cx)dx^3 + \frac{1}{20c}bx^4e\sqrt{\frac{c^2x^2}{c^2x^2-1}} + \frac{1}{40c^3}bex^2\sqrt{\frac{c^2x^2}{c^2x^2-1}} + \frac{1}{6c}bdx^2\sqrt{\frac{c^2x^2}{c^2x^2-1}} - \frac{1}{6c^3}bd\sqrt{\frac{c^2x^2}{c^2x^2-1}}$

**Maxima [A]** time = 0.986601, size = 313, normalized size = 1.94

$$\frac{1}{5}aex^5 + \frac{1}{3}adx^3 + \frac{1}{12} \left( 4x^3 \operatorname{arccsc}(cx) + \frac{2\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}\right) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1-1}\right)}{c^2}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} \right) bd + \frac{1}{80} \left( 16x^5 \operatorname{arccsc}(cx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{5}aex^5 + \frac{1}{3}adx^3 + \frac{1}{12}(4x^3\operatorname{arccsc}(cx) + \frac{(2\sqrt{-1/(c^2x^2)+1} + \log(\sqrt{-1/(c^2x^2)+1}) - \log(\sqrt{-1/(c^2x^2)+1-1}))}{c^2})bd + \frac{1}{80}(16x^5\operatorname{arccsc}(cx) - (2*(3*(-1/(c^2x^2)+1)^{3/2} - 5\sqrt{-1/(c^2x^2)+1}))/c^4 + (1/(c^2x^2)-1)^2 + 2*c^4*(1/(c^2x^2)-1) + c^4 - 3*\log(\sqrt{-1/(c^2x^2)+1}) + 1)/c^4 + 3*\log(\sqrt{-1/(c^2x^2)+1}) - 1)/c^4)c)*b*e$

**Fricas [A]** time = 3.56358, size = 398, normalized size = 2.47

$$\frac{24ac^5ex^5 + 40ac^5dx^3 + 8(3bc^5ex^5 + 5bc^5dx^3 - 5bc^5d - 3bc^5e)\operatorname{arccsc}(cx) - 16(5bc^5d + 3bc^5e)\operatorname{arctan}\left(-cx + \sqrt{c^2x^2-1}\right)}{120c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out]  $\frac{1}{120}(24a*c^5*e*x^5 + 40a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 - 5*b*c^5*d - 3*b*c^5*e)*\operatorname{arccsc}(c*x) - 16*(5*b*c^5*d + 3*b*c^5*e)*\operatorname{arctan}(-c*x + \sqrt{c^2*x^2 - 1}) - (20*b*c^2*d + 9*b*e)*\log(-c*x + \sqrt{c^2*x^2 - 1}))$

$$+ (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1)/c^5$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{acsc}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x\*\*2+d)\*(a+b\*acsc(c\*x)),x)

[Out] Integral(x\*\*2\*(a + b\*acsc(c\*x))\*(d + e\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arccsc}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*x^2, x)

### 3.78 $\int (d + ex^2) (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=109

$$dx (a + b \csc^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \csc^{-1}(cx)) + \frac{bx(6c^2d + e) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2 - 1}}\right)}{6c^2\sqrt{c^2x^2}} + \frac{bex^2\sqrt{c^2x^2 - 1}}{6c\sqrt{c^2x^2}}$$

```
[Out] (b*e*x^2*Sqrt[-1 + c^2*x^2])/(6*c*Sqrt[c^2*x^2]) + d*x*(a + b*ArcCsc[c*x])
+ (e*x^3*(a + b*ArcCsc[c*x]))/3 + (b*(6*c^2*d + e)*x*ArcTanh[(c*x)/Sqrt[-1
+ c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 0.0509346, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5229, 12, 388, 217, 206}

$$dx (a + b \csc^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \csc^{-1}(cx)) + \frac{bx(6c^2d + e) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2 - 1}}\right)}{6c^2\sqrt{c^2x^2}} + \frac{bex^2\sqrt{c^2x^2 - 1}}{6c\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (b*e*x^2*Sqrt[-1 + c^2*x^2])/(6*c*Sqrt[c^2*x^2]) + d*x*(a + b*ArcCsc[c*x])
+ (e*x^3*(a + b*ArcCsc[c*x]))/3 + (b*(6*c^2*d + e)*x*ArcTanh[(c*x)/Sqrt[-1
+ c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

#### Rule 5229

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])], x], x]
]; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int (d + ex^2)(a + b \operatorname{csc}^{-1}(cx)) dx &= dx(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{3}ex^3(a + b \operatorname{csc}^{-1}(cx)) + \frac{(bcx) \int \frac{3d+ex^2}{3\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
 &= dx(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{3}ex^3(a + b \operatorname{csc}^{-1}(cx)) + \frac{(bcx) \int \frac{3d+ex^2}{\sqrt{-1+c^2x^2}} dx}{3\sqrt{c^2x^2}} \\
 &= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{3}ex^3(a + b \operatorname{csc}^{-1}(cx)) - \frac{(b(-6c^2d - e)x)}{6c\sqrt{c^2x^2}} \\
 &= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{3}ex^3(a + b \operatorname{csc}^{-1}(cx)) - \frac{(b(-6c^2d - e)x)}{6c\sqrt{c^2x^2}} \\
 &= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{3}ex^3(a + b \operatorname{csc}^{-1}(cx)) + \frac{b(6c^2d + e)x \operatorname{atanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.261444, size = 149, normalized size = 1.37

$$adx + \frac{1}{3}aex^3 + \frac{bdx\sqrt{1 - \frac{1}{c^2x^2}} \operatorname{tanh}^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} + \frac{bex^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{6c} + \frac{be \log\left(x\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}} + 1\right)\right)}{6c^3} + bdx \operatorname{csc}^{-1}(cx) + \frac{1}{3}bex^3 \operatorname{csc}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] a\*d\*x + (a\*e\*x^3)/3 + (b\*e\*x^2\*sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])/(6\*c) + b\*d\*x\*ArcCsc[c\*x] + (b\*e\*x^3\*ArcCsc[c\*x])/3 + (b\*d\*sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTanh[(c\*x)/sqrt[-1 + c^2\*x^2]]/sqrt[-1 + c^2\*x^2] + (b\*e\*Log[x\*(1 + sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)])])/(6\*c^3)

**Maple [B]** time = 0.184, size = 194, normalized size = 1.8

$$\frac{ax^3e}{3} + adx + \frac{b \operatorname{arccsc}(cx) x^3 e}{3} + b \operatorname{arccsc}(cx) x d + \frac{bd}{c^2 x} \sqrt{c^2 x^2 - 1} \ln\left(cx + \sqrt{c^2 x^2 - 1}\right) \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} + \frac{bex^2}{6c} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}} - \frac{be}{6c^3} \frac{1}{\sqrt{\frac{c^2 x^2 - 1}{c^2 x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsc(c\*x)), x)

[Out] 1/3\*a\*x^3\*e+a\*d\*x+1/3\*b\*arccsc(c\*x)\*x^3\*e+b\*arccsc(c\*x)\*x\*d+1/c^2\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*(c^2\*x^2-1)^(1/2)\*d\*ln(cx+(c^2\*x^2-1)^(1/2))+1/6/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*e\*x^2-1/6/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*e+1/6/c^4\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*e\*ln(cx+(c^2\*x^2-1)^(1/2))

**Maxima [A]** time = 1.01685, size = 207, normalized size = 1.9

$$\frac{1}{3} a e x^3 + \frac{1}{12} \left( 4 x^3 \operatorname{arccsc}(c x) + \frac{2 \sqrt{-\frac{1}{c^2 x^2} + 1} \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right) - \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right)}{c^2 \left(\frac{1}{c^2 x^2} - 1\right) + c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right)}{c^2} \right) b e + a d x + \frac{(2 c x \operatorname{arccsc}(c x) + \log(\dots))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] 1/3\*a\*e\*x^3 + 1/12\*(4\*x^3\*arccsc(c\*x) + (2\*sqrt(-1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) - 1) + c^2) + log(sqrt(-1/(c^2\*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*b\*e + a\*d\*x + 1/2\*(2\*c\*x\*arccsc(c\*x) + log(sqrt(-1/(c^2\*x^2) + 1) + 1) - log(-sqrt(-1/(c^2\*x^2) + 1) + 1))\*b\*d/c

**Fricas [A]** time = 3.45268, size = 327, normalized size = 3.

$$\frac{2 a c^3 e x^3 + 6 a c^3 d x + \sqrt{c^2 x^2 - 1} b c e x + 2 (b c^3 e x^3 + 3 b c^3 d x - 3 b c^3 d - b c^3 e) \operatorname{arccsc}(c x) - 4 (3 b c^3 d + b c^3 e) \arctan(-c x + \sqrt{c^2 x^2 - 1})}{6 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] 1/6\*(2\*a\*c^3\*e\*x^3 + 6\*a\*c^3\*d\*x + sqrt(c^2\*x^2 - 1)\*b\*c\*e\*x + 2\*(b\*c^3\*e\*x^3 + 3\*b\*c^3\*d\*x - 3\*b\*c^3\*d - b\*c^3\*e)\*arccsc(c\*x) - 4\*(3\*b\*c^3\*d + b\*c^3\*e)\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) - (6\*b\*c^2\*d + b\*e)\*log(-c\*x + sqrt(c^2\*x^2 - 1)))/c^3

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(c x)) (d + e x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*arccsc(c\*x)),x)

[Out] Integral((a + b\*arccsc(c\*x))\*(d + e\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (e x^2 + d) (b \operatorname{arccsc}(c x) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a), x)

$$3.79 \quad \int \frac{(d+ex^2)(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=87

$$-\frac{d(a+b \operatorname{csc}^{-1}(cx))}{x} + ex(a+b \operatorname{csc}^{-1}(cx)) - \frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} + \frac{bex \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

[Out]  $-\left(\frac{b*c*d*\sqrt{-1 + c^2*x^2}}{\sqrt{c^2*x^2}}\right) - \left(\frac{d*(a + b*\operatorname{ArcCsc}[c*x])}{x} + e*x*(a + b*\operatorname{ArcCsc}[c*x]) + \left(\frac{b*e*x*\operatorname{ArcTanh}[(c*x)/\sqrt{-1 + c^2*x^2}]}{\sqrt{c^2*x^2}}\right)\right)$

**Rubi [A]** time = 0.0645933, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5239, 451, 217, 206}

$$-\frac{d(a+b \operatorname{csc}^{-1}(cx))}{x} + ex(a+b \operatorname{csc}^{-1}(cx)) - \frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} + \frac{bex \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x^2,x]

[Out]  $-\left(\frac{b*c*d*\sqrt{-1 + c^2*x^2}}{\sqrt{c^2*x^2}}\right) - \left(\frac{d*(a + b*\operatorname{ArcCsc}[c*x])}{x} + e*x*(a + b*\operatorname{ArcCsc}[c*x]) + \left(\frac{b*e*x*\operatorname{ArcTanh}[(c*x)/\sqrt{-1 + c^2*x^2}]}{\sqrt{c^2*x^2}}\right)\right)$

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 5239

Int[((a\_) + ArcCsc[(c\_)\*(x\_)])\*(b\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 451

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[d/e^n, Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n\*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

#### Rule 217



$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)(x_)^2], x\_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; } \text{FreeQ}\{a, b\}, x\} \ \&\& \ !\text{GtQ}[a, 0]$

### Rule 206

$\text{Int}[(a_) + (b_.)(x_)^2]^{-1}, x\_Symbol] \text{ :> } \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; } \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \csc^{-1}(cx))}{x^2} dx &= -\frac{d(a + b \csc^{-1}(cx))}{x} + ex(a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{-d+ex^2}{x^2 \sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a + b \csc^{-1}(cx))}{x} + ex(a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{1}{\sqrt{-1+c^2x^2}}}{\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a + b \csc^{-1}(cx))}{x} + ex(a + b \csc^{-1}(cx)) + \frac{(bcx) \text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a + b \csc^{-1}(cx))}{x} + ex(a + b \csc^{-1}(cx)) + \frac{bcx \tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.119431, size = 104, normalized size = 1.2

$$-\frac{ad}{x} + aex - bcd\sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{bcx\sqrt{1-\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} - \frac{bd \csc^{-1}(cx)}{x} + bcx \csc^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x^2,x]

[Out] -((a\*d)/x) + a\*e\*x - b\*c\*d\*Sqrt[(-1 + c^2\*x^2)/(c^2\*x^2)] - (b\*d\*ArcCsc[c\*x])/x + b\*e\*x\*ArcCsc[c\*x] + (b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/Sqrt[-1 + c^2\*x^2]

**Maple [A]** time = 0.186, size = 136, normalized size = 1.6

$$aex - \frac{ad}{x} + \text{barccsc}(cx)ex - \frac{\text{barccsc}(cx)d}{x} - bcd\frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bd}{cx^2}\frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{be}{c^2x}\sqrt{c^2x^2-1}\ln\left(cx + \sqrt{c^2x^2-1}\right)\frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^2,x)

[Out] a\*e\*x-a\*d/x+b\*arccsc(c\*x)\*e\*x-b\*arccsc(c\*x)\*d/x-c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d+b/c/x^2/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d+b/c^2\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*e\*ln(c\*x+(c^2\*x^2-1)^(1/2))

**Maxima [A]** time = 0.988959, size = 120, normalized size = 1.38

$$-\left(c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x}\right)bd + aex + \frac{\left(2cx \operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2c} - \frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^2,x, algorithm="maxima")

[Out] -(c\*sqrt(-1/(c^2\*x^2) + 1) + arccsc(c\*x)/x)\*b\*d + a\*e\*x + 1/2\*(2\*c\*x\*arccsc(c\*x) + log(sqrt(-1/(c^2\*x^2) + 1) + 1) - log(-sqrt(-1/(c^2\*x^2) + 1) + 1)) \*b\*e/c - a\*d/x

**Fricas [A]** time = 2.90754, size = 288, normalized size = 3.31

$$\frac{bc^2dx - acex^2 + bex \log(-cx + \sqrt{c^2x^2 - 1}) + \sqrt{c^2x^2 - 1}bcd + acd - 2(bcd - bce)x \arctan(-cx + \sqrt{c^2x^2 - 1}) - (bcex^2 - \dots)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^2,x, algorithm="fricas")

[Out] -(b\*c^2\*d\*x - a\*c\*e\*x^2 + b\*e\*x\*log(-c\*x + sqrt(c^2\*x^2 - 1)) + sqrt(c^2\*x^2 - 1)\*b\*c\*d + a\*c\*d - 2\*(b\*c\*d - b\*c\*e)\*x\*arctan(-c\*x + sqrt(c^2\*x^2 - 1)) - (b\*c\*e\*x^2 - b\*c\*d + (b\*c\*d - b\*c\*e)\*x)\*arccsc(c\*x))/(c\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))(d + ex^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsc(c\*x))/x\*\*2,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arccsc}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^2,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^2, x)

$$3.80 \quad \int \frac{(d+ex^2)(a+b \operatorname{csc}^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=105

$$\frac{d(a+b \operatorname{csc}^{-1}(cx))}{3x^3} - \frac{e(a+b \operatorname{csc}^{-1}(cx))}{x} - \frac{bc\sqrt{c^2x^2-1}(2c^2d+9e)}{9\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

[Out]  $-(b*c*(2*c^2*d + 9*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(9*\operatorname{Sqrt}[c^2*x^2]) - (b*c*d*\operatorname{Sqrt}[-1 + c^2*x^2])/(9*x^2*\operatorname{Sqrt}[c^2*x^2]) - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(3*x^3) - (e*(a + b*\operatorname{ArcCsc}[c*x]))/x$

**Rubi [A]** time = 0.0783417, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5239, 12, 453, 264}

$$\frac{d(a+b \operatorname{csc}^{-1}(cx))}{3x^3} - \frac{e(a+b \operatorname{csc}^{-1}(cx))}{x} - \frac{bc\sqrt{c^2x^2-1}(2c^2d+9e)}{9\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x^2)*(a + b*\operatorname{ArcCsc}[c*x])/x^4, x]$

[Out]  $-(b*c*(2*c^2*d + 9*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(9*\operatorname{Sqrt}[c^2*x^2]) - (b*c*d*\operatorname{Sqrt}[-1 + c^2*x^2])/(9*x^2*\operatorname{Sqrt}[c^2*x^2]) - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(3*x^3) - (e*(a + b*\operatorname{ArcCsc}[c*x]))/x$

#### Rule 14

$\operatorname{Int}[(u_*)*((c_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*u, x], x] /;$  FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 5239

$\operatorname{Int}[(a_*) + \operatorname{ArcCsc}[(c_*)*(x_*)*(b_*)]*((f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(p_*)}), x\_Symbol] \rightarrow \operatorname{With}[\{u = \operatorname{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \operatorname{Dist}[a + b*\operatorname{ArcCsc}[c*x], u, x] + \operatorname{Dist}[(b*c*x)/\operatorname{Sqrt}[c^2*x^2], \operatorname{Int}[\operatorname{SimplifyIntegrand}[u/(x*\operatorname{Sqrt}[c^2*x^2 - 1]), x], x], x] /;$  FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

$\operatorname{Int}[(a_*)*(u_*), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 453

$\operatorname{Int}[(e_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*e^{(m+1)}), x] + \operatorname{Dist}[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), \operatorname{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

**Rule 264**

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]`

**Rubi steps**

$$\begin{aligned} \int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4} dx &= -\frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x} + \frac{(bcx) \int \frac{-d-3ex^2}{3x^4\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= -\frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x} + \frac{(bcx) \int \frac{-d-3ex^2}{x^4\sqrt{-1+c^2x^2}} dx}{3\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x} + \frac{(bc(-2c^2d-9e)x) \int}{9\sqrt{c^2x^2}} \\ &= -\frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x} \end{aligned}$$

**Mathematica [A]** time = 0.103917, size = 69, normalized size = 0.66

$$\frac{3a(d+3ex^2) + bcx\sqrt{1-\frac{1}{c^2x^2}}(2c^2dx^2+d+9ex^2) + 3b\csc^{-1}(cx)(d+3ex^2)}{9x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x^4, x]

[Out] -(3\*a\*(d + 3\*e\*x^2) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + 2\*c^2\*d\*x^2 + 9\*e\*x^2) + 3\*b\*(d + 3\*e\*x^2)\*ArcCsc[c\*x])/(9\*x^3)

**Maple [A]** time = 0.187, size = 121, normalized size = 1.2

$$c^3 \left( \frac{a}{c^2} \left( -\frac{e}{cx} - \frac{d}{3cx^3} \right) + \frac{b}{c^2} \left( -\frac{\operatorname{arccsc}(cx)e}{cx} - \frac{\operatorname{arccsc}(cx)d}{3cx^3} - \frac{(c^2x^2-1)(2c^4dx^2+9c^2ex^2+c^2d)}{9c^4x^4} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^4, x)

[Out] c^3\*(a/c^2\*(-e/c/x-1/3\*c\*d/x^3)+b/c^2\*(-arccsc(c\*x)\*e/c/x-1/3\*arccsc(c\*x)/c\*d/x^3-1/9\*(c^2\*x^2-1)\*(2\*c^4\*d\*x^2+9\*c^2\*e\*x^2+c^2\*d)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4)

**Maxima [A]** time = 0.983086, size = 127, normalized size = 1.21

$$-\left( c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x} \right) be + \frac{1}{9} bd \left( \frac{c^4 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} - \frac{3 \operatorname{arccsc}(cx)}{x^3} \right) - \frac{ae}{x} - \frac{ad}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^4,x, algorithm="maxima")

[Out]  $-(c\sqrt{-1/(c^2x^2) + 1} + \operatorname{arccsc}(cx)/x)*b*e + 1/9*b*d*((c^4*(-1/(c^2x^2) + 1)^{(3/2)} - 3*c^4*\sqrt{-1/(c^2x^2) + 1}))/c - 3*\operatorname{arccsc}(cx)/x^3 - a*e/x - 1/3*a*d/x^3$

**Fricas [A]** time = 2.60323, size = 157, normalized size = 1.5

$$\frac{9 a e x^2 + 3 a d + 3 (3 b e x^2 + b d) \operatorname{arccsc}(c x) + \sqrt{c^2 x^2 - 1} ((2 b c^2 d + 9 b e) x^2 + b d)}{9 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^4,x, algorithm="fricas")

[Out]  $-1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*\operatorname{arccsc}(c*x) + \sqrt{c^2*x^2 - 1}*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))(d + ex^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsc(c\*x))/x\*\*4,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arccsc}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^4,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^4, x)

$$3.81 \quad \int \frac{(d+ex^2)(a+b \operatorname{csc}^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=152

$$\frac{d(a+b \operatorname{csc}^{-1}(cx))}{5x^5} - \frac{e(a+b \operatorname{csc}^{-1}(cx))}{3x^3} - \frac{2bc^3\sqrt{c^2x^2-1}(12c^2d+25e)}{225\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}(12c^2d+25e)}{225x^2\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}}$$

[Out]  $(-2*b*c^3*(12*c^2*d + 25*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(225*\operatorname{Sqrt}[c^2*x^2]) - (b*c*d*\operatorname{Sqrt}[-1 + c^2*x^2])/(25*x^4*\operatorname{Sqrt}[c^2*x^2]) - (b*c*(12*c^2*d + 25*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(225*x^2*\operatorname{Sqrt}[c^2*x^2]) - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(5*x^5) - (e*(a + b*\operatorname{ArcCsc}[c*x]))/(3*x^3)$

**Rubi [A]** time = 0.0921755, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {14, 5239, 12, 453, 271, 264}

$$\frac{d(a+b \operatorname{csc}^{-1}(cx))}{5x^5} - \frac{e(a+b \operatorname{csc}^{-1}(cx))}{3x^3} - \frac{2bc^3\sqrt{c^2x^2-1}(12c^2d+25e)}{225\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}(12c^2d+25e)}{225x^2\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] `Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]`

[Out]  $(-2*b*c^3*(12*c^2*d + 25*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(225*\operatorname{Sqrt}[c^2*x^2]) - (b*c*d*\operatorname{Sqrt}[-1 + c^2*x^2])/(25*x^4*\operatorname{Sqrt}[c^2*x^2]) - (b*c*(12*c^2*d + 25*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(225*x^2*\operatorname{Sqrt}[c^2*x^2]) - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(5*x^5) - (e*(a + b*\operatorname{ArcCsc}[c*x]))/(3*x^3)$

#### Rule 14

`Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

#### Rule 5239

`Int[((a_) + ArcCsc[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))`

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 453

`Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c`

- a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \csc^{-1}(cx))}{x^6} dx &= -\frac{d(a + b \csc^{-1}(cx))}{5x^5} - \frac{e(a + b \csc^{-1}(cx))}{3x^3} + \frac{(bcx) \int \frac{-3d-5ex^2}{15x^6\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= -\frac{d(a + b \csc^{-1}(cx))}{5x^5} - \frac{e(a + b \csc^{-1}(cx))}{3x^3} + \frac{(bcx) \int \frac{-3d-5ex^2}{x^6\sqrt{-1+c^2x^2}} dx}{15\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{d(a + b \csc^{-1}(cx))}{5x^5} - \frac{e(a + b \csc^{-1}(cx))}{3x^3} + \frac{bc(-12c^2d - 25e)}{75\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{bc(12c^2d + 25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a + b \csc^{-1}(cx))}{5x^5} - \frac{e(a + b \csc^{-1}(cx))}{3x^3} \\ &= -\frac{2bc^3(12c^2d + 25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{bc(12c^2d + 25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.159635, size = 94, normalized size = 0.62

$$\frac{15a(3d + 5ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(3d(8c^4x^4 + 4c^2x^2 + 3) + 25ex^2(2c^2x^2 + 1)) + 15b \csc^{-1}(cx)(3d + 5ex^2)}{225x^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x^6,x]

[Out] -(15\*a\*(3\*d + 5\*e\*x^2) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(25\*e\*x^2\*(1 + 2\*c^2\*x^2) + 3\*d\*(3 + 4\*c^2\*x^2 + 8\*c^4\*x^4)) + 15\*b\*(3\*d + 5\*e\*x^2)\*ArcCsc[c\*x])/(225\*x^5)

**Maple [A]** time = 0.185, size = 140, normalized size = 0.9

$$c^5 \left( \frac{a}{c^2} \left( -\frac{d}{5c^3x^5} - \frac{e}{3c^3x^3} \right) + \frac{b}{c^2} \left( -\frac{\operatorname{arccsc}(cx)d}{5c^3x^5} - \frac{\operatorname{arccsc}(cx)e}{3c^3x^3} - \frac{(c^2x^2 - 1)(24c^6dx^4 + 50c^4ex^4 + 12c^4dx^2 + 25c^2ex^2)}{225c^6x^6} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^6,x)

[Out]  $c^5*(a/c^2*(-1/5/c^3*d/x^5-1/3*e/c^3/x^3)+b/c^2*(-1/5*arccsc(c*x)/c^3*d/x^5-1/3*arccsc(c*x)*e/c^3/x^3-1/225*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+25*c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))$

**Maxima [A]** time = 0.972544, size = 185, normalized size = 1.22

$$-\frac{1}{75}bd\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{-\frac{1}{c^2x^2}+1}+15\arccsc(cx)}{c}+\frac{15\arccsc(cx)}{x^5}\right)+\frac{1}{9}be\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^6,x, algorithm="maxima")

[Out]  $-1/75*b*d*((3*c^6*(-1/(c^2*x^2)+1)^(5/2)-10*c^6*(-1/(c^2*x^2)+1)^(3/2))+15*c^6*sqrt(-1/(c^2*x^2)+1))/c+15*arccsc(c*x)/x^5+1/9*b*e*((c^4*(-1/(c^2*x^2)+1)^(3/2)-3*c^4*sqrt(-1/(c^2*x^2)+1))/c-3*arccsc(c*x)/x^3)-1/3*a*e/x^3-1/5*a*d/x^5$

**Fricas [A]** time = 2.62581, size = 217, normalized size = 1.43

$$\frac{75aex^2+45ad+15(5bex^2+3bd)\arccsc(cx)+\left(2(12bc^4d+25bc^2e)x^4+(12bc^2d+25be)x^2+9bd\right)\sqrt{c^2x^2-1}}{225x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^6,x, algorithm="fricas")

[Out]  $-1/225*(75*a*e*x^2+45*a*d+15*(5*b*e*x^2+3*b*d)*arccsc(c*x)+(2*(12*b*c^4*d+25*b*c^2*e)*x^4+(12*b*c^2*d+25*b*e)*x^2+9*b*d)*sqrt(c^2*x^2-1))/x^5$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{arccsc}(cx))(d+ex^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsc(c\*x))/x\*\*6,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)/x\*\*6, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)(b\operatorname{arccsc}(cx)+a)}{x^6} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)/x^6, x)
```

$$3.82 \quad \int \frac{(d+ex^2)(a+b \operatorname{csc}^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=197

$$\frac{d(a+b \operatorname{csc}^{-1}(cx))}{7x^7} - \frac{e(a+b \operatorname{csc}^{-1}(cx))}{5x^5} - \frac{8bc^5\sqrt{c^2x^2-1}(30c^2d+49e)}{3675\sqrt{c^2x^2}} - \frac{4bc^3\sqrt{c^2x^2-1}(30c^2d+49e)}{3675x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}}{1225x^4}$$

[Out]  $(-8*b*c^5*(30*c^2*d + 49*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(3675*\operatorname{Sqrt}[c^2*x^2]) - (b*c*d*\operatorname{Sqrt}[-1 + c^2*x^2])/(49*x^6*\operatorname{Sqrt}[c^2*x^2]) - (b*c*(30*c^2*d + 49*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(1225*x^4*\operatorname{Sqrt}[c^2*x^2]) - (4*b*c^3*(30*c^2*d + 49*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(3675*x^2*\operatorname{Sqrt}[c^2*x^2]) - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(7*x^7) - (e*(a + b*\operatorname{ArcCsc}[c*x]))/(5*x^5)$

**Rubi [A]** time = 0.118038, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {14, 5239, 12, 453, 271, 264}

$$\frac{d(a+b \operatorname{csc}^{-1}(cx))}{7x^7} - \frac{e(a+b \operatorname{csc}^{-1}(cx))}{5x^5} - \frac{8bc^5\sqrt{c^2x^2-1}(30c^2d+49e)}{3675\sqrt{c^2x^2}} - \frac{4bc^3\sqrt{c^2x^2-1}(30c^2d+49e)}{3675x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}}{1225x^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x^2)*(a + b*\operatorname{ArcCsc}[c*x])/x^8, x]$

[Out]  $(-8*b*c^5*(30*c^2*d + 49*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(3675*\operatorname{Sqrt}[c^2*x^2]) - (b*c*d*\operatorname{Sqrt}[-1 + c^2*x^2])/(49*x^6*\operatorname{Sqrt}[c^2*x^2]) - (b*c*(30*c^2*d + 49*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(1225*x^4*\operatorname{Sqrt}[c^2*x^2]) - (4*b*c^3*(30*c^2*d + 49*e)*\operatorname{Sqrt}[-1 + c^2*x^2])/(3675*x^2*\operatorname{Sqrt}[c^2*x^2]) - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(7*x^7) - (e*(a + b*\operatorname{ArcCsc}[c*x]))/(5*x^5)$

#### Rule 14

$\operatorname{Int}[(u_*)((c_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*u, x], x] /;$  FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 5239

$\operatorname{Int}[(a_ + \operatorname{ArcCsc}[(c_)*(x_)]*(b_))*((f_)*(x_))^{(m_)}*((d_ + (e_)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{u = \operatorname{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \operatorname{Dist}[a + b*\operatorname{ArcCsc}[c*x], u, x] + \operatorname{Dist}[(b*c*x)/\operatorname{Sqrt}[c^2*x^2], \operatorname{Int}[\operatorname{SimplifyIntegrand}[u/(x*\operatorname{Sqrt}[c^2*x^2 - 1]), x], x], x] /;$  FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

$\operatorname{Int}[(a_)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 453

$\operatorname{Int}[(e_)*(x_))^{(m_)}*((a_ + (b_)*(x_)^{(n_))^{(p_)}*((c_ + (d_)*(x_)^{(n_)})), x\_Symbol] \rightarrow \operatorname{Simp}[(c*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*e*(m+1)),$

$x] + \text{Dist}[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), \text{Int}[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{IntegerQ}[n] \|\| \text{GtQ}[e, 0]) \&\& ((\text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1]) \|\| (\text{LtQ}[n, 0] \&\& \text{GtQ}[m + n, -1])) \&\& !\text{ILtQ}[p, -1]$

### Rule 271

$\text{Int}[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x\_Symbol] := \text{Simp}[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - \text{Dist}[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), \text{Int}[x^(m + n)*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{ILtQ}[\text{Simplify}[(m + 1)/n + p + 1], 0] \&\& \text{NeQ}[m, -1]$

### Rule 264

$\text{Int}[(c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x\_Symbol] := \text{Simp}[(c*x)^(m + 1)*(a + b*x^n)^(p + 1)/(a*c*(m + 1)), x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x\} \&\& \text{EqQ}[(m + 1)/n + p + 1, 0] \&\& \text{NeQ}[m, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \csc^{-1}(cx))}{x^8} dx &= -\frac{d(a + b \csc^{-1}(cx))}{7x^7} - \frac{e(a + b \csc^{-1}(cx))}{5x^5} + \frac{(bcx) \int \frac{-5d-7ex^2}{35x^8\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= -\frac{d(a + b \csc^{-1}(cx))}{7x^7} - \frac{e(a + b \csc^{-1}(cx))}{5x^5} + \frac{(bcx) \int \frac{-5d-7ex^2}{x^8\sqrt{-1+c^2x^2}} dx}{35\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{d(a + b \csc^{-1}(cx))}{7x^7} - \frac{e(a + b \csc^{-1}(cx))}{5x^5} + \frac{(bc(-30c^2d - 49e))}{245\sqrt{c^2x^2}} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d + 49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{d(a + b \csc^{-1}(cx))}{7x^7} - \frac{e(a + b \csc^{-1}(cx))}{5x^5} \\ &= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d + 49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{4bc^3(30c^2d + 49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}} \\ &= -\frac{8bc^5(30c^2d + 49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d + 49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.165466, size = 110, normalized size = 0.56

$$\frac{105a(5d + 7ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(15d(16c^6x^6 + 8c^4x^4 + 6c^2x^2 + 5) + 49ex^2(8c^4x^4 + 4c^2x^2 + 3)) + 105b \csc^{-1}(cx)}{3675x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x^8, x]

[Out]  $-(105*a*(5*d + 7*e*x^2) + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) + 105*b*(5*d + 7*e*x^2)*\text{ArcCsc}[c*x])/(3675*x^7)$

**Maple [A]** time = 0.19, size = 158, normalized size = 0.8

$$c^7 \left( \frac{a}{c^2} \left( -\frac{d}{7c^5x^7} - \frac{e}{5c^5x^5} \right) + \frac{b}{c^2} \left( -\frac{\operatorname{arccsc}(cx)d}{7c^5x^7} - \frac{\operatorname{arccsc}(cx)e}{5c^5x^5} - \frac{(c^2x^2 - 1)(240c^8dx^6 + 392c^6ex^6 + 120c^6dx^4 + 196c^4d^2x^2 + 147c^2e^2x^2 + 75c^2d^2)}{3675c^8x^8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^8,x)

[Out] c^7\*(a/c^2\*(-1/7/c^5\*d/x^7-1/5\*e/c^5/x^5)+b/c^2\*(-1/7\*arccsc(c\*x)/c^5\*d/x^7-1/5\*arccsc(c\*x)\*e/c^5/x^5-1/3675\*(c^2\*x^2-1)\*(240\*c^8\*d\*x^6+392\*c^6\*e\*x^6+120\*c^6\*d\*x^4+196\*c^4\*e\*x^4+90\*c^4\*d\*x^2+147\*c^2\*e\*x^2+75\*c^2\*d^2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^8/x^8))

**Maxima [A]** time = 0.979352, size = 232, normalized size = 1.18

$$\frac{1}{245} bd \left( \frac{5c^8 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{7}{2}} - 21c^8 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} + 35c^8 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 35c^8 \sqrt{-\frac{1}{c^2x^2} + 1} - \frac{35 \operatorname{arccsc}(cx)}{x^7}}{c} \right) - \frac{1}{75} be \left( \frac{3c^6 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} - 15c^6 \sqrt{-\frac{1}{c^2x^2} + 1} - \frac{15 \operatorname{arccsc}(cx)}{x^5}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^8,x, algorithm="maxima")

[Out] 1/245\*b\*d\*((5\*c^8\*(-1/(c^2\*x^2) + 1)^(7/2) - 21\*c^8\*(-1/(c^2\*x^2) + 1)^(5/2) + 35\*c^8\*(-1/(c^2\*x^2) + 1)^(3/2) - 35\*c^8\*sqrt(-1/(c^2\*x^2) + 1))/c - 35\*arccsc(c\*x)/x^7) - 1/75\*b\*e\*((3\*c^6\*(-1/(c^2\*x^2) + 1)^(5/2) - 10\*c^6\*(-1/(c^2\*x^2) + 1)^(3/2) + 15\*c^6\*sqrt(-1/(c^2\*x^2) + 1))/c + 15\*arccsc(c\*x)/x^5) - 1/5\*a\*e/x^5 - 1/7\*a\*d/x^7

**Fricas [A]** time = 2.52314, size = 273, normalized size = 1.39

$$\frac{735 aex^2 + 525 ad + 105 (7 bex^2 + 5 bd) \operatorname{arccsc}(cx) + (8 (30 bc^6d + 49 bc^4e)x^6 + 4 (30 bc^4d + 49 bc^2e)x^4 + 3 (30 bc^2d + 49 b^2e)x^2 + 75 b^2d) \sqrt{c^2x^2 - 1}}{3675 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^8,x, algorithm="fricas")

[Out] -1/3675\*(735\*a\*e\*x^2 + 525\*a\*d + 105\*(7\*b\*e\*x^2 + 5\*b\*d)\*arccsc(c\*x) + (8\*(30\*b\*c^6\*d + 49\*b\*c^4\*e)\*x^6 + 4\*(30\*b\*c^4\*d + 49\*b\*c^2\*e)\*x^4 + 3\*(30\*b\*c^2\*d + 49\*b^2\*e)\*x^2 + 75\*b\*d)\*sqrt(c^2\*x^2 - 1)/x^7

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))(d + ex^2)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsc(c\*x))/x\*\*8,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)/x\*\*8, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arccsc}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^8,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^8, x)

### 3.83 $\int x^5 (d + ex^2) (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=196

$$\frac{1}{6}dx^6(a + b \csc^{-1}(cx)) + \frac{1}{8}ex^8(a + b \csc^{-1}(cx)) + \frac{bx(c^2x^2 - 1)^{5/2}(4c^2d + 9e)}{120c^7\sqrt{c^2x^2}} + \frac{bx(c^2x^2 - 1)^{3/2}(8c^2d + 9e)}{72c^7\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2}}{24}$$

[Out] (b\*(4\*c^2\*d + 3\*e)\*x\*Sqrt[-1 + c^2\*x^2])/(24\*c^7\*Sqrt[c^2\*x^2]) + (b\*(8\*c^2\*d + 9\*e)\*x\*(-1 + c^2\*x^2)^(3/2))/(72\*c^7\*Sqrt[c^2\*x^2]) + (b\*(4\*c^2\*d + 9\*e)\*x\*(-1 + c^2\*x^2)^(5/2))/(120\*c^7\*Sqrt[c^2\*x^2]) + (b\*e\*x\*(-1 + c^2\*x^2)^(7/2))/(56\*c^7\*Sqrt[c^2\*x^2]) + (d\*x^6\*(a + b\*ArcCsc[c\*x]))/6 + (e\*x^8\*(a + b\*ArcCsc[c\*x]))/8

**Rubi [A]** time = 0.150333, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5239, 12, 446, 77}

$$\frac{1}{6}dx^6(a + b \csc^{-1}(cx)) + \frac{1}{8}ex^8(a + b \csc^{-1}(cx)) + \frac{bx(c^2x^2 - 1)^{5/2}(4c^2d + 9e)}{120c^7\sqrt{c^2x^2}} + \frac{bx(c^2x^2 - 1)^{3/2}(8c^2d + 9e)}{72c^7\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2}}{24}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*(4\*c^2\*d + 3\*e)\*x\*Sqrt[-1 + c^2\*x^2])/(24\*c^7\*Sqrt[c^2\*x^2]) + (b\*(8\*c^2\*d + 9\*e)\*x\*(-1 + c^2\*x^2)^(3/2))/(72\*c^7\*Sqrt[c^2\*x^2]) + (b\*(4\*c^2\*d + 9\*e)\*x\*(-1 + c^2\*x^2)^(5/2))/(120\*c^7\*Sqrt[c^2\*x^2]) + (b\*e\*x\*(-1 + c^2\*x^2)^(7/2))/(56\*c^7\*Sqrt[c^2\*x^2]) + (d\*x^6\*(a + b\*ArcCsc[c\*x]))/6 + (e\*x^8\*(a + b\*ArcCsc[c\*x]))/8

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_)\*(x\_)]\*(b\_)))\*((f\_)\*(x\_))^(m\_)\*((d\_.) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 77

$\text{Int}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) \|\ \text{EqQ}[p, 1] \|\ (\text{IGtQ}[p, 0] \&\& (\text{IntegerQ}[n] \|\ \text{LeQ}[9*p + 5*(n + 2), 0] \|\ \text{GeQ}[n + p + 1, 0] \|\ (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

### Rubi steps

$$\begin{aligned} \int x^5 (d + ex^2) (a + b \csc^{-1}(cx)) dx &= \frac{1}{6} dx^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^5(4d+3ex^2)}{24\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= \frac{1}{6} dx^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^5(4d+3ex^2)}{\sqrt{-1+c^2x^2}} dx}{24\sqrt{c^2x^2}} \\ &= \frac{1}{6} dx^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \csc^{-1}(cx)) + \frac{(bcx) \text{Subst} \left( \int \frac{x^2(4d+3ex)}{\sqrt{-1+c^2x}} dx, \right)}{48\sqrt{c^2x^2}} \\ &= \frac{1}{6} dx^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \csc^{-1}(cx)) + \frac{(bcx) \text{Subst} \left( \int \left( \frac{4c^2d+3e}{c^6\sqrt{-1+c^2x}} + \right) dx, \right)}{48\sqrt{c^2x^2}} \\ &= \frac{b(4c^2d+3e)x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}} + \frac{b(8c^2d+9e)x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}} + \frac{b(4c^2d+9e)x(-1+c^2x^2)^{3/2}}{120c^7\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.226948, size = 115, normalized size = 0.59

$$\frac{x \left( 105ax^5 (4d + 3ex^2) + \frac{b\sqrt{1-\frac{1}{c^2x^2}}(c^6(84dx^4+45ex^6)+2c^4(56dx^2+27ex^4)+8c^2(28d+9ex^2)+144e)}{c^7} + 105bx^5 \csc^{-1}(cx) (4d + 3ex^2) \right)}{2520}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] (x\*(105\*a\*x^5\*(4\*d + 3\*e\*x^2) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*(144\*e + 8\*c^2\*(28\*d + 9\*e\*x^2) + 2\*c^4\*(56\*d\*x^2 + 27\*e\*x^4) + c^6\*(84\*d\*x^4 + 45\*e\*x^6))))/c^7 + 105\*b\*x^5\*(4\*d + 3\*e\*x^2)\*ArcCsc[c\*x])/2520

**Maple [A]** time = 0.182, size = 152, normalized size = 0.8

$$\frac{1}{c^6} \left( \frac{a}{c^2} \left( \frac{ec^8x^8}{8} + \frac{c^8x^6d}{6} \right) + \frac{b}{c^2} \left( \frac{\text{arccsc}(cx)ec^8x^8}{8} + \frac{\text{arccsc}(cx)c^8x^6d}{6} + \frac{(c^2x^2 - 1)(45c^6ex^6 + 84c^6dx^4 + 54c^4ex^4 + 11c^2d + 9e)}{2520c^7} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x)`

[Out]  $\frac{1}{c^6} \left( \frac{a}{c^2} \left( \frac{1}{8} e c^8 x^8 + \frac{1}{6} c^8 x^6 d \right) + \frac{b}{c^2} \left( \frac{1}{8} \operatorname{arccsc}(c x) e c^8 x^8 + \frac{1}{6} \operatorname{arccsc}(c x) c^8 x^6 d + \frac{1}{2520} (c^2 x^2 - 1) (45 c^6 e x^6 + 84 c^6 d x^4 + 54 c^4 e x^4 + 112 c^4 d x^2 + 72 c^2 e x^2 + 224 c^2 d + 144 e) \right) \right) / \left( \frac{c^2 x^2 - 1}{c^2 x^2} \right)^{(1/2)} / c x$

**Maxima [A]** time = 0.979867, size = 247, normalized size = 1.26

$$\frac{1}{8} a e x^8 + \frac{1}{6} a d x^6 + \frac{1}{90} \left( 15 x^6 \operatorname{arccsc}(c x) + \frac{3 c^4 x^5 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} + 10 c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 15 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^5} \right) b d + \frac{1}{280} \left( 35 x^8 \operatorname{arccsc}(c x) + (5 c^6 x^7 + 21 c^4 x^5 + 35 c^2 x^3 + 35 x) \sqrt{-\frac{1}{c^2 x^2} + 1} \right) / c^7 b e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{8} a e x^8 + \frac{1}{6} a d x^6 + \frac{1}{90} (15 x^6 \operatorname{arccsc}(c x) + (3 c^4 x^5 (-1/(c^2 x^2) + 1)^{(5/2)} + 10 c^2 x^3 (-1/(c^2 x^2) + 1)^{(3/2)} + 15 x \sqrt{-1/(c^2 x^2) + 1}) / c^5) b d + \frac{1}{280} (35 x^8 \operatorname{arccsc}(c x) + (5 c^6 x^7 + 21 c^4 x^5 + 35 c^2 x^3 + 35 x) \sqrt{-1/(c^2 x^2) + 1}) / c^7 b e$

**Fricas [A]** time = 3.00844, size = 304, normalized size = 1.55

$$\frac{315 a c^8 e x^8 + 420 a c^8 d x^6 + 105 (3 b c^8 e x^8 + 4 b c^8 d x^6) \operatorname{arccsc}(c x) + (45 b c^6 e x^6 + 6 (14 b c^6 d + 9 b c^4 e) x^4 + 224 b c^2 d + 8 (14 b c^4 d + 9 b c^2 e) x^2 + 144 b e) \sqrt{c^2 x^2 - 1}}{2520 c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out]  $\frac{1}{2520} (315 a c^8 e x^8 + 420 a c^8 d x^6 + 105 (3 b c^8 e x^8 + 4 b c^8 d x^6) \operatorname{arccsc}(c x) + (45 b c^6 e x^6 + 6 (14 b c^6 d + 9 b c^4 e) x^4 + 224 b c^2 d + 8 (14 b c^4 d + 9 b c^2 e) x^2 + 144 b e) \sqrt{c^2 x^2 - 1}) / c^8$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{acsc}(c x)) (d + e x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(e*x**2+d)*(a+b*acsc(c*x)),x)`

[Out] `Integral(x**5*(a + b*acsc(c*x))*(d + e*x**2), x)`



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arccsc}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)*x^5, x)
```

### 3.84 $\int x^3 (d + ex^2) (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=153

$$\frac{1}{4}dx^4(a + b \csc^{-1}(cx)) + \frac{1}{6}ex^6(a + b \csc^{-1}(cx)) + \frac{bx(c^2x^2 - 1)^{3/2}(3c^2d + 4e)}{36c^5\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2 - 1}(3c^2d + 2e)}{12c^5\sqrt{c^2x^2}} + \frac{bex(c^2x^2 - 1)}{30c^5\sqrt{c^2x^2}}$$

[Out] (b\*(3\*c^2\*d + 2\*e)\*x\*Sqrt[-1 + c^2\*x^2])/(12\*c^5\*Sqrt[c^2\*x^2]) + (b\*(3\*c^2\*d + 4\*e)\*x\*(-1 + c^2\*x^2)^(3/2))/(36\*c^5\*Sqrt[c^2\*x^2]) + (b\*e\*x\*(-1 + c^2\*x^2)^(5/2))/(30\*c^5\*Sqrt[c^2\*x^2]) + (d\*x^4\*(a + b\*ArcCsc[c\*x]))/4 + (e\*x^6\*(a + b\*ArcCsc[c\*x]))/6

**Rubi [A]** time = 0.121809, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 5239, 12, 446, 77}

$$\frac{1}{4}dx^4(a + b \csc^{-1}(cx)) + \frac{1}{6}ex^6(a + b \csc^{-1}(cx)) + \frac{bx(c^2x^2 - 1)^{3/2}(3c^2d + 4e)}{36c^5\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2 - 1}(3c^2d + 2e)}{12c^5\sqrt{c^2x^2}} + \frac{bex(c^2x^2 - 1)}{30c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*(3\*c^2\*d + 2\*e)\*x\*Sqrt[-1 + c^2\*x^2])/(12\*c^5\*Sqrt[c^2\*x^2]) + (b\*(3\*c^2\*d + 4\*e)\*x\*(-1 + c^2\*x^2)^(3/2))/(36\*c^5\*Sqrt[c^2\*x^2]) + (b\*e\*x\*(-1 + c^2\*x^2)^(5/2))/(30\*c^5\*Sqrt[c^2\*x^2]) + (d\*x^4\*(a + b\*ArcCsc[c\*x]))/4 + (e\*x^6\*(a + b\*ArcCsc[c\*x]))/6

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 5239

Int[((a\_) + ArcCsc[(c\_)\*(x\_)])\*(b\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 77

```
Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rubi steps

$$\begin{aligned} \int x^3 (d + ex^2) (a + b \csc^{-1}(cx)) dx &= \frac{1}{4} dx^4 (a + b \csc^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^3(3d+2ex^2)}{12\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= \frac{1}{4} dx^4 (a + b \csc^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{x^3(3d+2ex^2)}{\sqrt{-1+c^2x^2}} dx}{12\sqrt{c^2x^2}} \\ &= \frac{1}{4} dx^4 (a + b \csc^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \csc^{-1}(cx)) + \frac{(bcx) \operatorname{Subst}\left(\int \frac{x(3d+2ex)}{\sqrt{-1+c^2x}} dx, x, cx\right)}{24\sqrt{c^2x^2}} \\ &= \frac{1}{4} dx^4 (a + b \csc^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \csc^{-1}(cx)) + \frac{(bcx) \operatorname{Subst}\left(\int \left(\frac{3c^2d+2e}{c^4\sqrt{-1+c^2x}} + \frac{x(3d+2ex)}{c^4\sqrt{-1+c^2x}}\right) dx, x, cx\right)}{24\sqrt{c^2x^2}} \\ &= \frac{b(3c^2d+2e)x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}} + \frac{b(3c^2d+4e)x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}} + \frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.205599, size = 97, normalized size = 0.63

$$\frac{1}{180} x \left( 15ax^3(3d+2ex^2) + \frac{b\sqrt{1-\frac{1}{c^2x^2}}(3c^4(5dx^2+2ex^4)+c^2(30d+8ex^2)+16e)}{c^5} + 15bx^3 \csc^{-1}(cx)(3d+2ex^2) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d + 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*ArcCsc[c*x])/180
```

**Maple [A]** time = 0.183, size = 134, normalized size = 0.9

$$\frac{1}{c^4} \left( \frac{a}{c^2} \left( \frac{ec^6x^6}{6} + \frac{x^4c^6d}{4} \right) + \frac{b}{c^2} \left( \frac{\operatorname{arccsc}(cx)ec^6x^6}{6} + \frac{\operatorname{arccsc}(cx)c^6x^4d}{4} + \frac{(c^2x^2-1)(6c^4ex^4+15c^4dx^2+8c^2ex^2+30c^2d)}{180cx} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccsc(c*x)), x)
```

```
[Out] 1/c^4*(a/c^2*(1/6*e*c^6*x^6+1/4*x^4*c^6*d)+b/c^2*(1/6*arccsc(c*x)*e*c^6*x^6+1/4*arccsc(c*x)*c^6*x^4*d+1/180*(c^2*x^2-1)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2
```

$$2*e*x^2+30*c^2*d+16*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))$$

**Maxima [A]** time = 0.991776, size = 192, normalized size = 1.25

$$\frac{1}{6} aex^6 + \frac{1}{4} adx^4 + \frac{1}{12} \left( 3x^4 \operatorname{arccsc}(cx) + \frac{c^2x^3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 3x \sqrt{-\frac{1}{c^2x^2} + 1}}{c^3} \right) bd + \frac{1}{90} \left( 15x^6 \operatorname{arccsc}(cx) + \frac{3c^4x^5 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} + 10c^2x^3 \sqrt{-\frac{1}{c^2x^2} + 1}}{c^5} \right) b^2e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] 1/6\*a\*e\*x^6 + 1/4\*a\*d\*x^4 + 1/12\*(3\*x^4\*arccsc(c\*x) + (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b\*d + 1/90\*(15\*x^6\*arccsc(c\*x) + (3\*c^4\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 10\*c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 15\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^5)\*b^2e

**Fricas [A]** time = 2.91554, size = 247, normalized size = 1.61

$$\frac{30ac^6ex^6 + 45ac^6dx^4 + 15(2bc^6ex^6 + 3bc^6dx^4) \operatorname{arccsc}(cx) + (6bc^4ex^4 + 30bc^2d + (15bc^4d + 8bc^2e)x^2 + 16be) \sqrt{c^2x^2 - 1}}{180c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] 1/180\*(30\*a\*c^6\*e\*x^6 + 45\*a\*c^6\*d\*x^4 + 15\*(2\*b\*c^6\*e\*x^6 + 3\*b\*c^6\*d\*x^4)\*arccsc(c\*x) + (6\*b\*c^4\*e\*x^4 + 30\*b\*c^2\*d + (15\*b\*c^4\*d + 8\*b\*c^2\*e)\*x^2 + 16\*b\*e)\*sqrt(c^2\*x^2 - 1))/c^6

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arccsc}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x\*\*2+d)\*(a+b\*arccsc(c\*x)),x)

[Out] Integral(x\*\*3\*(a + b\*arccsc(c\*x))\*(d + e\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arccsc}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*x^3, x)

### 3.85 $\int x (d + ex^2) (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=138

$$\frac{(d + ex^2)^2 (a + b \csc^{-1}(cx))}{4e} + \frac{bcd^2 x \tan^{-1}(\sqrt{c^2 x^2 - 1})}{4e\sqrt{c^2 x^2}} + \frac{bx\sqrt{c^2 x^2 - 1}(2c^2 d + e)}{4c^3\sqrt{c^2 x^2}} + \frac{bex(c^2 x^2 - 1)^{3/2}}{12c^3\sqrt{c^2 x^2}}$$

```
[Out] (b*(2*c^2*d + e)*x*Sqrt[-1 + c^2*x^2])/(4*c^3*Sqrt[c^2*x^2]) + (b*e*x*(-1 +
c^2*x^2)^(3/2))/(12*c^3*Sqrt[c^2*x^2]) + ((d + e*x^2)^2*(a + b*ArcCsc[c*x]
))/(4*e) + (b*c*d^2*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(4*e*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 0.0957639, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5237, 446, 88, 63, 205}

$$\frac{(d + ex^2)^2 (a + b \csc^{-1}(cx))}{4e} + \frac{bcd^2 x \tan^{-1}(\sqrt{c^2 x^2 - 1})}{4e\sqrt{c^2 x^2}} + \frac{bx\sqrt{c^2 x^2 - 1}(2c^2 d + e)}{4c^3\sqrt{c^2 x^2}} + \frac{bex(c^2 x^2 - 1)^{3/2}}{12c^3\sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(2*c^2*d + e)*x*Sqrt[-1 + c^2*x^2])/(4*c^3*Sqrt[c^2*x^2]) + (b*e*x*(-1 +
c^2*x^2)^(3/2))/(12*c^3*Sqrt[c^2*x^2]) + ((d + e*x^2)^2*(a + b*ArcCsc[c*x]
))/(4*e) + (b*c*d^2*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(4*e*Sqrt[c^2*x^2])
```

#### Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 88

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x
_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

**Rule 205**

$\text{Int}[(a + (b \cdot x^2)^{-1}), x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[a, b], x] \ \&\& \ \text{PosQ}[a/b]$

**Rubi steps**

$$\begin{aligned} \int x(d + ex^2)(a + b \csc^{-1}(cx)) dx &= \frac{(d + ex^2)^2(a + b \csc^{-1}(cx))}{4e} + \frac{(bcx) \int \frac{(d+ex^2)^2}{x\sqrt{-1+c^2x^2}} dx}{4e\sqrt{c^2x^2}} \\ &= \frac{(d + ex^2)^2(a + b \csc^{-1}(cx))}{4e} + \frac{(bcx) \text{Subst}\left(\int \frac{(d+ex^2)^2}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{8e\sqrt{c^2x^2}} \\ &= \frac{(d + ex^2)^2(a + b \csc^{-1}(cx))}{4e} + \frac{(bcx) \text{Subst}\left(\int \left(\frac{e(2c^2d+e)}{c^2\sqrt{-1+c^2x}} + \frac{d^2}{x\sqrt{-1+c^2x}} + \frac{e^2\sqrt{-1+c^2x}}{c^2}\right) dx, x, x^2\right)}{8e\sqrt{c^2x^2}} \\ &= \frac{b(2c^2d + e)x\sqrt{-1 + c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex(-1 + c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d + ex^2)^2(a + b \csc^{-1}(cx))}{4e} + \dots \\ &= \frac{b(2c^2d + e)x\sqrt{-1 + c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex(-1 + c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d + ex^2)^2(a + b \csc^{-1}(cx))}{4e} + \dots \\ &= \frac{b(2c^2d + e)x\sqrt{-1 + c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex(-1 + c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d + ex^2)^2(a + b \csc^{-1}(cx))}{4e} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.0952709, size = 78, normalized size = 0.57

$$\frac{x\left(3ac^3x(2d + ex^2) + b\sqrt{1 - \frac{1}{c^2x^2}}(c^2(6d + ex^2) + 2e) + 3bc^3x \csc^{-1}(cx)(2d + ex^2)\right)}{12c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (x\*(3\*a\*c^3\*x\*(2\*d + e\*x^2) + b\*Sqrt[1 - 1/(c^2\*x^2)]\*(2\*e + c^2\*(6\*d + e\*x^2)) + 3\*b\*c^3\*x\*ArcCsc[c\*x])/(12\*c^3)

**Maple [A]** time = 0.178, size = 115, normalized size = 0.8

$$\frac{1}{c^2} \left( \frac{a}{c^2} \left( \frac{ec^4x^4}{4} + \frac{x^2c^4d}{2} \right) + \frac{b}{c^2} \left( \frac{\text{arccsc}(cx)ec^4x^4}{4} + \frac{\text{arccsc}(cx)dc^4x^2}{2} + \frac{(c^2x^2 - 1)(c^2ex^2 + 6c^2d + 2e)}{12cx} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x)

[Out] 1/c^2\*(a/c^2\*(1/4\*e\*c^4\*x^4+1/2\*x^2\*c^4\*d)+b/c^2\*(1/4\*arccsc(c\*x)\*e\*c^4\*x^4+1/2\*arccsc(c\*x)\*d\*c^4\*x^2+1/12\*(c^2\*x^2-1)\*(c^2\*e\*x^2+6\*c^2\*d+2\*e)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c/x)

---

**Maxima [A]** time = 1.00324, size = 132, normalized size = 0.96

$$\frac{1}{4} aex^4 + \frac{1}{2} adx^2 + \frac{1}{2} \left( x^2 \operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2} + 1}}{c} \right) bd + \frac{1}{12} \left( 3x^4 \operatorname{arccsc}(cx) + \frac{c^2x^3 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 3x\sqrt{-\frac{1}{c^2x^2} + 1}}{c^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] 1/4\*a\*e\*x^4 + 1/2\*a\*d\*x^2 + 1/2\*(x^2\*arccsc(c\*x) + x\*sqrt(-1/(c^2\*x^2) + 1)/c)\*b\*d + 1/12\*(3\*x^4\*arccsc(c\*x) + (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b\*e

---

**Fricas [A]** time = 2.78292, size = 192, normalized size = 1.39

$$\frac{3ac^4ex^4 + 6ac^4dx^2 + 3(bc^4ex^4 + 2bc^4dx^2) \operatorname{arccsc}(cx) + (bc^2ex^2 + 6bc^2d + 2be)\sqrt{c^2x^2 - 1}}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] 1/12\*(3\*a\*c^4\*e\*x^4 + 6\*a\*c^4\*d\*x^2 + 3\*(b\*c^4\*e\*x^4 + 2\*b\*c^4\*d\*x^2)\*arccsc(c\*x) + (b\*c^2\*e\*x^2 + 6\*b\*c^2\*d + 2\*b\*e)\*sqrt(c^2\*x^2 - 1))/c^4

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x(a + b \operatorname{arccsc}(cx))(d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x\*\*2+d)\*(a+b\*arccsc(c\*x)),x)

[Out] Integral(x\*(a + b\*arccsc(c\*x))\*(d + e\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arccsc}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*x, x)

$$3.86 \quad \int \frac{(d+ex^2)(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

**Optimal.** Leaf size=124

$$\frac{1}{2}ibd \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right) - d \log\left(\frac{1}{x}\right)(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{2}ex^2(a + b \operatorname{csc}^{-1}(cx)) + \frac{bex\sqrt{1 - \frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ibd \operatorname{csc}^{-1}(cx)^2 -$$

[Out] (b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(2\*c) + (I/2)\*b\*d\*ArcCsc[c\*x]^2 + (e\*x^2\*(a + b\*ArcCsc[c\*x]))/2 - b\*d\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + b\*d\*ArcCsc[c\*x]\*Log[x^(-1)] - d\*(a + b\*ArcCsc[c\*x])\*Log[x^(-1)] + (I/2)\*b\*d\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]

**Rubi [A]** time = 0.286194, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$ , Rules used = {5241, 14, 4731, 6742, 264, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2}ibd \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right) - d \log\left(\frac{1}{x}\right)(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{2}ex^2(a + b \operatorname{csc}^{-1}(cx)) + \frac{bex\sqrt{1 - \frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ibd \operatorname{csc}^{-1}(cx)^2 -$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x, x]

[Out] (b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/(2\*c) + (I/2)\*b\*d\*ArcCsc[c\*x]^2 + (e\*x^2\*(a + b\*ArcCsc[c\*x]))/2 - b\*d\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + b\*d\*ArcCsc[c\*x]\*Log[x^(-1)] - d\*(a + b\*ArcCsc[c\*x])\*Log[x^(-1)] + (I/2)\*b\*d\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]

#### Rule 5241

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSin[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 4731

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSin[c\*x], u, x] - Dist[b\*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2\*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2\*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

#### Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]



Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 2326

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(ArcSin[(Rt[-e, 2]\*x)/Sqrt[d]]\*(a + b\*Log[c\*x^n]))/Rt[-e, 2], x] - Dist[(b\*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]\*x)/Sqrt[d]]/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]

Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3717

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m+1))/(d\*(m+1)), x] - Dist[2\*I, Int[((c + d\*x)^m \* E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi) \* E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 2190

Int((((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.)))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1) \* Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(a + b \csc^{-1}(cx))}{x} dx &= -\text{Subst} \left( \int \frac{(e + dx^2)(a + b \sin^{-1}(\frac{x}{c}))}{x^3} dx, x, \frac{1}{x} \right) \\
&= \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) - d(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \frac{-\frac{e}{2x^2} + d \log(x)}{\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{c} \\
&= \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) - d(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \left( -\frac{e}{2x^2 \sqrt{1-\frac{x^2}{c^2}}} + \frac{d \log(x)}{\sqrt{1-\frac{x^2}{c^2}}} \right) dx, x, \frac{1}{x} \right)}{c} \\
&= \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) - d(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{(bd) \text{Subst} \left( \int \frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{c} \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) + bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) + bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ibd \csc^{-1}(cx)^2 + \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) + bd \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - d(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ibd \csc^{-1}(cx)^2 + \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) - bd \csc^{-1}(cx) \log\left(1 - e^{2i \csc^{-1}(cx)}\right) \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ibd \csc^{-1}(cx)^2 + \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) - bd \csc^{-1}(cx) \log\left(1 - e^{2i \csc^{-1}(cx)}\right) \\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ibd \csc^{-1}(cx)^2 + \frac{1}{2}ex^2 (a + b \csc^{-1}(cx)) - bd \csc^{-1}(cx) \log\left(1 - e^{2i \csc^{-1}(cx)}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.128069, size = 103, normalized size = 0.83

$$\frac{ibcd \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) + 2acd \log(x) + acex^2 + bex\sqrt{1-\frac{1}{c^2x^2}} + bc \csc^{-1}(cx) \left(ex^2 - 2d \log\left(1 - e^{2i \csc^{-1}(cx)}\right)\right) + ibcd \csc^{-1}(cx) \log\left(\frac{1}{x}\right)}{2c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x,x]

[Out] (b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x + a\*c\*e\*x^2 + I\*b\*c\*d\*ArcCsc[c\*x]^2 + b\*c\*ArcCsc[c\*x]\*(e\*x^2 - 2\*d\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])]) + 2\*a\*c\*d\*Log[x] + I\*b\*c\*d\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/(2\*c)

**Maple [A]** time = 0.554, size = 198, normalized size = 1.6

$$\frac{ax^2e}{2} + ad \ln(cx) + \frac{i}{2}bd (\text{arccsc}(cx))^2 + \frac{b \text{arccsc}(cx) x^2 e}{2} + \frac{bx e \sqrt{c^2 x^2 - 1}}{2c} - \frac{i}{2} \frac{be}{c^2} - b d \text{arccsc}(cx) \ln\left(1 + \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x,x)

[Out]  $\frac{1}{2}ax^2e + ad \ln(cx) + \frac{1}{2}I^2 b^2 d \operatorname{arccsc}(cx)^2 + \frac{1}{2}b \operatorname{arccsc}(cx) x^2 e + \frac{1}{2} \frac{b}{c} \left( \frac{c^2 x^2 - 1}{c^2/x^2} \right)^{1/2} x e - \frac{1}{2} I^2 b/c^2 e - b^2 d \operatorname{arccsc}(cx) \ln(1 + I/c/x + (1 - 1/c^2/x^2)^{1/2}) - b^2 d \operatorname{arccsc}(cx) \ln(1 - I/c/x - (1 - 1/c^2/x^2)^{1/2}) + I^2 b^2 d \operatorname{polylog}(2, -I/c/x - (1 - 1/c^2/x^2)^{1/2}) + I^2 b^2 d \operatorname{polylog}(2, I/c/x + (1 - 1/c^2/x^2)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} aex^2 + ad \log(x) + \frac{2i bc^2 d \log(-cx + 1) \log(x) + 2i bc^2 d \log(x)^2 + 2i bc^2 d \operatorname{Li}_2(cx) + 2i bc^2 d \operatorname{Li}_2(-cx) + 2(bc^2 \arctan(1, \sqrt{cx + 1}) \sqrt{cx - 1})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x,x, algorithm="maxima")

[Out]  $\frac{1}{2} a e x^2 + a d \log(x) + \frac{1}{4} (2 I^2 b^2 c^2 d \log(-c x + 1) \log(x) + 2 I^2 b^2 c^2 d \log(x)^2 + 2 I^2 b^2 c^2 d \operatorname{dilog}(c x) + 2 I^2 b^2 c^2 d \operatorname{dilog}(-c x) + 2 (b^2 c^2 \arctan(1, \sqrt{c x + 1}) \sqrt{c x - 1}) + I^2 b^2 c^2 \log(c)) e x^2 - I^2 (b^2 e (\log(c x + 1)/c^2 + \log(c x - 1)/c^2) + 8 b^2 d \operatorname{integrate}(1/2 \log(x)/(c^2 x^3 - x), x)) c^2 + 4 c^2 \operatorname{integrate}(1/2 (b^2 e x^2 + 2 b^2 d \log(x)) \sqrt{c x + 1} \sqrt{c x - 1}/(c^2 x^3 - x), x) + I^2 b^2 e \log(c x - 1) + (-I^2 b^2 c^2 e x^2 - 2 I^2 b^2 c^2 d \log(x)) \log(c^2 x^2) + (2 I^2 b^2 c^2 d \log(x) + I^2 b^2 e) \log(c x + 1) + (2 I^2 b^2 c^2 e x^2 + 4 (b^2 c^2 \arctan(1, \sqrt{c x + 1}) \sqrt{c x - 1}) + I^2 b^2 c^2 \log(c)) d \log(x) / c^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsc(c\*x))/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))(d + ex^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsc(c\*x))/x,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arccsc}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

$$3.87 \quad \int \frac{(d+ex^2)(a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=137

$$\frac{1}{2} i b e \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right) - \frac{d(a+b \operatorname{csc}^{-1}(cx))}{2x^2} - e \log\left(\frac{1}{x}\right)(a+b \operatorname{csc}^{-1}(cx)) - \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4} bc^2 d \operatorname{csc}^{-1}(cx) +$$

[Out]  $-(b*c*d*\operatorname{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) + (b*c^2*d*\operatorname{ArcCsc}[c*x])/4 + (I/2)*b*e*\operatorname{ArcCsc}[c*x]^2 - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(2*x^2) - b*e*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcCsc}[c*x])}] + b*e*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[x^{(-1)}] - e*(a + b*\operatorname{ArcCsc}[c*x])* \operatorname{Log}[x^{(-1)}] + (I/2)*b*e*\operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcCsc}[c*x])}]$

**Rubi [A]** time = 0.302397, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$ , Rules used = {5241, 14, 4731, 12, 6742, 321, 216, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2} i b e \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right) - \frac{d(a+b \operatorname{csc}^{-1}(cx))}{2x^2} - e \log\left(\frac{1}{x}\right)(a+b \operatorname{csc}^{-1}(cx)) - \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4} bc^2 d \operatorname{csc}^{-1}(cx) +$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x^2)*(a + b*\operatorname{ArcCsc}[c*x])/x^3, x]$

[Out]  $-(b*c*d*\operatorname{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) + (b*c^2*d*\operatorname{ArcCsc}[c*x])/4 + (I/2)*b*e*\operatorname{ArcCsc}[c*x]^2 - (d*(a + b*\operatorname{ArcCsc}[c*x]))/(2*x^2) - b*e*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcCsc}[c*x])}] + b*e*\operatorname{ArcCsc}[c*x]*\operatorname{Log}[x^{(-1)}] - e*(a + b*\operatorname{ArcCsc}[c*x])* \operatorname{Log}[x^{(-1)}] + (I/2)*b*e*\operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcCsc}[c*x])}]$

#### Rule 5241

$\operatorname{Int}[(a + \operatorname{ArcCsc}[c*(x)]*(b))^n*(x)^m*((d) + (e)*(x))^2]^p, x\_Symbol] \rightarrow -\operatorname{Subst}[\operatorname{Int}[(e + d*x^2)^p*(a + b*\operatorname{ArcSin}[x/c])^n]/x^{(m + 2*(p + 1))}, x], x, 1/x] /; \operatorname{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{IntegerQ}[p]$

#### Rule 14

$\operatorname{Int}[(u)*(c*(x))^m], x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*u, x], x] /; \operatorname{FreeQ}\{c, m\}, x \ \&\& \operatorname{SumQ}[u] \ \&\& \operatorname{!LinearQ}[u, x] \ \&\& \operatorname{!MatchQ}[u, (a) + (b)*(v)] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{InverseFunctionQ}[v]$

#### Rule 4731

$\operatorname{Int}[(a + \operatorname{ArcSin}[c*(x)]*(b))*(f*(x))^m*((d) + (e)*(x))^2]^p, x\_Symbol] \rightarrow \operatorname{With}\{u = \operatorname{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \operatorname{Dist}[a + b*\operatorname{ArcSin}[c*x], u, x] - \operatorname{Dist}[b*c, \operatorname{Int}[\operatorname{SimplifyIntegrand}[u/\operatorname{Sqrt}[1 - c^2*x^2], x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \operatorname{NeQ}[c^2*d + e, 0] \ \&\& \operatorname{IntegerQ}[p] \ \&\& (\operatorname{GtQ}[p, 0] \ || \ (\operatorname{IGtQ}[(m - 1)/2, 0] \ \&\& \operatorname{LeQ}[m + p, 0]))$

#### Rule 12

$\operatorname{Int}[(a)*(u), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \operatorname{!MatchQ}[u, (b)*(v)] /; \operatorname{FreeQ}[b, x]$

Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]

Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(  
n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[  
(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x],  
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p  
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[(Rt[-b, 2]\*x)/Sqr  
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2326

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symb  
ol] := Simp[(ArcSin[(Rt[-e, 2]\*x)/Sqrt[d]]\*(a + b\*Log[c\*x^n]))/Rt[-e, 2], x  
] - Dist[(b\*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]\*x)/Sqrt[d]]/x, x], x] /; Fr  
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]

Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(  
a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3717

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol  
] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)  
^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x],  
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.))/  
((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)  
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol]  
:= Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(a + b \csc^{-1}(cx))}{x^3} dx &= -\text{Subst} \left( \int \frac{(e + dx^2)(a + b \sin^{-1}(\frac{x}{c}))}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{d(a + b \csc^{-1}(cx))}{2x^2} - e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \frac{dx^2 + 2e \log(x)}{2\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{c} \\
&= -\frac{d(a + b \csc^{-1}(cx))}{2x^2} - e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \frac{dx^2 + 2e \log(x)}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2c} \\
&= -\frac{d(a + b \csc^{-1}(cx))}{2x^2} - e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \left( \frac{dx^2}{\sqrt{1 - \frac{x^2}{c^2}}} + \frac{2e \log(x)}{\sqrt{1 - \frac{x^2}{c^2}}} \right) dx, x, \frac{1}{x} \right)}{2c} \\
&= -\frac{d(a + b \csc^{-1}(cx))}{2x^2} - e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{(bd) \text{Subst} \left( \int \frac{x^2}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2c} \\
&= -\frac{bcd\sqrt{1 - \frac{1}{c^2x^2}}}{4x} - \frac{d(a + b \csc^{-1}(cx))}{2x^2} + be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= -\frac{bcd\sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d \csc^{-1}(cx) - \frac{d(a + b \csc^{-1}(cx))}{2x^2} + be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= -\frac{bcd\sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d \csc^{-1}(cx) + \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \csc^{-1}(cx))}{2x^2} + be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) - e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= -\frac{bcd\sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d \csc^{-1}(cx) + \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \csc^{-1}(cx))}{2x^2} - be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) + e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= -\frac{bcd\sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d \csc^{-1}(cx) + \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \csc^{-1}(cx))}{2x^2} - be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) + e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= -\frac{bcd\sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d \csc^{-1}(cx) + \frac{1}{2}ibe \csc^{-1}(cx)^2 - \frac{d(a + b \csc^{-1}(cx))}{2x^2} - be \csc^{-1}(cx) \log\left(\frac{1}{x}\right) + e(a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.413998, size = 137, normalized size = 1.

$$\frac{1}{4} \left( 2ibe \left( \csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right)\right) - \frac{2ad}{x^2} + 4ae \log(x) - \frac{bcd\sqrt{1 - \frac{1}{c^2x^2}} \left( \frac{c^2x^2 \tanh^{-1}\left(\sqrt{1 - c^2x^2}\right)}{\sqrt{1 - c^2x^2}} + 1 \right)}{x} - \frac{2bd \csc^{-1}(cx)}{x^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/x^3, x]

[Out] ((-2\*a\*d)/x^2 - (2\*b\*d\*ArcCsc[c\*x])/x^2 - (b\*c\*d\*Sqrt[1 - 1/(c^2\*x^2)]\*(1 + (c^2\*x^2\*ArcTanh[Sqrt[1 - c^2\*x^2]])/Sqrt[1 - c^2\*x^2]))/x - 4\*b\*e\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + 4\*a\*e\*Log[x] + (2\*I)\*b\*e\*(ArcCsc[c\*x]^2 + PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]))/4

---

**Maple [A]** time = 0.384, size = 201, normalized size = 1.5

$$-\frac{ad}{2x^2} + ae \ln(cx) + \frac{i}{2} be (\operatorname{arccsc}(cx))^2 - \frac{bcd}{4x} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} + \frac{bc^2 d \operatorname{arccsc}(cx)}{4} - \frac{b \operatorname{arccsc}(cx) d}{2x^2} - b \operatorname{arccsc}(cx) \ln\left(1 + \frac{i}{cx} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x)`

[Out] `-1/2*a*d/x^2+a*e*ln(c*x)+1/2*I*b*e*arccsc(c*x)^2-1/4*c*b*d/x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*c^2*d*arccsc(c*x)-1/2*b*arccsc(c*x)*d/x^2-b*e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-b*e*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*e*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+I*b*e*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( c^2 \int \frac{\sqrt{cx+1}\sqrt{cx-1} \log(x)}{c^4x^3 - c^2x} dx + \arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) \log(x) \right) be + \frac{1}{4} bd \left( \frac{c^4x \sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2 \left(\frac{1}{c^2x^2}-1\right)-1} - c^3 \arctan\left(cx \sqrt{-\frac{1}{c^2x^2}+1}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")`

[Out] `(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e + 1/4*b*d*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) + a*e*log(x) - 1/2*a*d/x^2`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))/x^3, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))(d + ex^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((e\*x\*\*2+d)\*(a+b\*acsc(c\*x))/x\*\*3,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arccsc}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsc(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^3, x)

### 3.88 $\int x^2 (d + ex^2)^2 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=252

$$\frac{1}{3}d^2x^3(a + b \csc^{-1}(cx)) + \frac{2}{5}dex^5(a + b \csc^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \csc^{-1}(cx)) + \frac{bx^2\sqrt{c^2x^2-1}(280c^4d^2 + 252c^2de + 75e^2)}{1680c^5\sqrt{c^2x^2}}$$

[Out] (b\*(280\*c^4\*d^2 + 252\*c^2\*d\*e + 75\*e^2)\*x^2\*Sqrt[-1 + c^2\*x^2])/(1680\*c^5\*Sqrt[c^2\*x^2]) + (b\*e\*(84\*c^2\*d + 25\*e)\*x^4\*Sqrt[-1 + c^2\*x^2])/(840\*c^3\*Sqrt[c^2\*x^2]) + (b\*e^2\*x^6\*Sqrt[-1 + c^2\*x^2])/(42\*c\*Sqrt[c^2\*x^2]) + (d^2\*x^3\*(a + b\*ArcCsc[c\*x]))/3 + (2\*d\*e\*x^5\*(a + b\*ArcCsc[c\*x]))/5 + (e^2\*x^7\*(a + b\*ArcCsc[c\*x]))/7 + (b\*(280\*c^4\*d^2 + 252\*c^2\*d\*e + 75\*e^2)\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(1680\*c^6\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.237142, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {270, 5239, 12, 1267, 459, 321, 217, 206}

$$\frac{1}{3}d^2x^3(a + b \csc^{-1}(cx)) + \frac{2}{5}dex^5(a + b \csc^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \csc^{-1}(cx)) + \frac{bx^2\sqrt{c^2x^2-1}(280c^4d^2 + 252c^2de + 75e^2)}{1680c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*(280\*c^4\*d^2 + 252\*c^2\*d\*e + 75\*e^2)\*x^2\*Sqrt[-1 + c^2\*x^2])/(1680\*c^5\*Sqrt[c^2\*x^2]) + (b\*e\*(84\*c^2\*d + 25\*e)\*x^4\*Sqrt[-1 + c^2\*x^2])/(840\*c^3\*Sqrt[c^2\*x^2]) + (b\*e^2\*x^6\*Sqrt[-1 + c^2\*x^2])/(42\*c\*Sqrt[c^2\*x^2]) + (d^2\*x^3\*(a + b\*ArcCsc[c\*x]))/3 + (2\*d\*e\*x^5\*(a + b\*ArcCsc[c\*x]))/5 + (e^2\*x^7\*(a + b\*ArcCsc[c\*x]))/7 + (b\*(280\*c^4\*d^2 + 252\*c^2\*d\*e + 75\*e^2)\*x\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(1680\*c^6\*Sqrt[c^2\*x^2])

#### Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 1267

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^(
q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
+ 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

#### Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

#### Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

#### Rubi steps

$$\begin{aligned}
\int x^2 (d + ex^2)^2 (a + b \csc^{-1}(cx)) dx &= \frac{1}{3}d^2x^3 (a + b \csc^{-1}(cx)) + \frac{2}{5}dex^5 (a + b \csc^{-1}(cx)) + \frac{1}{7}e^2x^7 (a + b \csc^{-1}(cx)) + \dots \\
&= \frac{1}{3}d^2x^3 (a + b \csc^{-1}(cx)) + \frac{2}{5}dex^5 (a + b \csc^{-1}(cx)) + \frac{1}{7}e^2x^7 (a + b \csc^{-1}(cx)) + \dots \\
&= \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3 (a + b \csc^{-1}(cx)) + \frac{2}{5}dex^5 (a + b \csc^{-1}(cx)) + \frac{1}{7}e^2x^7 (a + b \csc^{-1}(cx)) + \dots \\
&= \frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3 (a + b \csc^{-1}(cx)) + \dots \\
&= \frac{b(280c^4d^2+252c^2de+75e^2)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \dots \\
&= \frac{b(280c^4d^2+252c^2de+75e^2)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \dots \\
&= \frac{b(280c^4d^2+252c^2de+75e^2)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.350116, size = 184, normalized size = 0.73

$$\frac{c^2x^2 \left( 16ac^5x(35d^2 + 42dex^2 + 15e^2x^4) + b\sqrt{1 - \frac{1}{c^2x^2}} (8c^4(35d^2 + 21dex^2 + 5e^2x^4) + 2c^2e(126d + 25ex^2) + 75e^2) \right) + b(280c^4d^2 + 252c^2de + 75e^2)x^2\sqrt{-1+c^2x^2}}{1680c^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]), x]

[Out] (c^2\*x^2\*(16\*a\*c^5\*x\*(35\*d^2 + 42\*d\*e\*x^2 + 15\*e^2\*x^4) + b\*Sqrt[1 - 1/(c^2\*x^2)]\*(75\*e^2 + 2\*c^2\*e\*(126\*d + 25\*e\*x^2) + 8\*c^4\*(35\*d^2 + 21\*d\*e\*x^2 + 5\*e^2\*x^4))) + 16\*b\*c^7\*x^3\*(35\*d^2 + 42\*d\*e\*x^2 + 15\*e^2\*x^4)\*ArcCsc[c\*x] + b\*(280\*c^4\*d^2 + 252\*c^2\*d\*e + 75\*e^2)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x]/(1680\*c^7)

**Maple [B]** time = 0.184, size = 494, normalized size = 2.

$$\frac{ae^2x^7}{7} + \frac{2aedx^5}{5} + \frac{ax^3d^2}{3} + \frac{\text{barccsc}(cx)e^2x^7}{7} + \frac{2\text{barccsc}(cx)edx^5}{5} + \frac{\text{barccsc}(cx)x^3d^2}{3} + \frac{bx^6e^2}{42c} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bx^4e^2}{168c^3} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)), x)

[Out] 1/7\*a\*e^2\*x^7+2/5\*a\*e\*d\*x^5+1/3\*a\*x^3\*d^2+1/7\*b\*arccsc(c\*x)\*e^2\*x^7+2/5\*b\*a\*arccsc(c\*x)\*e\*d\*x^5+1/3\*b\*arccsc(c\*x)\*x^3\*d^2+1/42/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^6\*e^2+1/168/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4\*e^2+1/10/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^4\*e\*d+1/20/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x^2\*e\*d+1/6/c\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d^2\*x^2-1/6/c^3\*b/((c^2\*x^2-1)/c^2/x^2)^(1/2)\*d^2+1/6/c^4\*b\*(c^2\*x^2-1)^(1/2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/x\*d

$$\begin{aligned} &^2 \ln(cx + (c^2x^2 - 1)^{1/2}) + 5/336/c^5 b / ((c^2x^2 - 1)/c^2/x^2)^{1/2} * x^2 * e^{2-3/20/c^5 b / ((c^2x^2 - 1)/c^2/x^2)^{1/2} * e * d + 3/20/c^6 b * (c^2x^2 - 1)^{1/2} / ((c^2x^2 - 1)/c^2/x^2)^{1/2} / x * e * d * \ln(cx + (c^2x^2 - 1)^{1/2}) - 5/112/c^7 b / ((c^2x^2 - 1)/c^2/x^2)^{1/2} * e^2 + 5/112/c^8 b * (c^2x^2 - 1)^{1/2} / ((c^2x^2 - 1)/c^2/x^2)^{1/2} / x * e^2 * \ln(cx + (c^2x^2 - 1)^{1/2}) \end{aligned}$$

**Maxima [A]** time = 1.03673, size = 545, normalized size = 2.16

$$\frac{1}{7} a e^2 x^7 + \frac{2}{5} a d e x^5 + \frac{1}{3} a d^2 x^3 + \frac{1}{12} \left( 4 x^3 \operatorname{arccsc}(c x) + \frac{2 \sqrt{-\frac{1}{c^2 x^2} + 1} + \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right) - \log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right)}{c^2 \left(\frac{1}{c^2 x^2} - 1\right) + c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2 x^2} + 1}\right)}{c^2} \right) b d^2 + \frac{1}{40} \left( 16 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{7} a e^2 x^7 + \frac{2}{5} a d e x^5 + \frac{1}{3} a d^2 x^3 + \frac{1}{12} (4 x^3 \operatorname{arccsc}(c x) + (2 \sqrt{-1/(c^2 x^2) + 1} / (c^2 (1/(c^2 x^2) - 1) + c^2) + \log(\sqrt{-1/(c^2 x^2) + 1} + 1) / c^2 - \log(\sqrt{-1/(c^2 x^2) + 1} - 1) / c^2) / c) * b d^2 + \frac{1}{40} (16 x^5 \operatorname{arccsc}(c x) - (2 * (3 * (-1/(c^2 x^2) + 1)^{3/2} - 5 * \sqrt{-1/(c^2 x^2) + 1})) / (c^4 * (1/(c^2 x^2) - 1)^2 + 2 * c^4 * (1/(c^2 x^2) - 1) + c^4) - 3 * \log(\sqrt{-1/(c^2 x^2) + 1} + 1) / c^4 + 3 * \log(\sqrt{-1/(c^2 x^2) + 1} - 1) / c^4) / c) * b * d * e + \frac{1}{672} (96 x^7 \operatorname{arccsc}(c x) + (2 * (15 * (-1/(c^2 x^2) + 1)^{5/2} - 40 * (-1/(c^2 x^2) + 1)^{3/2} + 33 * \sqrt{-1/(c^2 x^2) + 1})) / (c^6 * (1/(c^2 x^2) - 1)^3 + 3 * c^6 * (1/(c^2 x^2) - 1)^2 + 3 * c^6 * (1/(c^2 x^2) - 1) + c^6) + 15 * \log(\sqrt{-1/(c^2 x^2) + 1} + 1) / c^6 - 15 * \log(\sqrt{-1/(c^2 x^2) + 1} - 1) / c^6) / c) * b * e^2$

**Fricas [A]** time = 6.06912, size = 641, normalized size = 2.54

$$240 a c^7 e^2 x^7 + 672 a c^7 d e x^5 + 560 a c^7 d^2 x^3 + 16 (15 b c^7 e^2 x^7 + 42 b c^7 d e x^5 + 35 b c^7 d^2 x^3 - 35 b c^7 d^2 - 42 b c^7 d e - 15 b c^7 e^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out]  $\frac{1}{1680} (240 a c^7 e^2 x^7 + 672 a c^7 d e x^5 + 560 a c^7 d^2 x^3 + 16 (15 b c^7 e^2 x^7 + 42 b c^7 d e x^5 + 35 b c^7 d^2 x^3 - 35 b c^7 d^2 - 42 b c^7 d e - 15 b c^7 e^2) * \operatorname{arccsc}(c x) - 32 (35 b c^7 d^2 + 42 b c^7 d e + 15 b c^7 e^2) * \arctan(-c x + \sqrt{c^2 x^2 - 1}) - (280 b c^4 d^2 + 252 b c^2 d e + 75 b e^2) * \log(-c x + \sqrt{c^2 x^2 - 1}) + (40 b c^5 e^2 x^5 + 2 * (84 b c^5 d e + 25 b c^3 e^2) * x^3 + (280 b c^5 d^2 + 252 b c^3 d e + 75 b c e^2) * x) * \sqrt{c^2 x^2 - 1}) / c^7$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arccsc}(c x)) (d + e x^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x\*\*2+d)\*\*2\*(a+b\*acsc(c\*x)),x)

[Out] Integral(x\*\*2\*(a + b\*acsc(c\*x))\*(d + e\*x\*\*2)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a)\*x^2, x)

### 3.89 $\int (d + ex^2)^2 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=191

$$d^2x(a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3(a + b \csc^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b \csc^{-1}(cx)) + \frac{bx(120c^4d^2 + 40c^2de + 9e^2) \tanh^{-1}\left(\frac{c}{\sqrt{c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

```
[Out] (b*e*(40*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*
e^2*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + d^2*x*(a + b*ArcCsc[c*x]
) + (2*d*e*x^3*(a + b*ArcCsc[c*x]))/3 + (e^2*x^5*(a + b*ArcCsc[c*x]))/5 + (
b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(
120*c^4*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 0.114112, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {194, 5229, 12, 1159, 388, 217, 206}

$$d^2x(a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3(a + b \csc^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b \csc^{-1}(cx)) + \frac{bx(120c^4d^2 + 40c^2de + 9e^2) \tanh^{-1}\left(\frac{c}{\sqrt{c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (b*e*(40*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*
e^2*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + d^2*x*(a + b*ArcCsc[c*x]
) + (2*d*e*x^3*(a + b*ArcCsc[c*x]))/3 + (e^2*x^5*(a + b*ArcCsc[c*x]))/5 + (
b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(
120*c^4*Sqrt[c^2*x^2])
```

#### Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

#### Rule 5229

```
Int[((a_) + ArcCsc[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
```

\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + b \csc^{-1}(cx)) dx &= d^2x (a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \csc^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int}{5} \\ &= d^2x (a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \csc^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int}{5} \\ &= \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \csc^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + b \csc^{-1}(cx)) \\ &= \frac{be(40c^2d + 9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \csc^{-1}(cx)) \\ &= \frac{be(40c^2d + 9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \csc^{-1}(cx)) \\ &= \frac{be(40c^2d + 9e)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x (a + b \csc^{-1}(cx)) + \frac{2}{3}dex^3 (a + b \csc^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.236115, size = 151, normalized size = 0.79

$$\frac{c^2x \left( 8ac^3 (15d^2 + 10dex^2 + 3e^2x^4) + bex\sqrt{1 - \frac{1}{c^2x^2}} (c^2(40d + 6ex^2) + 9e) \right) + b(120c^4d^2 + 40c^2de + 9e^2) \log \left( x \left( \sqrt{1 - \frac{1}{c^2x^2}} \right) \right)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]),x]

[Out] (c^2\*x\*(8\*a\*c^3\*(15\*d^2 + 10\*d\*e\*x^2 + 3\*e^2\*x^4) + b\*e\*sqrt[1 - 1/(c^2\*x^2)])\*x\*(9\*e + c^2\*(40\*d + 6\*e\*x^2))) + 8\*b\*c^5\*x\*(15\*d^2 + 10\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsc[c\*x] + b\*(120\*c^4\*d^2 + 40\*c^2\*d\*e + 9\*e^2)\*Log[(1 + Sqrt[1 - 1/(c^2\*x^2)])\*x]/(120\*c^5)





$$3bc^5e^2 \operatorname{arccsc}(cx) - 16(15b^2c^5d^2 + 10b^2c^5de + 3b^2c^5e^2) \operatorname{arctan}(-cx + \sqrt{c^2x^2 - 1}) - (120b^4c^4d^2 + 40b^4c^2de + 9b^4e^2) \log(-cx + \sqrt{c^2x^2 - 1}) + (6b^3c^3e^2x^3 + (40b^3c^3de + 9b^3c^3e^2)x) \sqrt{c^2x^2 - 1} / c^5$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*arccsc(c\*x)),x)

[Out] Integral((a + b\*arccsc(c\*x))\*(d + e\*x\*\*2)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a), x)

$$3.90 \quad \int \frac{(d+ex^2)^2 (a+b \csc^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=163

$$-\frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \csc^{-1}(cx)) - \frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} + \frac{bex(12c^2d+e)\tanh^{-1}\left(\frac{d+ex^2}{c}\right)}{6c^2\sqrt{c^2x^2}}$$

[Out]  $-\left(\frac{b^2cd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}\right) + \left(\frac{b^2e^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}\right) - \left(\frac{d^2(a+b\text{ArcCsc}[c*x])}{x} + 2d^2e^2x^3(a+b\text{ArcCsc}[c*x])\right) + \left(\frac{e^2x^3(a+b\text{ArcCsc}[c*x])}{3} + \frac{b^2e^2(12c^2d+e)x\text{ArcTanh}\left[\frac{d+ex^2}{c}\right]}{6c^2\sqrt{c^2x^2}}\right)$

**Rubi [A]** time = 0.129523, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 5239, 12, 1265, 388, 217, 206}

$$-\frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \csc^{-1}(cx)) - \frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} + \frac{bex(12c^2d+e)\tanh^{-1}\left(\frac{d+ex^2}{c}\right)}{6c^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]))/x^2,x]

[Out]  $-\left(\frac{b^2cd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}\right) + \left(\frac{b^2e^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}\right) - \left(\frac{d^2(a+b\text{ArcCsc}[c*x])}{x} + 2d^2e^2x^3(a+b\text{ArcCsc}[c*x])\right) + \left(\frac{e^2x^3(a+b\text{ArcCsc}[c*x])}{3} + \frac{b^2e^2(12c^2d+e)x\text{ArcTanh}\left[\frac{d+ex^2}{c}\right]}{6c^2\sqrt{c^2x^2}}\right)$

#### Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.))\*((f\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 1265

Int[((f\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(q\_.))\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] :> With[{Qx = PolynomialQuotient[(a + b\*x^2 + c\*x^4)^p, f\*x, x], R = PolynomialRemainder[(a + b\*x^2 + c\*x^4)^p, f\*x, x]},

Simp[(R\*(f\*x)^(m + 1)\*(d + e\*x^2)^(q + 1))/(d\*f\*(m + 1)), x] + Dist[1/(d\*f^2\*(m + 1)), Int[(f\*x)^(m + 2)\*(d + e\*x^2)^q\*ExpandToSum[(d\*f\*(m + 1)\*Qx)/x - e\*R\*(m + 2\*q + 3), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && LtQ[m, -1]

### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \csc^{-1}(cx))}{x^2} dx &= -\frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int -}{(bcx) \int -} \\ &= -\frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \csc^{-1}(cx)) + \frac{(bcx) \int -}{(bcx) \int -} \\ &= -\frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \csc^{-1}(cx)) \\ &= -\frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1 + c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) \\ &= -\frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1 + c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) \\ &= -\frac{bcd^2\sqrt{-1 + c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1 + c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{x} + 2dex (a + b \csc^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.193874, size = 134, normalized size = 0.82

$$\frac{c^2 \left( 2ac \left( -3d^2 + 6dex^2 + e^2x^4 \right) + bx \sqrt{1 - \frac{1}{c^2x^2}} \left( e^2x^2 - 6c^2d^2 \right) \right) + 2bc^3 \csc^{-1}(cx) \left( -3d^2 + 6dex^2 + e^2x^4 \right) + bex \left( 12c^2d + e \right) \log \left( \frac{d + ex^2 + \sqrt{d^2 + ex^2}}{d - ex^2 + \sqrt{d^2 + ex^2}} \right)}{6c^3x}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]))/x^2,x]

[Out] (c^2\*(b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(-6\*c^2\*d^2 + e^2\*x^2) + 2\*a\*c\*(-3\*d^2 + 6\*d\*e\*x^2 + e^2\*x^4)) + 2\*b\*c^3\*(-3\*d^2 + 6\*d\*e\*x^2 + e^2\*x^4)\*ArcCsc[c\*x] +

$$b * e * (12 * c^2 * d + e) * x * \text{Log}[(1 + \text{Sqrt}[1 - 1/(c^2 * x^2)]) * x] / (6 * c^3 * x)$$

**Maple [A]** time = 0.187, size = 286, normalized size = 1.8

$$\frac{ae^2x^3}{3} + 2aedx - \frac{ad^2}{x} + \frac{be^2 \arccsc(cx)x^3}{3} + 2 \arccsc(cx)edx - \frac{\arccsc(cx)d^2}{x} - cbd^2 \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{bd^2}{cx^2} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + 2 \frac{b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^2,x)

[Out]  $\frac{1}{3}ae^2x^3 + 2aedx - \frac{ad^2}{x} + \frac{1}{3}b^2e^2 \arccsc(cx)x^3 + 2b \arccsc(cx) * e * d * x - b \arccsc(cx) * d^2 / x - c * b / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} * d^2 + b / c / x^2 / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} * d^2 + 2 * b / c^2 * ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / x * e * d * \ln(c * x + ((c^2 * x^2 - 1) / c^2)^{(1/2)}) + 1 / 6 * c * b * e^2 / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} * x^2 - 1 / 6 * c^3 * b * e^2 / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} + 1 / 6 * c^4 * b * e^2 * ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / ((c^2 * x^2 - 1) / c^2 / x^2)^{(1/2)} / x * \ln(c * x + ((c^2 * x^2 - 1) / c^2)^{(1/2)})$

**Maxima [A]** time = 0.977365, size = 266, normalized size = 1.63

$$\frac{1}{3}ae^2x^3 - \left( c \sqrt{-\frac{1}{c^2x^2} + 1} + \frac{\arccsc(cx)}{x} \right) bd^2 + \frac{1}{12} \left( 4x^3 \arccsc(cx) + \frac{2\sqrt{-\frac{1}{c^2x^2} + 1} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} - 1\right)}{c^2\left(\frac{1}{c^2x^2} - 1\right) + c^2}}{c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} - 1\right)}{c^2} \right) / c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}ae^2x^3 - (c * \text{sqrt}(-1/(c^2 * x^2) + 1) + \arccsc(cx)/x) * b * d^2 + 1/12 * (4 * x^3 * \arccsc(cx) + (2 * \text{sqrt}(-1/(c^2 * x^2) + 1) / (c^2 * (1/(c^2 * x^2) - 1) + c^2) + \log(\text{sqrt}(-1/(c^2 * x^2) + 1) + 1) / c^2 - \log(\text{sqrt}(-1/(c^2 * x^2) + 1) - 1) / c^2) / c) * b * e^2 + 2 * a * d * e * x + (2 * c * x * \arccsc(cx) + \log(\text{sqrt}(-1/(c^2 * x^2) + 1) + 1) - \log(-\text{sqrt}(-1/(c^2 * x^2) + 1) + 1)) * b * d * e / c - a * d^2 / x$

**Fricas [A]** time = 3.89202, size = 502, normalized size = 3.08

$$\frac{2ac^3e^2x^4 - 6bc^4d^2x + 12ac^3dex^2 - 6ac^3d^2 + 4(3bc^3d^2 - 6bc^3de - bc^3e^2)x \arctan(-cx + \sqrt{c^2x^2 - 1}) - (12bc^2de + b^2e^2)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^2,x, algorithm="fricas")

[Out]  $\frac{1}{6} * (2 * a * c^3 * e^2 * x^4 - 6 * b * c^4 * d^2 * x + 12 * a * c^3 * d * e * x^2 - 6 * a * c^3 * d^2 + 4 * (3 * b * c^3 * d^2 - 6 * b * c^3 * d * e - b * c^3 * e^2) * x * \arctan(-c * x + \text{sqrt}(c^2 * x^2 - 1)) - (12 * b * c^2 * d * e + b * e^2) * x * \log(-c * x + \text{sqrt}(c^2 * x^2 - 1)) + 2 * (b * c^3 * e^2 * x^4 + 6 * b * c^3 * d * e * x^2 - 3 * b * c^3 * d^2 + (3 * b * c^3 * d^2 - 6 * b * c^3 * d * e - b * c^3 * e^2) * x$

)\*arccsc(c\*x) - (6\*b\*c^3\*d^2 - b\*c\*e^2\*x^2)\*sqrt(c^2\*x^2 - 1))/(c^3\*x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))(d + ex^2)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*arccsc(c\*x))/x\*\*2,x)

[Out] Integral((a + b\*arccsc(c\*x))\*(d + e\*x\*\*2)\*\*2/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^2,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a)/x^2, x)

$$3.91 \quad \int \frac{(d+ex^2)^2 (a+b \csc^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=157

$$\frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} + e^2 x (a + b \csc^{-1}(cx)) - \frac{bcd^2 \sqrt{c^2 x^2 - 1}}{9x^2 \sqrt{c^2 x^2}} - \frac{2bcd \sqrt{c^2 x^2 - 1} (c^2 d + 9e)}{9 \sqrt{c^2 x^2}} +$$

[Out]  $(-2*b*c*d*(c^2*d + 9*e)*\text{Sqrt}[-1 + c^2*x^2])/(9*\text{Sqrt}[c^2*x^2]) - (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(9*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcCsc}[c*x]))/(3*x^3) - (2*d*e*(a + b*\text{ArcCsc}[c*x]))/x + e^2*x*(a + b*\text{ArcCsc}[c*x]) + (b*e^2*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]])/\text{Sqrt}[c^2*x^2]$

**Rubi [A]** time = 0.132483, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 5239, 12, 1265, 451, 217, 206}

$$\frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} + e^2 x (a + b \csc^{-1}(cx)) - \frac{bcd^2 \sqrt{c^2 x^2 - 1}}{9x^2 \sqrt{c^2 x^2}} - \frac{2bcd \sqrt{c^2 x^2 - 1} (c^2 d + 9e)}{9 \sqrt{c^2 x^2}} +$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x^2)^2*(a + b*\text{ArcCsc}[c*x])/x^4, x]$

[Out]  $(-2*b*c*d*(c^2*d + 9*e)*\text{Sqrt}[-1 + c^2*x^2])/(9*\text{Sqrt}[c^2*x^2]) - (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(9*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcCsc}[c*x]))/(3*x^3) - (2*d*e*(a + b*\text{ArcCsc}[c*x]))/x + e^2*x*(a + b*\text{ArcCsc}[c*x]) + (b*e^2*x*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]])/\text{Sqrt}[c^2*x^2]$

#### Rule 270

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, n\}, x \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 5239

$\text{Int}[(a_*) + \text{ArcCsc}[(c_*)*(x_*)*(b_*)]*((f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(p_*)}, x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcCsc}[c*x], u, x] + \text{Dist}[(b*c*x)/\text{Sqrt}[c^2*x^2], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[c^2*x^2 - 1]), x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x \ \&\& \ ((\text{IGtQ}[p, 0] \ \&\& \ !(\text{ILtQ}[(m - 1)/2, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{IGtQ}[(m + 1)/2, 0] \ \&\& \ !(\text{ILtQ}[p, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{ILtQ}[(m + 2*p + 1)/2, 0] \ \&\& \ !\text{ILtQ}[(m - 1)/2, 0]))$

#### Rule 12

$\text{Int}[(a_*)*(u_*)], x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_*)] /; \text{FreeQ}[b, x]$

#### Rule 1265

$\text{Int}[(f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(q_*)}*((a_*) + (b_*)*(x_*)^2 + (c_*)*(x_*)^4)^{(p_*)}, x\_Symbol] \rightarrow \text{With}\{Qx = \text{PolynomialQuotient}[(a + b*x^2 + c*x^4)^p, f*x, x], R = \text{PolynomialRemainder}[(a + b*x^2 + c*x^4)^p, f*x, x]\},$

```
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

### Rule 451

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \csc^{-1}(cx))}{x^4} dx &= -\frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} + e^2 x (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{-d^2 -}{3x^4} \sqrt{a + bx^2}}{\sqrt{a + bx^2}} \\ &= -\frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} + e^2 x (a + b \csc^{-1}(cx)) + \frac{(bcx) \int \frac{-d^2 -}{x^4} \sqrt{a + bx^2}}{3\sqrt{a + bx^2}} \\ &= -\frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} + e^2 x (a + b \csc^{-1}(cx)) \\ &= -\frac{2bcd (c^2 d + 9e) \sqrt{-1 + c^2 x^2}}{9\sqrt{c^2 x^2}} - \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} \\ &= -\frac{2bcd (c^2 d + 9e) \sqrt{-1 + c^2 x^2}}{9\sqrt{c^2 x^2}} - \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} \\ &= -\frac{2bcd (c^2 d + 9e) \sqrt{-1 + c^2 x^2}}{9\sqrt{c^2 x^2}} - \frac{bcd^2 \sqrt{-1 + c^2 x^2}}{9x^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2de (a + b \csc^{-1}(cx))}{x} \end{aligned}$$

**Mathematica [A]** time = 0.267873, size = 125, normalized size = 0.8

$$-\frac{3a(d^2 + 6dex^2 - 3e^2x^4) + bc dx \sqrt{1 - \frac{1}{c^2 x^2}} (2c^2 dx^2 + d + 18ex^2)}{9x^3} + \frac{be^2 \log\left(x \left(\sqrt{1 - \frac{1}{c^2 x^2}} + 1\right)\right)}{c} - \frac{b \csc^{-1}(cx) (d^2 + 6dex^2)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```



[Out]  $-(b*c*d*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) + 3*a*(d^2 + 6*d*e*x^2 - 3*e^2*x^4))/(9*x^3) - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*\text{ArcCsc}[c*x])/(3*x^3) + (b*e^2*\text{Log}[(1 + \text{Sqrt}[1 - 1/(c^2*x^2)])*x])/c$

**Maple [A]** time = 0.189, size = 254, normalized size = 1.6

$$axe^2 - 2 \frac{aed}{x} - \frac{ad^2}{3x^3} + b \operatorname{arccsc}(cx) xe^2 - 2 \frac{b \operatorname{arccsc}(cx) ed}{x} - \frac{b \operatorname{arccsc}(cx) d^2}{3x^3} - \frac{2c^3bd^2}{9} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{cbd^2}{9x^2} \frac{1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - 2bc$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x)`

[Out]  $a*x*e^{2-2*a*e*d/x-1/3*a*d^2/x^3} + b*\operatorname{arccsc}(c*x)*x*e^{2-2*b*\operatorname{arccsc}(c*x)*e*d/x-1/3*b*\operatorname{arccsc}(c*x)*d^2/x^3} - 2/9*c^3*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*d^2 + 1/9*c*b/x^2/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*d^2 - 2*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}*e*d + 2/c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^2*e*d + 1/9*c*b/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x^4*d^2 + 1/c^2*b*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*e^2*\ln(c*x + (c^2*x^2-1)^{(1/2)})$

**Maxima [A]** time = 0.994512, size = 213, normalized size = 1.36

$$-2 \left( c \sqrt{-\frac{1}{c^2x^2} + 1} + \frac{\operatorname{arccsc}(cx)}{x} \right) bde + ae^2x + \frac{1}{9} bd^2 \left( \frac{c^4 \left( -\frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} - \frac{3 \operatorname{arccsc}(cx)}{x^3} \right) + \frac{(2cx \operatorname{arccsc}(cx))}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")`

[Out]  $-2*(c*\text{sqrt}(-1/(c^2*x^2) + 1) + \operatorname{arccsc}(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*(c^4*(-1/(c^2*x^2) + 1)^{(3/2)} - 3*c^4*\text{sqrt}(-1/(c^2*x^2) + 1))/c - 3*\operatorname{arccsc}(c*x)/x^3 + 1/2*(2*c*x*\operatorname{arccsc}(c*x) + \log(\text{sqrt}(-1/(c^2*x^2) + 1) + 1) - \log(-\text{sqrt}(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3$

**Fricas [A]** time = 3.29647, size = 501, normalized size = 3.19

$$9ace^2x^4 - 9be^2x^3 \log(-cx + \sqrt{c^2x^2 - 1}) - 18acdex^2 + 6(bcd^2 + 6bcde - 3bce^2)x^3 \arctan(-cx + \sqrt{c^2x^2 - 1}) - 3acd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")`

[Out]  $1/9*(9*a*c*e^{2*x^4} - 9*b*e^{2*x^3}*\log(-c*x + \text{sqrt}(c^2*x^2 - 1)) - 18*a*c*d*e*x^2 + 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*\arctan(-c*x + \text{sqrt}(c^2*x^2 - 1)) - 3*a*c*d^2 - 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^{2*x^4} - 6*b*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*\operatorname{arccsc}(c*x) - (b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*\text{sqrt}(c^2*x^2 - 1))/(c*x^3)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx))(d + ex^2)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*acsc(c\*x))/x\*\*4,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)\*\*2/x\*\*4, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^4,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a)/x^4, x)

$$3.92 \quad \int \frac{(d+ex^2)^2 (a+b \csc^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=183

$$\frac{d^2 (a + b \csc^{-1}(cx))}{5x^5} - \frac{2de (a + b \csc^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \csc^{-1}(cx))}{x} - \frac{bc\sqrt{c^2x^2 - 1} (24c^4d^2 + 100c^2de + 225e^2)}{225\sqrt{c^2x^2}} - \frac{bcd}{2}$$

[Out]  $-(b*c*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/(225*\text{Sqrt}[c^2*x^2]) - (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(25*x^4*\text{Sqrt}[c^2*x^2]) - (2*b*c*d*(6*c^2*d + 25*e)*\text{Sqrt}[-1 + c^2*x^2])/(225*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcCsc}[c*x]))/(5*x^5) - (2*d*e*(a + b*\text{ArcCsc}[c*x]))/(3*x^3) - (e^2*(a + b*\text{ArcCsc}[c*x]))/x$

**Rubi [A]** time = 0.160178, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {270, 5239, 12, 1265, 453, 264}

$$\frac{d^2 (a + b \csc^{-1}(cx))}{5x^5} - \frac{2de (a + b \csc^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \csc^{-1}(cx))}{x} - \frac{bc\sqrt{c^2x^2 - 1} (24c^4d^2 + 100c^2de + 225e^2)}{225\sqrt{c^2x^2}} - \frac{bcd}{2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x^2)^2*(a + b*\text{ArcCsc}[c*x])/x^6, x]$

[Out]  $-(b*c*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/(225*\text{Sqrt}[c^2*x^2]) - (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(25*x^4*\text{Sqrt}[c^2*x^2]) - (2*b*c*d*(6*c^2*d + 25*e)*\text{Sqrt}[-1 + c^2*x^2])/(225*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcCsc}[c*x]))/(5*x^5) - (2*d*e*(a + b*\text{ArcCsc}[c*x]))/(3*x^3) - (e^2*(a + b*\text{ArcCsc}[c*x]))/x$

#### Rule 270

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, n\}, x \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 5239

$\text{Int}[(a_*) + \text{ArcCsc}[(c_*)*(x_*)*(b_*)]*((f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(p_*)}, x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcCsc}[c*x], u, x] + \text{Dist}[(b*c*x)/\text{Sqrt}[c^2*x^2], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[c^2*x^2 - 1]), x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x \ \&\& \ ((\text{IGtQ}[p, 0] \ \&\& \ !(\text{ILtQ}[(m - 1)/2, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{IGtQ}[(m + 1)/2, 0] \ \&\& \ !(\text{ILtQ}[p, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{ILtQ}[(m + 2*p + 1)/2, 0] \ \&\& \ !\text{ILtQ}[(m - 1)/2, 0]))$

#### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 1265

$\text{Int}[(f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(q_*)}*((a_*) + (b_*)*(x_*)^2 + (c_*)*(x_*)^4)^{(p_*)}, x\_Symbol] \rightarrow \text{With}\{Qx = \text{PolynomialQuotient}[a + b*x^2 +$

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x],
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

### Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 264

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \csc^{-1}(cx))}{x^6} dx &= -\frac{d^2 (a + b \csc^{-1}(cx))}{5x^5} - \frac{2de (a + b \csc^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \csc^{-1}(cx))}{x} + \frac{(bcx) \int \frac{-3d^2 - 15x}{\sqrt{-1 + c^2x^2}}}{15x} \\ &= -\frac{d^2 (a + b \csc^{-1}(cx))}{5x^5} - \frac{2de (a + b \csc^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \csc^{-1}(cx))}{x} + \frac{(bcx) \int \frac{-3d^2 - 15x}{x^6 \sqrt{-1 + c^2x^2}}}{15x} \\ &= -\frac{bcd^2 \sqrt{-1 + c^2x^2}}{25x^4 \sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{5x^5} - \frac{2de (a + b \csc^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \csc^{-1}(cx))}{x} \\ &= -\frac{bcd^2 \sqrt{-1 + c^2x^2}}{25x^4 \sqrt{c^2x^2}} - \frac{2bcd (6c^2d + 25e) \sqrt{-1 + c^2x^2}}{225x^2 \sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{5x^5} - \frac{2de (a + b \csc^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \csc^{-1}(cx))}{x} \\ &= -\frac{bc (225e^2 + 4c^2d (6c^2d + 25e)) \sqrt{-1 + c^2x^2}}{225 \sqrt{c^2x^2}} - \frac{bcd^2 \sqrt{-1 + c^2x^2}}{25x^4 \sqrt{c^2x^2}} - \frac{2bcd (6c^2d + 25e) \sqrt{-1 + c^2x^2}}{225x^2 \sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{5x^5} - \frac{2de (a + b \csc^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \csc^{-1}(cx))}{x} \end{aligned}$$

**Mathematica [A]** time = 0.262065, size = 127, normalized size = 0.69

$$\frac{15a(3d^2 + 10dex^2 + 15e^2x^4) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(3d^2(8c^4x^4 + 4c^2x^2 + 3) + 50dex^2(2c^2x^2 + 1) + 225e^2x^4) + 15b \csc^{-1}(cx)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6, x]
```

```
[Out] -(15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(225
*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4))
+ 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x])/(225*x^5)
```

**Maple [A]** time = 0.194, size = 191, normalized size = 1.

$$c^5 \left( \frac{a}{c^4} \left( -\frac{e^2}{cx} - \frac{d^2}{5cx^5} - \frac{2de}{3cx^3} \right) + \frac{b}{c^4} \left( -\frac{\operatorname{arccsc}(cx)e^2}{cx} - \frac{\operatorname{arccsc}(cx)d^2}{5cx^5} - \frac{2\operatorname{arccsc}(cx)ed}{3cx^3} - \frac{(c^2x^2-1)(24x^4c^8d^2+100}{c^2x^2-1} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^6,x)

[Out] c^5\*(a/c^4\*(-e^2/c/x-1/5\*d^2/c/x^5-2/3\*c\*e\*d/x^3)+b/c^4\*(-arccsc(c\*x)\*e^2/c/x-1/5\*arccsc(c\*x)\*d^2/c/x^5-2/3\*arccsc(c\*x)/c\*e\*d/x^3-1/225\*(c^2\*x^2-1)\*(24\*c^8\*d^2\*x^4+100\*c^6\*d\*e\*x^4+12\*c^6\*d^2\*x^2+225\*c^4\*e^2\*x^4+50\*c^4\*d\*e\*x^2+9\*c^4\*d^2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))

**Maxima [A]** time = 1.02725, size = 244, normalized size = 1.33

$$-\left( c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x} \right) be^2 - \frac{1}{75} bd^2 \left( \frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{15\operatorname{arccsc}(cx)}{x^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^6,x, algorithm="maxima")

[Out] -(c\*sqrt(-1/(c^2\*x^2)+1)+arccsc(c\*x)/x)\*b\*e^2-1/75\*b\*d^2\*((3\*c^6\*(-1/(c^2\*x^2)+1)^(5/2)-10\*c^6\*(-1/(c^2\*x^2)+1)^(3/2)+15\*c^6\*sqrt(-1/(c^2\*x^2)+1))/c+15\*arccsc(c\*x)/x^5)+2/9\*b\*d\*e\*((c^4\*(-1/(c^2\*x^2)+1)^(3/2)-3\*c^4\*sqrt(-1/(c^2\*x^2)+1))/c-3\*arccsc(c\*x)/x^3)-a\*e^2/x-2/3\*a\*d\*e/x^3-1/5\*a\*d^2/x^5)

**Fricas [A]** time = 2.69368, size = 302, normalized size = 1.65

$$\frac{225ae^2x^4 + 150adex^2 + 45ad^2 + 15(15be^2x^4 + 10bdex^2 + 3bd^2)\operatorname{arccsc}(cx) + ((24bc^4d^2 + 100bc^2de + 225be^2)x^4)}{225x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^6,x, algorithm="fricas")

[Out] -1/225\*(225\*a\*e^2\*x^4+150\*a\*d\*e\*x^2+45\*a\*d^2+15\*(15\*b\*e^2\*x^4+10\*b\*d\*e\*x^2+3\*b\*d^2)\*arccsc(c\*x)+((24\*b\*c^4\*d^2+100\*b\*c^2\*d\*e+225\*b\*e^2)\*x^4+9\*b\*d^2+2\*(6\*b\*c^2\*d^2+25\*b\*d\*e)\*x^2)\*sqrt(c^2\*x^2-1)/x^5)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))(d + ex^2)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*acsc(c\*x))/x\*\*6,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)\*\*2/x\*\*6, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^6,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a)/x^6, x)

$$3.93 \quad \int \frac{(d+ex^2)^2 (a+b \csc^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=241

$$\frac{d^2 (a + b \csc^{-1}(cx))}{7x^7} - \frac{2de (a + b \csc^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2bc^3 \sqrt{c^2 x^2 - 1} (360c^4 d^2 + 1176c^2 de + 1225e^2)}{11025 \sqrt{c^2 x^2}}$$

[Out]  $(-2*b*c^3*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/(11025*\text{Sqrt}[c^2*x^2]) - (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(49*x^6*\text{Sqrt}[c^2*x^2]) - (2*b*c*d*(15*c^2*d + 49*e)*\text{Sqrt}[-1 + c^2*x^2])/(1225*x^4*\text{Sqrt}[c^2*x^2]) - (b*c*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/(11025*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcCsc}[c*x]))/(7*x^7) - (2*d*e*(a + b*\text{ArcCsc}[c*x]))/(5*x^5) - (e^2*(a + b*\text{ArcCsc}[c*x]))/(3*x^3)$

**Rubi [A]** time = 0.196919, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 5239, 12, 1265, 453, 271, 264}

$$\frac{d^2 (a + b \csc^{-1}(cx))}{7x^7} - \frac{2de (a + b \csc^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \csc^{-1}(cx))}{3x^3} - \frac{2bc^3 \sqrt{c^2 x^2 - 1} (360c^4 d^2 + 1176c^2 de + 1225e^2)}{11025 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x^2)^2*(a + b*\text{ArcCsc}[c*x])/x^8, x]$

[Out]  $(-2*b*c^3*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/(11025*\text{Sqrt}[c^2*x^2]) - (b*c*d^2*\text{Sqrt}[-1 + c^2*x^2])/(49*x^6*\text{Sqrt}[c^2*x^2]) - (2*b*c*d*(15*c^2*d + 49*e)*\text{Sqrt}[-1 + c^2*x^2])/(1225*x^4*\text{Sqrt}[c^2*x^2]) - (b*c*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 + c^2*x^2])/(11025*x^2*\text{Sqrt}[c^2*x^2]) - (d^2*(a + b*\text{ArcCsc}[c*x]))/(7*x^7) - (2*d*e*(a + b*\text{ArcCsc}[c*x]))/(5*x^5) - (e^2*(a + b*\text{ArcCsc}[c*x]))/(3*x^3)$

#### Rule 270

$\text{Int}[(c_*)*(x_*)^{(m_*)}((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, m, n\}, x \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 5239

$\text{Int}[(a_*) + \text{ArcCsc}[(c_*)*(x_*)*(b_*)]*((f_*)*(x_*)^{(m_*)}((d_*) + (e_*)*(x_*)^2)^{(p_*)}, x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcCsc}[c*x], u, x] + \text{Dist}[(b*c*x)/\text{Sqrt}[c^2*x^2], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[c^2*x^2 - 1]), x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x \ \&\& \ ((\text{IGtQ}[p, 0] \ \&\& \ !(\text{ILtQ}[(m - 1)/2, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{IGtQ}[(m + 1)/2, 0] \ \&\& \ !(\text{ILtQ}[p, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{ILtQ}[(m + 2*p + 1)/2, 0] \ \&\& \ !\text{ILtQ}[(m - 1)/2, 0]))$

#### Rule 12

$\text{Int}[(a_*)*(u_*)], x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_*) /; \text{FreeQ}[b, x]]$

#### Rule 1265

```
Int[((f_)*(x_)^(m_)*((d_)+(e_)*(x_)^2)^(q_)*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

### Rule 453

```
Int[((e_)*(x_)^(m_)*((a_)+(b_)*(x_)^(n_))^(p_)*((c_)+(d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 271

```
Int[(x_)^(m_)*((a_)+(b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

### Rule 264

```
Int[((c_)*(x_)^(m_)*((a_)+(b_)*(x_)^(n_))^(p_)), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \csc^{-1}(cx))}{x^8} dx &= -\frac{d^2 (a + b \csc^{-1}(cx))}{7x^7} - \frac{2de (a + b \csc^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \csc^{-1}(cx))}{3x^3} + \frac{(bcx) \int \frac{-15d^2}{105 \sqrt{-1 + c^2x^2}}}{x^3} \\ &= -\frac{d^2 (a + b \csc^{-1}(cx))}{7x^7} - \frac{2de (a + b \csc^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \csc^{-1}(cx))}{3x^3} + \frac{(bcx) \int \frac{-15d^2}{105 \sqrt{-1 + c^2x^2}}}{x^3} \\ &= -\frac{bcd^2 \sqrt{-1 + c^2x^2}}{49x^6 \sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{7x^7} - \frac{2de (a + b \csc^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \csc^{-1}(cx))}{3x^3} \\ &= -\frac{bcd^2 \sqrt{-1 + c^2x^2}}{49x^6 \sqrt{c^2x^2}} - \frac{2bcd (15c^2d + 49e) \sqrt{-1 + c^2x^2}}{1225x^4 \sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{7x^7} - \frac{2de (a + b \csc^{-1}(cx))}{5x^5} \\ &= -\frac{bcd^2 \sqrt{-1 + c^2x^2}}{49x^6 \sqrt{c^2x^2}} - \frac{2bcd (15c^2d + 49e) \sqrt{-1 + c^2x^2}}{1225x^4 \sqrt{c^2x^2}} - \frac{bc (1225e^2 + 24c^2d (15c^2d + 49e)) \sqrt{-1 + c^2x^2}}{11025x^2 \sqrt{c^2x^2}} - \frac{11025x^2 \sqrt{c^2x^2}}{1225x^4} \\ &= -\frac{2bc^3 (1225e^2 + 24c^2d (15c^2d + 49e)) \sqrt{-1 + c^2x^2}}{11025 \sqrt{c^2x^2}} - \frac{bcd^2 \sqrt{-1 + c^2x^2}}{49x^6 \sqrt{c^2x^2}} - \frac{2bcd (15c^2d + 49e) \sqrt{-1 + c^2x^2}}{1225x^4 \sqrt{c^2x^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{7x^7} - \frac{2de (a + b \csc^{-1}(cx))}{5x^5} - \frac{e^2 (a + b \csc^{-1}(cx))}{3x^3} \end{aligned}$$

**Mathematica [A]** time = 0.261088, size = 153, normalized size = 0.63

$$\frac{105a(15d^2 + 42dex^2 + 35e^2x^4) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(45d^2(16c^6x^6 + 8c^4x^4 + 6c^2x^2 + 5) + 294dex^2(8c^4x^4 + 4c^2x^2 + 3) + 1225e^2x^4)}{11025x^7}$$

Antiderivative was successfully verified.



[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]))/x^8,x]

[Out]  $-(105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) + 105*b*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4)*\text{ArcCsc}[c*x])/(11025*x^7)$

**Maple [A]** time = 0.191, size = 223, normalized size = 0.9

$$c^7 \left( \frac{a}{c^4} \left( -\frac{d^2}{7c^3x^7} - \frac{2de}{5c^3x^5} - \frac{e^2}{3c^3x^3} \right) + \frac{b}{c^4} \left( -\frac{\text{arccsc}(cx)d^2}{7c^3x^7} - \frac{2\text{arccsc}(cx)ed}{5c^3x^5} - \frac{\text{arccsc}(cx)e^2}{3c^3x^3} - \frac{(c^2x^2 - 1)(720c^{10}d^2x^6 + 2352c^8d^2e^2x^4 + 360c^8d^2e^2x^4 + 2450c^6e^2x^6 + 1176c^6d^2e^2x^4 + 270c^6d^2e^2x^2 + 1225c^4e^2x^4 + 882c^4d^2e^2x^2 + 225c^4d^2)}{(c^2x^2 - 1)/c^2/x^2} \right)^{1/2} / c^8/x^8 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^8,x)

[Out]  $c^7*(a/c^4*(-1/7*d^2/c^3/x^7-2/5/c^3*e*d/x^5-1/3*e^2/c^3/x^3)+b/c^4*(-1/7*arccsc(c*x)*d^2/c^3/x^7-2/5*arccsc(c*x)/c^3*e*d/x^5-1/3*arccsc(c*x)*e^2/c^3/x^3-1/11025*(c^2*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d^2*e*x^4+360*c^8*d^2*x^4+2450*c^6*e^2*x^6+1176*c^6*d^2*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d^2*e*x^2+225*c^4*d^2)/(c^2*x^2-1)/c^2/x^2)^{1/2}/c^8/x^8)$

**Maxima [A]** time = 1.01665, size = 325, normalized size = 1.35

$$\frac{1}{245} bd^2 \left( \frac{5c^8 \left( -\frac{1}{c^2x^2} + 1 \right)^{7/2} - 21c^8 \left( -\frac{1}{c^2x^2} + 1 \right)^{5/2} + 35c^8 \left( -\frac{1}{c^2x^2} + 1 \right)^{3/2} - 35c^8 \sqrt{-\frac{1}{c^2x^2} + 1} - \frac{35 \text{arccsc}(cx)}{x^7}}{c} \right) - \frac{2}{75} bde \left( \frac{3c^6 \left( -\frac{1}{c^2x^2} + 1 \right)^{5/2} - 10c^6 \left( -\frac{1}{c^2x^2} + 1 \right)^{3/2} + 15c^6 \sqrt{-\frac{1}{c^2x^2} + 1}}{c} + 15 \text{arccsc}(cx) \right) / x^5 + 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^8,x, algorithm="maxima")

[Out]  $1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^{7/2} - 21*c^8*(-1/(c^2*x^2) + 1)^{5/2} + 35*c^8*(-1/(c^2*x^2) + 1)^{3/2} - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c - 35*arccsc(c*x)/x^7) - 2/75*b*d*e*((3*c^6*(-1/(c^2*x^2) + 1)^{5/2} - 10*c^6*(-1/(c^2*x^2) + 1)^{3/2} + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^{3/2} - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7$

**Fricas [A]** time = 2.56862, size = 401, normalized size = 1.66

$$\frac{3675 a^2 x^4 + 4410 a d e x^2 + 1575 a d^2 + 105 (35 b e^2 x^4 + 42 b d e x^2 + 15 b d^2) \text{arccsc}(cx) + (2 (360 b c^6 d^2 + 1176 b c^4 d e + 11025 x^7))}{11025 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^8,x, algorithm="fricas")

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*arccsc(c*x) + (2*(360*b*c^6*d^2 + 1176*b*c^4*d*e
+ 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x^4
+ 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^7
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccsc}(cx))(d + ex^2)^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*arccsc(c*x))/x**8,x)
```

```
[Out] Integral((a + b*arccsc(c*x))*(d + e*x**2)**2/x**8, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)/x^8, x)
```

### 3.94 $\int x^3 (d + ex^2)^2 (a + b \operatorname{csc}^{-1}(cx)) dx$

**Optimal.** Leaf size=242

$$\frac{1}{4}d^2x^4(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{3}dex^6(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{8}e^2x^8(a + b \operatorname{csc}^{-1}(cx)) + \frac{bx(c^2x^2 - 1)^{3/2}(6c^4d^2 + 16c^2de + 9e^2)}{72c^7\sqrt{c^2x^2}}$$

```
[Out] (b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x^2]) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*Sqrt[c^2*x^2]) + (b*e*(8*c^2*d + 9*e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt[c^2*x^2]) + (b*e^2*x*(-1 + c^2*x^2)^(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d^2*x^4*(a + b*ArcCsc[c*x]))/4 + (d*e*x^6*(a + b*ArcCsc[c*x]))/3 + (e^2*x^8*(a + b*ArcCsc[c*x]))/8
```

**Rubi [A]** time = 0.223124, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {266, 43, 5239, 12, 1251, 771}

$$\frac{1}{4}d^2x^4(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{3}dex^6(a + b \operatorname{csc}^{-1}(cx)) + \frac{1}{8}e^2x^8(a + b \operatorname{csc}^{-1}(cx)) + \frac{bx(c^2x^2 - 1)^{3/2}(6c^4d^2 + 16c^2de + 9e^2)}{72c^7\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x^2]) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*Sqrt[c^2*x^2]) + (b*e*(8*c^2*d + 9*e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt[c^2*x^2]) + (b*e^2*x*(-1 + c^2*x^2)^(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d^2*x^4*(a + b*ArcCsc[c*x]))/4 + (d*e*x^6*(a + b*ArcCsc[c*x]))/3 + (e^2*x^8*(a + b*ArcCsc[c*x]))/8
```

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5239

```
Int[((a_) + ArcCsc[(c_)*(x_)])*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)
^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (
c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned} \int x^3 (d + ex^2)^2 (a + b \csc^{-1}(cx)) dx &= \frac{1}{4} d^2 x^4 (a + b \csc^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \csc^{-1}(cx)) + \dots \\ &= \frac{1}{4} d^2 x^4 (a + b \csc^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \csc^{-1}(cx)) + \dots \\ &= \frac{1}{4} d^2 x^4 (a + b \csc^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \csc^{-1}(cx)) + \dots \\ &= \frac{1}{4} d^2 x^4 (a + b \csc^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \csc^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \csc^{-1}(cx)) + \dots \\ &= \frac{b(6c^4 d^2 + 8c^2 de + 3e^2)x\sqrt{-1 + c^2 x^2}}{24c^7 \sqrt{c^2 x^2}} + \frac{b(6c^4 d^2 + 16c^2 de + 9e^2)x(-1 + c^2 x^2)^{3/2}}{72c^7 \sqrt{c^2 x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.319317, size = 159, normalized size = 0.66

$$x \left( 105ax^3 (6d^2 + 8dex^2 + 3e^2x^4) + \frac{b\sqrt{1 - \frac{1}{c^2x^2}} (3c^6(70d^2x^2 + 56dex^4 + 15e^2x^6) + c^4(420d^2 + 224dex^2 + 54e^2x^4) + 8c^2e(56d + 9ex^2) + 144e^2)}{c^7} + 105bx^3 \csc^{-1}(cx) \right)$$

2520

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(1
44*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^4
) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^2 +
8*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c*x])/2520
```

**Maple [A]** time = 0.186, size = 214, normalized size = 0.9

$$\frac{1}{c^4} \left( \frac{a}{c^4} \left( \frac{e^2 c^8 x^8}{8} + \frac{c^8 e d x^6}{3} + \frac{x^4 c^8 d^2}{4} \right) + \frac{b}{c^4} \left( \frac{\operatorname{arccsc}(cx) e^2 c^8 x^8}{8} + \frac{\operatorname{arccsc}(cx) c^8 e d x^6}{3} + \frac{\operatorname{arccsc}(cx) c^8 x^4 d^2}{4} + \frac{(c^2 x^2 - 1)}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x)

[Out] 1/c^4\*(a/c^4\*(1/8\*e^2\*c^8\*x^8+1/3\*c^8\*e\*d\*x^6+1/4\*x^4\*c^8\*d^2)+b/c^4\*(1/8\*arccsc(c\*x)\*e^2\*c^8\*x^8+1/3\*arccsc(c\*x)\*c^8\*e\*d\*x^6+1/4\*arccsc(c\*x)\*c^8\*x^4\*d^2+1/2520\*(c^2\*x^2-1)\*(45\*c^6\*e^2\*x^6+168\*c^6\*d\*e\*x^4+210\*c^6\*d^2\*x^2+54\*c^4\*e^2\*x^4+224\*c^4\*d\*e\*x^2+420\*c^4\*d^2+72\*c^2\*e^2\*x^2+448\*c^2\*d\*e+144\*e^2)/((c^2\*x^2-1)/c^2/x^2)^(1/2)/c/x))

**Maxima [A]** time = 1.01405, size = 342, normalized size = 1.41

$$\frac{1}{8} a e^2 x^8 + \frac{1}{3} a d e x^6 + \frac{1}{4} a d^2 x^4 + \frac{1}{12} \left( 3 x^4 \operatorname{arccsc}(cx) + \frac{c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 3 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^3} \right) b d^2 + \frac{1}{45} \left( 15 x^6 \operatorname{arccsc}(cx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] 1/8\*a\*e^2\*x^8 + 1/3\*a\*d\*e\*x^6 + 1/4\*a\*d^2\*x^4 + 1/12\*(3\*x^4\*arccsc(c\*x) + (c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 3\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^3)\*b\*d^2 + 1/45\*(15\*x^6\*arccsc(c\*x) + (3\*c^4\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 10\*c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 15\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^5)\*b\*d\*e + 1/280\*(35\*x^8\*arccsc(c\*x) + (5\*c^6\*x^7\*(-1/(c^2\*x^2) + 1)^(7/2) + 21\*c^4\*x^5\*(-1/(c^2\*x^2) + 1)^(5/2) + 35\*c^2\*x^3\*(-1/(c^2\*x^2) + 1)^(3/2) + 35\*x\*sqrt(-1/(c^2\*x^2) + 1))/c^7)\*b\*e^2

**Fricas [A]** time = 3.3606, size = 431, normalized size = 1.78

$$\frac{315 a c^8 e^2 x^8 + 840 a c^8 d e x^6 + 630 a c^8 d^2 x^4 + 105 (3 b c^8 e^2 x^8 + 8 b c^8 d e x^6 + 6 b c^8 d^2 x^4) \operatorname{arccsc}(cx) + (45 b c^6 e^2 x^6 + 420 b c^6 d e x^4 + 144 b c^6 d^2 x^2 + 2 (105 b c^6 d^2 + 112 b c^4 d e + 36 b c^2 e^2) x^2) \sqrt{(c^2 x^2 - 1)}}{2520 c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] 1/2520\*(315\*a\*c^8\*e^2\*x^8 + 840\*a\*c^8\*d\*e\*x^6 + 630\*a\*c^8\*d^2\*x^4 + 105\*(3\*b\*c^8\*e^2\*x^8 + 8\*b\*c^8\*d\*e\*x^6 + 6\*b\*c^8\*d^2\*x^4)\*arccsc(c\*x) + (45\*b\*c^6\*e^2\*x^6 + 420\*b\*c^6\*d\*e\*x^4 + 144\*b\*c^6\*d^2\*x^2 + 2\*(105\*b\*c^6\*d^2 + 112\*b\*c^4\*d\*e + 36\*b\*c^2\*e^2)\*x^2)\*sqrt((c^2\*x^2 - 1))/c^8

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{acsc}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x\*\*2+d)\*\*2\*(a+b\*acsc(c\*x)),x)

[Out] Integral(x\*\*3\*(a + b\*acsc(c\*x))\*(d + e\*x\*\*2)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a)\*x^3, x)

### 3.95 $\int x (d + ex^2)^2 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=195

$$\frac{(d + ex^2)^3 (a + b \csc^{-1}(cx))}{6e} + \frac{bx\sqrt{c^2x^2 - 1} (3c^4d^2 + 3c^2de + e^2)}{6c^5\sqrt{c^2x^2}} + \frac{bcd^3x \tan^{-1}(\sqrt{c^2x^2 - 1})}{6e\sqrt{c^2x^2}} + \frac{bex(c^2x^2 - 1)^{3/2} (3c^2d)}{18c^5\sqrt{c^2x^2}}$$

```
[Out] (b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*x*Sqrt[-1 + c^2*x^2])/(6*c^5*Sqrt[c^2*x^2])
+ (b*e*(3*c^2*d + 2*e)*x*(-1 + c^2*x^2)^(3/2))/(18*c^5*Sqrt[c^2*x^2]) + (
b*e^2*x*(-1 + c^2*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + ((d + e*x^2)^3*(a +
b*ArcCsc[c*x]))/(6*e) + (b*c*d^3*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*e*Sqrt[c^
2*x^2])
```

**Rubi [A]** time = 0.144222, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5237, 446, 88, 63, 205}

$$\frac{(d + ex^2)^3 (a + b \csc^{-1}(cx))}{6e} + \frac{bx\sqrt{c^2x^2 - 1} (3c^4d^2 + 3c^2de + e^2)}{6c^5\sqrt{c^2x^2}} + \frac{bcd^3x \tan^{-1}(\sqrt{c^2x^2 - 1})}{6e\sqrt{c^2x^2}} + \frac{bex(c^2x^2 - 1)^{3/2} (3c^2d)}{18c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*x*Sqrt[-1 + c^2*x^2])/(6*c^5*Sqrt[c^2*x^2])
+ (b*e*(3*c^2*d + 2*e)*x*(-1 + c^2*x^2)^(3/2))/(18*c^5*Sqrt[c^2*x^2]) + (
b*e^2*x*(-1 + c^2*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + ((d + e*x^2)^3*(a +
b*ArcCsc[c*x]))/(6*e) + (b*c*d^3*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*e*Sqrt[c^
2*x^2])
```

#### Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 88

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x
_)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 205

$\text{Int}[(a_ + (b_ )*(x_ )^2)^{-1}, x\_Symbol] :> \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

### Rubi steps

$$\begin{aligned} \int x(d+ex^2)^2(a+b\csc^{-1}(cx))dx &= \frac{(d+ex^2)^3(a+b\csc^{-1}(cx))}{6e} + \frac{(bcx)\int\frac{(d+ex^2)^3}{x\sqrt{-1+c^2x^2}}dx}{6e\sqrt{c^2x^2}} \\ &= \frac{(d+ex^2)^3(a+b\csc^{-1}(cx))}{6e} + \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex^2)^3}{x\sqrt{-1+c^2x^2}}dx, x, x^2\right)}{12e\sqrt{c^2x^2}} \\ &= \frac{(d+ex^2)^3(a+b\csc^{-1}(cx))}{6e} + \frac{(bcx)\text{Subst}\left(\int\left(\frac{e(3c^4d^2+3c^2de+e^2)}{c^4\sqrt{-1+c^2x^2}} + \frac{d^3}{x\sqrt{-1+c^2x^2}} + \frac{e^2(3c^2d+2e)}{c^4\sqrt{-1+c^2x^2}}\right)dx, x, x^2\right)}{12e\sqrt{c^2x^2}} \\ &= \frac{b(3c^4d^2+3c^2de+e^2)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{3/2}}{30c^5\sqrt{c^2x^2}} \\ &= \frac{b(3c^4d^2+3c^2de+e^2)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{3/2}}{30c^5\sqrt{c^2x^2}} \\ &= \frac{b(3c^4d^2+3c^2de+e^2)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{3/2}}{30c^5\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.28052, size = 124, normalized size = 0.64

$$\frac{1}{90}x\left(15ax(3d^2+3dex^2+e^2x^4) + \frac{b\sqrt{1-\frac{1}{c^2x^2}}(3c^4(15d^2+5dex^2+e^2x^4)+2c^2e(15d+2ex^2)+8e^2)}{c^5} + 15bx\csc^{-1}(cx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]), x]

[Out] (x\*(15\*a\*x\*(3\*d^2 + 3\*d\*e\*x^2 + e^2\*x^4) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*(8\*e^2 + 2\*c^2\*e\*(15\*d + 2\*e\*x^2) + 3\*c^4\*(15\*d^2 + 5\*d\*e\*x^2 + e^2\*x^4))))/c^5 + 15\*b\*x\*(3\*d^2 + 3\*d\*e\*x^2 + e^2\*x^4)\*ArcCsc[c\*x])/90

**Maple [A]** time = 0.188, size = 182, normalized size = 0.9

$$\frac{1}{c^2}\left(\frac{a}{c^4}\left(\frac{e^2c^6x^6}{6} + \frac{c^6x^4de}{2} + \frac{x^2c^6d^2}{2}\right) + \frac{b}{c^4}\left(\frac{\text{arccsc}(cx)e^2c^6x^6}{6} + \frac{\text{arccsc}(cx)c^6x^4de}{2} + \frac{\text{arccsc}(cx)c^6x^2d^2}{2} + \frac{(c^2x^2-1)(3c^4d^2+3c^2de+e^2)}{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)`

[Out]  $\frac{1}{c^2} \left( \frac{a}{c^4} \left( \frac{1}{6} e^2 c^6 x^6 + \frac{1}{2} c^6 x^4 d e + \frac{1}{2} x^2 c^6 d^2 \right) + \frac{b}{c^4} \left( \frac{1}{6} a \operatorname{arccsc}(c x) e^2 c^6 x^6 + \frac{1}{2} \operatorname{arccsc}(c x) c^6 x^4 d e + \frac{1}{2} \operatorname{arccsc}(c x) c^6 x^2 d^2 + \frac{1}{90} (c^2 x^2 - 1) (3 c^4 e^2 x^4 + 15 c^4 d e x^2 + 45 c^4 d^2 + 4 c^2 e^2 x^2 + 30 c^2 d e + 8 e^2) \right) \right) / \left( \frac{c^2 x^2 - 1}{c^2 x^2} \right)^{(1/2)} / c/x$

**Maxima [A]** time = 1.00152, size = 255, normalized size = 1.31

$$\frac{1}{6} a e^2 x^6 + \frac{1}{2} a d e x^4 + \frac{1}{2} a d^2 x^2 + \frac{1}{2} \left( x^2 \operatorname{arccsc}(c x) + \frac{x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c} \right) b d^2 + \frac{1}{6} \left( 3 x^4 \operatorname{arccsc}(c x) + \frac{c^2 x^3 \left( -\frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 3 x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{6} a e^2 x^6 + \frac{1}{2} a d e x^4 + \frac{1}{2} a d^2 x^2 + \frac{1}{2} (x^2 \operatorname{arccsc}(c x) + x \sqrt{-1/(c^2 x^2) + 1}/c) b d^2 + \frac{1}{6} (3 x^4 \operatorname{arccsc}(c x) + (c^2 x^3 (-1/(c^2 x^2) + 1)^{(3/2)} + 3 x \sqrt{-1/(c^2 x^2) + 1})/c^3) b d e + \frac{1}{90} (15 x^6 \operatorname{arccsc}(c x) + (3 c^4 x^5 (-1/(c^2 x^2) + 1)^{(5/2)} + 10 c^2 x^3 (-1/(c^2 x^2) + 1)^{(3/2)} + 15 x \sqrt{-1/(c^2 x^2) + 1})/c^5) b e^2$

**Fricas [A]** time = 3.04265, size = 336, normalized size = 1.72

$$\frac{15 a c^6 e^2 x^6 + 45 a c^6 d e x^4 + 45 a c^6 d^2 x^2 + 15 (b c^6 e^2 x^6 + 3 b c^6 d e x^4 + 3 b c^6 d^2 x^2) \operatorname{arccsc}(c x) + (3 b c^4 e^2 x^4 + 45 b c^4 d^2 + 30 b c^4 d e + 8 b e^2 + (15 b c^4 d e + 4 b c^2 e^2) x^2) \sqrt{c^2 x^2 - 1}}{90 c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out]  $\frac{1}{90} (15 a c^6 e^2 x^6 + 45 a c^6 d e x^4 + 45 a c^6 d^2 x^2 + 15 (b c^6 e^2 x^6 + 3 b c^6 d e x^4 + 3 b c^6 d^2 x^2) \operatorname{arccsc}(c x) + (3 b c^4 e^2 x^4 + 45 b c^4 d^2 + 30 b c^4 d e + 8 b e^2 + (15 b c^4 d e + 4 b c^2 e^2) x^2) \sqrt{c^2 x^2 - 1}) / c^6$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{acsc}(c x)) (d + e x^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d)**2*(a+b*acsc(c*x)),x)`

[Out] `Integral(x*(a + b*acsc(c*x))*(d + e*x**2)**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)*x, x)
```

$$3.96 \quad \int \frac{(d+ex^2)^2 (a+b \csc^{-1}(cx))}{x} dx$$

**Optimal.** Leaf size=186

$$\frac{1}{2} ibd^2 \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) - d^2 \log\left(\frac{1}{x}\right) (a + b \csc^{-1}(cx)) + dex^2 (a + b \csc^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + b \csc^{-1}(cx)) + \frac{bex}{4}$$

```
[Out] (b*e*(6*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c) + (I/2)*b*d^2*ArcCsc[c*x]^2 + d*e*x^2*(a + b*ArcCsc[c*x]) + (e^2*x^4*(a + b*ArcCsc[c*x]))/4 - b*d^2*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + b*d^2*ArcCsc[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsc[c*x])*Log[x^(-1)] + (I/2)*b*d^2*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

**Rubi [A]** time = 0.422263, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {5241, 266, 43, 4731, 6742, 453, 264, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2} ibd^2 \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) - d^2 \log\left(\frac{1}{x}\right) (a + b \csc^{-1}(cx)) + dex^2 (a + b \csc^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + b \csc^{-1}(cx)) + \frac{bex}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (b*e*(6*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c) + (I/2)*b*d^2*ArcCsc[c*x]^2 + d*e*x^2*(a + b*ArcCsc[c*x]) + (e^2*x^4*(a + b*ArcCsc[c*x]))/4 - b*d^2*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + b*d^2*ArcCsc[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsc[c*x])*Log[x^(-1)] + (I/2)*b*d^2*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

#### Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 4731

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

$[a + b \cdot \text{ArcSin}[c \cdot x], u, x] - \text{Dist}[b \cdot c, \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2 \cdot x^2], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2\*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

#### Rule 6742

$\text{Int}[u_, x\_Symbol] := \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$   
]

#### Rule 453

$\text{Int}[\{(e\_)\cdot(x\_)\}^{(m\_)}\cdot\{(a\_)+(b\_)\cdot(x\_)^{(n\_)}\}^{(p\_)}\cdot\{(c\_)+(d\_)\cdot(x\_)^{(n\_)}\}, x\_Symbol] := \text{Simp}[(c\cdot(e\cdot x)^{(m+1)}\cdot(a+b\cdot x^n)^{(p+1)})/(a\cdot e^{(m+1)}), x] + \text{Dist}[(a\cdot d\cdot(m+1) - b\cdot c\cdot(m+n\cdot(p+1)+1))/(a\cdot e^n\cdot(m+1)), \text{Int}[(e\cdot x)^{(m+n)}\cdot(a+b\cdot x^n)^p, x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

#### Rule 264

$\text{Int}[\{(c\_)\cdot(x\_)\}^{(m\_)}\cdot\{(a\_)+(b\_)\cdot(x\_)^{(n\_)}\}^{(p\_)}, x\_Symbol] := \text{Simp}[(c\cdot x)^{(m+1)}\cdot(a+b\cdot x^n)^{(p+1)})/(a\cdot c\cdot(m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 2326

$\text{Int}[\{(a\_)+\text{Log}[(c\_)\cdot(x\_)^{(n\_)}]\cdot(b\_)\}/\text{Sqrt}[(d\_)+(e\_)\cdot(x\_)^2], x\_Symbol] := \text{Simp}[(\text{ArcSin}[(\text{Rt}[-e, 2]\cdot x)/\text{Sqrt}[d]]\cdot(a+b\cdot \text{Log}[c\cdot x^n]))/\text{Rt}[-e, 2], x] - \text{Dist}[(b\cdot n)/\text{Rt}[-e, 2], \text{Int}[\text{ArcSin}[(\text{Rt}[-e, 2]\cdot x)/\text{Sqrt}[d]]/x, x], x] /;$  FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]

#### Rule 4625

$\text{Int}[\{(a\_)+\text{ArcSin}[(c\_)\cdot(x\_)]\cdot(b\_)\}^{(n\_)}(x\_), x\_Symbol] := \text{Subst}[\text{Int}[(a+b\cdot x)^n/\text{Tan}[x], x], x, \text{ArcSin}[c\cdot x] /;$  FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 3717

$\text{Int}[\{(c\_)+(d\_)\cdot(x\_)\}^{(m\_)}\cdot \text{tan}[(e\_)+\text{Pi}\cdot(k\_)+(f\_)\cdot(x\_)], x\_Symbol] := \text{Simp}[(I\cdot(c+d\cdot x)^{(m+1)})/(d\cdot(m+1)), x] - \text{Dist}[2\cdot I, \text{Int}[(c+d\cdot x)^m\cdot E^{(2\cdot I\cdot k\cdot \text{Pi})}\cdot E^{(2\cdot I\cdot(e+f\cdot x))})/(1+E^{(2\cdot I\cdot k\cdot \text{Pi})}\cdot E^{(2\cdot I\cdot(e+f\cdot x))}), x], x] /;$  FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 2190

$\text{Int}[\{((F\_)^{(g\_)\cdot\{(e\_)+(f\_)\cdot(x\_)\}})^{(n\_)}\cdot\{(c\_)+(d\_)\cdot(x\_)\}^{(m\_)}\}/\{(a\_)+(b\_)\cdot\{(F\_)^{(g\_)\cdot\{(e\_)+(f\_)\cdot(x\_)\}})^{(n\_)}\}, x\_Symbol] := \text{Simp}[(c+d\cdot x)^m\cdot \text{Log}[1+(b\cdot(F^{(g\cdot(e+f\cdot x))})^n)/a]]/(b\cdot f\cdot g\cdot n\cdot \text{Log}[F]), x] - \text{Dist}[(d\cdot m)/(b\cdot f\cdot g\cdot n\cdot \text{Log}[F]), \text{Int}[(c+d\cdot x)^{(m-1)}\cdot \text{Log}[1+(b\cdot(F^{(g\cdot(e+f\cdot x))})^n)/a], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

$\text{Int}[\text{Log}[\{(a\_)+(b\_)\cdot\{(F\_)^{(e\_)\cdot\{(c\_)+(d\_)\cdot(x\_)\}})^{(n\_)}\}], x\_Symbol] := \text{Dist}[1/(d\cdot e\cdot n\cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a+b\cdot x]/x, x], x, (F^{(e\cdot(c+d\cdot x))})^n], x] /;$  FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^2 (a+b \csc^{-1}(cx))}{x} dx &= -\text{Subst} \left( \int \frac{(e+dx^2)^2 (a+b \sin^{-1}(\frac{x}{c}))}{x^5} dx, x, \frac{1}{x} \right) \\ &= dex^2 (a+b \csc^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b \csc^{-1}(cx)) - d^2 (a+b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \\ &= dex^2 (a+b \csc^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b \csc^{-1}(cx)) - d^2 (a+b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \\ &= dex^2 (a+b \csc^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b \csc^{-1}(cx)) - d^2 (a+b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \\ &= \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + dex^2 (a+b \csc^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b \csc^{-1}(cx)) + bd^2 \csc^{-1}(cx) \\ &= \frac{be(6c^2d+e) \sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + dex^2 (a+b \csc^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b \csc^{-1}(cx)) \\ &= \frac{be(6c^2d+e) \sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+b \csc^{-1}(cx)) \\ &= \frac{be(6c^2d+e) \sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+b \csc^{-1}(cx)) \\ &= \frac{be(6c^2d+e) \sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+b \csc^{-1}(cx)) \\ &= \frac{be(6c^2d+e) \sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2 \sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}ibd^2 \csc^{-1}(cx)^2 + dex^2 (a+b \csc^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.364491, size = 157, normalized size = 0.84

$$\frac{1}{2}ibd^2 \left( \text{PolyLog}\left(2, e^{2i \csc^{-1}(cx)}\right) + \csc^{-1}(cx) \left( \csc^{-1}(cx) + 2i \log\left(1 - e^{2i \csc^{-1}(cx)}\right)\right) \right) + ad^2 \log(x) + adex^2 + \frac{1}{4}ae^2x^4 + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]))/x, x]

[Out] a\*d\*e\*x^2 + (a\*e^2\*x^4)/4 + (b\*d\*e\*x\*(Sqrt[1 - 1/(c^2\*x^2)] + c\*x\*ArcCsc[c\*x]))/c + (b\*e^2\*x\*(Sqrt[1 - 1/(c^2\*x^2)]\*(2 + c^2\*x^2) + 3\*c^3\*x^3\*ArcCsc[c\*x]))/(12\*c^3) + a\*d^2\*Log[x] + (I/2)\*b\*d^2\*(ArcCsc[c\*x]\*(ArcCsc[c\*x] + (2\*I)\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])]) + PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])

---

**Maple [A]** time = 1.038, size = 301, normalized size = 1.6

$$\frac{ax^4e^2}{4} + ax^2de + ad^2 \ln(cx) + \frac{i}{2}bd^2 (\operatorname{arccsc}(cx))^2 + \frac{\operatorname{arccsc}(cx)e^2x^4}{4} + \operatorname{arccsc}(cx)x^2de + \frac{bx^3e^2}{12c} \sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{bxde}{c} \sqrt{\frac{c^2x^2-1}{c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x)`

[Out] `1/4*a*x^4*e^2+a*x^2*d*e+a*d^2*ln(c*x)+1/2*I*b*d^2*arccsc(c*x)^2+1/4*b*arccsc(c*c*x)*e^2*x^4+b*arccsc(c*x)*x^2*d*e+1/12*b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*x^3*e^2+b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d*e-I*b/c^2*d*e+1/6*b/c^3*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e^2-1/6*I*b/c^4*e^2-b*d^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-b*d^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*d^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+I*b*d^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="maxima")`

[Out] `1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + 1/8*(4*I*b*c^4*d^2*log(-c*x + 1)*log(x) + 4*I*b*c^4*d^2*log(x)^2 + 4*I*b*c^4*d^2*dilog(c*x) + 4*I*b*c^4*d^2*dilog(-c*x) + 2*(b*c^4*arctan2(1, sqrt(c*x + 1))*sqrt(c*x - 1)) + I*b*c^4*log(c))*e^2*x^4 - I*(b*e^2*(x^2/c^2 + log(c*x + 1)/c^4 + log(c*x - 1)/c^4) + 4*b*d*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 32*b*d^2*integrate(1/4*log(x)/(c^2*x^3 - x), x))*c^4 + 8*c^4*integrate(1/4*(b*e^2*x^4 + 4*b*d*e*x^2 + 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) + (I*b*c^2*e^2 + 8*(b*c^4*arctan2(1, sqrt(c*x + 1))*sqrt(c*x - 1)) + I*b*c^4*log(c))*d*e*x^2 + (-I*b*c^4*e^2*x^4 - 4*I*b*c^4*d*e*x^2 - 4*I*b*c^4*d^2*log(x))*log(c^2*x^2) + (4*I*b*c^4*d^2*log(x) + 4*I*b*c^2*d*e + I*b*e^2)*log(c*x + 1) + (4*I*b*c^2*d*e + I*b*e^2)*log(c*x - 1) + (2*I*b*c^4*e^2*x^4 + 8*I*b*c^4*d*e*x^2 + 8*(b*c^4*arctan2(1, sqrt(c*x + 1))*sqrt(c*x - 1)) + I*b*c^4*log(c))*d^2*log(x))/c^4`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2)\operatorname{arccsc}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="fricas")`

[Out] `integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arccsc(c*x))/x, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx)) (d + ex^2)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*acsc(c\*x))/x,x)

[Out] Integral((a + b\*acsc(c\*x))\*(d + e\*x\*\*2)\*\*2/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arccsc}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a)/x, x)

$$3.97 \quad \int \frac{(d+ex^2)^2 (a+b \csc^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=189

$$ibdePolyLog\left(2, e^{2i \csc^{-1}(cx)}\right) - \frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} - 2de \log\left(\frac{1}{x}\right) (a + b \csc^{-1}(cx)) + \frac{1}{2} e^{2x^2} (a + b \csc^{-1}(cx)) - \frac{bcd^2 \sqrt{1 - \frac{1}{c^2 x^2}}}{4x}$$

[Out]  $-(b*c*d^2*\text{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) + (b*e^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(2*c) + (b*c^2*d^2*\text{ArcCsc}[c*x])/4 + I*b*d*e*\text{ArcCsc}[c*x]^2 - (d^2*(a + b*\text{ArcCsc}[c*x]))/(2*x^2) + (e^2*x^2*(a + b*\text{ArcCsc}[c*x]))/2 - 2*b*d*e*\text{ArcCsc}[c*x]*\text{Log}[1 - E^((2*I)*\text{ArcCsc}[c*x])] + 2*b*d*e*\text{ArcCsc}[c*x]*\text{Log}[x^(-1)] - 2*d*e*(a + b*\text{ArcCsc}[c*x])*\text{Log}[x^(-1)] + I*b*d*e*\text{PolyLog}[2, E^((2*I)*\text{ArcCsc}[c*x])]$

**Rubi [A]** time = 0.429612, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5241, 266, 43, 4731, 12, 6742, 264, 321, 216, 2326, 4625, 3717, 2190, 2279, 2391}

$$ibdePolyLog\left(2, e^{2i \csc^{-1}(cx)}\right) - \frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} - 2de \log\left(\frac{1}{x}\right) (a + b \csc^{-1}(cx)) + \frac{1}{2} e^{2x^2} (a + b \csc^{-1}(cx)) - \frac{bcd^2 \sqrt{1 - \frac{1}{c^2 x^2}}}{4x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x^2)^2*(a + b*\text{ArcCsc}[c*x])/x^3, x]$

[Out]  $-(b*c*d^2*\text{Sqrt}[1 - 1/(c^2*x^2)])/(4*x) + (b*e^2*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)/(2*c) + (b*c^2*d^2*\text{ArcCsc}[c*x])/4 + I*b*d*e*\text{ArcCsc}[c*x]^2 - (d^2*(a + b*\text{ArcCsc}[c*x]))/(2*x^2) + (e^2*x^2*(a + b*\text{ArcCsc}[c*x]))/2 - 2*b*d*e*\text{ArcCsc}[c*x]*\text{Log}[1 - E^((2*I)*\text{ArcCsc}[c*x])] + 2*b*d*e*\text{ArcCsc}[c*x]*\text{Log}[x^(-1)] - 2*d*e*(a + b*\text{ArcCsc}[c*x])*\text{Log}[x^(-1)] + I*b*d*e*\text{PolyLog}[2, E^((2*I)*\text{ArcCsc}[c*x])]$

#### Rule 5241

$\text{Int}[(a + \text{ArcCsc}[c*(x)]*(b))^n*(x)^m*((d) + (e)*(x)^2)^p, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcSin}[x/c])^n/x^{m+2*(p+1)}, x], x, 1/x] /;$  FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rule 266

$\text{Int}[(x)^m*((a) + (b)*(x)^n)^p, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(Simplify[(m+1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /;$  FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

#### Rule 43

$\text{Int}[(a + (b)*(x))^m*((c) + (d)*(x))^n, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

#### Rule 4731

$\text{Int}[(a + \text{ArcSin}[c*(x)]*(b))*(f*(x))^m*((d) + (e)*(x)^2)^p, x\_Symbol] \rightarrow \text{With}[u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x], \text{Dist}$



$[a + b \cdot \text{ArcSin}[c \cdot x], u, x] - \text{Dist}[b \cdot c, \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2 \cdot x^2], x], x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2\*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

### Rule 12

$\text{Int}[(a\_)(u\_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\\_)(v\\_)] /; FreeQ[b, x]

### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

### Rule 264

$\text{Int}[(c\_)(x\_)^{(m\_)}((a\_)+(b\_)(x\_)^{(n\_)})^{(p\_)}, x\_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{(m+1)}(a + b \cdot x^n)^{(p+1)}/(a \cdot c \cdot (m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

### Rule 321

$\text{Int}[(c\_)(x\_)^{(m\_)}((a\_)+(b\_)(x\_)^{(n\_)})^{(p\_)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}(c \cdot x)^{(m-n+1)}(a + b \cdot x^n)^{(p+1)})/(b \cdot (m+n \cdot p+1)), x] - \text{Dist}[(a \cdot c^n \cdot (m-n+1))/(b \cdot (m+n \cdot p+1)), \text{Int}[(c \cdot x)^{(m-n)}(a + b \cdot x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n \cdot p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 216

$\text{Int}[1/\text{Sqrt}[(a\_)+(b\_)(x_)^2], x\_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2] \cdot x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$  FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

### Rule 2326

$\text{Int}[(a\_)+\text{Log}[(c\_)(x_)^{(n\_)}](b\_)]/\text{Sqrt}[(d\_)+(e\_)(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{ArcSin}[(\text{Rt}[-e, 2] \cdot x)/\text{Sqrt}[d]] \cdot (a + b \cdot \text{Log}[c \cdot x^n]))/\text{Rt}[-e, 2], x] - \text{Dist}[(b \cdot n)/\text{Rt}[-e, 2], \text{Int}[\text{ArcSin}[(\text{Rt}[-e, 2] \cdot x)/\text{Sqrt}[d]]/x, x], x] /;$  FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]

### Rule 4625

$\text{Int}[(a\_)+\text{ArcSin}[(c\_)(x_)](b_)^{(n\_)}(x_), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b \cdot x)^n/\text{Tan}[x], x], x, \text{ArcSin}[c \cdot x]] /;$  FreeQ[{a, b, c}, x] && IGtQ[n, 0]

### Rule 3717

$\text{Int}[(c\_)+(d\_)(x_)^{(m\_)}\text{tan}[(e\_)+\text{Pi} \cdot (k\_)+(f\_)(x_)], x\_Symbol] \rightarrow \text{Simp}[(I \cdot (c + d \cdot x)^{(m+1)})/(d \cdot (m+1)), x] - \text{Dist}[2 \cdot I, \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))})/(1 + E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}), x], x] /;$  FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

$\text{Int}[(F\_)^{((g\_)((e\_)+(f\_)(x_)))^{(n\_)}((c\_)+(d\_)(x_)^{(m\_)}))}/((a\_)+(b\_)(F\_)^{((g\_)((e\_)+(f\_)(x_)))^{(n\_)})), x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x)))^n})/a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x)))^n})], x], x] /;$

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\int \frac{(d + ex^2)^2 (a + b \csc^{-1}(cx))}{x^3} dx = -\text{Subst} \left( \int \frac{(e + dx^2)^2 (a + b \sin^{-1}(\frac{x}{c}))}{x^3} dx, x, \frac{1}{x} \right)$$

$$= -\frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a + b \csc^{-1}(cx)) - 2de (a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \dots$$

$$= -\frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a + b \csc^{-1}(cx)) - 2de (a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \dots$$

$$= -\frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a + b \csc^{-1}(cx)) - 2de (a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \dots$$

$$= -\frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a + b \csc^{-1}(cx)) - 2de (a + b \csc^{-1}(cx)) \log\left(\frac{1}{x}\right) + \dots$$

$$= -\frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{be^2 \sqrt{1 - \frac{1}{c^2x^2}}x}{2c} - \frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a + b \csc^{-1}(cx)) + \dots$$

$$= -\frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{be^2 \sqrt{1 - \frac{1}{c^2x^2}}x}{2c} + \frac{1}{4}bc^2d^2 \csc^{-1}(cx) - \frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2 (a + b \csc^{-1}(cx)) + \dots$$

$$= -\frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{be^2 \sqrt{1 - \frac{1}{c^2x^2}}x}{2c} + \frac{1}{4}bc^2d^2 \csc^{-1}(cx) + ibde \csc^{-1}(cx)^2 - \frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \dots$$

$$= -\frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{be^2 \sqrt{1 - \frac{1}{c^2x^2}}x}{2c} + \frac{1}{4}bc^2d^2 \csc^{-1}(cx) + ibde \csc^{-1}(cx)^2 - \frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \dots$$

$$= -\frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}}}{4x} + \frac{be^2 \sqrt{1 - \frac{1}{c^2x^2}}x}{2c} + \frac{1}{4}bc^2d^2 \csc^{-1}(cx) + ibde \csc^{-1}(cx)^2 - \frac{d^2 (a + b \csc^{-1}(cx))}{2x^2} + \dots$$

**Mathematica [A]** time = 0.640438, size = 187, normalized size = 0.99

$$\frac{1}{4} \left( 4ibde \left( \csc^{-1}(cx)^2 + \text{PolyLog} \left( 2, e^{2i \csc^{-1}(cx)} \right) \right) - \frac{2ad^2}{x^2} + 8ade \log(x) + 2ae^2x^2 - \frac{bcd^2 \sqrt{1 - \frac{1}{c^2x^2}} \left( \frac{c^2x^2 \tanh^{-1}(\sqrt{1-c^2x^2})}{\sqrt{1-c^2x^2}} + \dots \right)}{x} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]))/x^3,x]

[Out] ((-2\*a\*d^2)/x^2 + 2\*a\*e^2\*x^2 - (2\*b\*d^2\*ArcCsc[c\*x])/x^2 + (2\*b\*e^2\*x\*(Sqrt[1 - 1/(c^2\*x^2)] + c\*x\*ArcCsc[c\*x]))/c - (b\*c\*d^2\*Sqrt[1 - 1/(c^2\*x^2)]\*(1 + (c^2\*x^2\*ArcTanh[Sqrt[1 - c^2\*x^2]])/Sqrt[1 - c^2\*x^2]))/x - 8\*b\*d\*e\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] + 8\*a\*d\*e\*Log[x] + (4\*I)\*b\*d\*e\*(ArcCsc[c\*x]^2 + PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])]))/4

**Maple [A]** time = 0.7, size = 276, normalized size = 1.5

$$\frac{ax^2e^2}{2} - \frac{ad^2}{2x^2} + 2aed \ln(cx) + ibde (\operatorname{arccsc}(cx))^2 - \frac{cbd^2}{4x} \sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{bc^2d^2 \operatorname{arccsc}(cx)}{4} - \frac{b \operatorname{arccsc}(cx) d^2}{2x^2} + \frac{b \operatorname{arccsc}(cx) d^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^3,x)

[Out] 1/2\*a\*x^2\*e^2-1/2\*a\*d^2/x^2+2\*a\*e\*d\*ln(c\*x)+I\*b\*d\*e\*arccsc(c\*x)^2-1/4\*c\*b\*d^2/x\*((c^2\*x^2-1)/c^2/x^2)^(1/2)+1/4\*b\*c^2\*d^2\*arccsc(c\*x)-1/2\*b\*arccsc(c\*x)\*d^2/x^2+1/2\*b\*arccsc(c\*x)\*x^2\*e^2+1/2/c\*b\*((c^2\*x^2-1)/c^2/x^2)^(1/2)\*x\*e^2-1/2\*I/c^2\*b\*e^2-2\*b\*e\*d\*arccsc(c\*x)\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2\*b\*e\*d\*arccsc(c\*x)\*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2\*I\*b\*e\*d\*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+2\*I\*b\*e\*d\*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} ae^2x^2 + \frac{1}{4} bd^2 \left( \frac{c^4x \sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2 \left( \frac{1}{c^2x^2}-1 \right)^{-1}} - c^3 \arctan \left( cx \sqrt{-\frac{1}{c^2x^2}+1} \right) \right) - \frac{2 \operatorname{arccsc}(cx)}{x^2} + 2ade \log(x) - \frac{ad^2}{2x^2} + \frac{4i bc^2de \log(-cx)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsc(c\*x))/x^3,x, algorithm="maxima")

[Out] 1/2\*a\*e^2\*x^2 + 1/4\*b\*d^2\*((c^4\*x\*sqrt(-1/(c^2\*x^2) + 1))/(c^2\*x^2\*(1/(c^2\*x^2) - 1) - 1) - c^3\*arctan(c\*x\*sqrt(-1/(c^2\*x^2) + 1)))/c - 2\*arccsc(c\*x)/x^2 + 2\*a\*d\*e\*log(x) - 1/2\*a\*d^2/x^2 + 1/4\*(4\*I\*b\*c^2\*d\*e\*log(-c\*x + 1)\*log(x) + 4\*I\*b\*c^2\*d\*e\*log(x)^2 + 4\*I\*b\*c^2\*d\*e\*dilog(c\*x) + 4\*I\*b\*c^2\*d\*e\*dilog(-c\*x) + 2\*(b\*c^2\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + I\*b\*c^2\*log(c)))\*e^2\*x^2 + I\*b\*e^2\*log(c\*x - 1) - I\*(b\*e^2\*(log(c\*x + 1)/c^2 + log(c\*x - 1)/c^2) + 16\*b\*d\*e\*integrate(1/2\*log(x)/(c^2\*x^3 - x), x))\*c^2 + 4\*c^2\*inte

```
grate(1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3
- x), x) + (-I*b*c^2*e^2*x^2 - 4*I*b*c^2*d*e*log(x))*log(c^2*x^2) + (4*I*b
*c^2*d*e*log(x) + I*b*e^2)*log(c*x + 1) + (2*I*b*c^2*e^2*x^2 + 8*(b*c^2*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^2*log(c))*d*e*log(x))/c^2
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2)\text{arccsc}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsc(c*x))/x^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \text{acsc}(cx))(d + ex^2)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2(b \text{arccsc}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)/x^3, x)
```

$$3.98 \quad \int \frac{x^2(a+b \csc^{-1}(cx))}{d+ex^2} dx$$

**Optimal.** Leaf size=565

$$\frac{ib\sqrt{-d}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{ib\sqrt{-d}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}$$

[Out] (x\*(a + b\*ArcCsc[c\*x]))/e + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(c\*e) - (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/(2\*e^(3/2)) + (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/(2\*e^(3/2)) - (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/(2\*e^(3/2)) + (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/(2\*e^(3/2)) - ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/e^(3/2) + ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/e^(3/2) - ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/e^(3/2) + ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/e^(3/2)

**Rubi [A]** time = 1.25703, antiderivative size = 565, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5241, 4733, 4627, 266, 63, 208, 4667, 4741, 4519, 2190, 2279, 2391}

$$\frac{ib\sqrt{-d}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{ib\sqrt{-d}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

[Out] (x\*(a + b\*ArcCsc[c\*x]))/e + (b\*ArcTanh[Sqrt[1 - 1/(c^2\*x^2)]])/(c\*e) - (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/(2\*e^(3/2)) + (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/(2\*e^(3/2)) - (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/(2\*e^(3/2)) + (Sqrt[-d]\*(a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/(2\*e^(3/2)) - ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/e^(3/2) + ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] - Sqrt[c^2\*d + e]))/e^(3/2) - ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/e^(3/2) + ((I/2)\*b\*Sqrt[-d]\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))]/(Sqrt[e] + Sqrt[c^2\*d + e]))/e^(3/2)

#### Rule 5241

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_.)]\*(b\_.))^n\_.\*(x\_.)^(m\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSin[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

### Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || IGtQ[n, 0])
```

### Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

### Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2] - I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x]
```

))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol]  
 :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
 )^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2  
 , -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2 (a + b \csc^{-1}(cx))}{d + ex^2} dx &= -\text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x^2 (e + dx^2)} dx, x, \frac{1}{x} \right) \\
 &= -\text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{ex^2} - \frac{d (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
 &= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e} + \frac{d \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
 &= \frac{x (a + b \csc^{-1}(cx))}{e} - \frac{b \text{Subst} \left( \int \frac{1}{x \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{ce} + \frac{d \text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{e} \\
 &= \frac{x (a + b \csc^{-1}(cx))}{e} + \frac{d \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{3/2}} + \frac{d \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \csc^{-1}(cx))}{e} + \frac{d \text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2e^{3/2}} + \frac{d \text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \csc^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} + \frac{d \text{Subst} \left( \int \frac{e^{ix} (a+bx)}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2 d + e}}{c} - i \sqrt{-d} e^{ix}} dx, x, \csc^{-1}(cx) \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \csc^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} - \frac{\sqrt{-d} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic \sqrt{-d} e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \csc^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} - \frac{\sqrt{-d} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic \sqrt{-d} e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \csc^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce} - \frac{\sqrt{-d} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic \sqrt{-d} e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^{3/2}}
 \end{aligned}$$

**Mathematica [B]** time = 1.69576, size = 1260, normalized size = 2.23

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

[Out] ((I/4)\*((-4\*I)\*a\*c\*Sqrt[e]\*x - (4\*I)\*b\*c\*Sqrt[e]\*x\*ArcCsc[c\*x] + (4\*I)\*a\*c\*Sqrt[d]\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]] + (8\*I)\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[(((-I)\*c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + 2\*ArcCsc[c\*x])/4])/Sqrt[c^2\*d + e]] - (8\*I)\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[((I\*c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + 2\*ArcCsc[c\*x])/4])/Sqrt[c^2\*d + e]] + b\*c\*Sqrt[d]\*Pi\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - 2\*b\*c\*Sqrt[d]\*ArcCsc[c\*x]\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - b\*c\*Sqrt[d]\*Pi\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + 2\*b\*c\*Sqrt[d]\*ArcCsc[c\*x]\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - b\*c\*Sqrt[d]\*Pi\*Log[1 - (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + 2\*b\*c\*Sqrt[d]\*ArcCsc[c\*x]\*Log[1 - (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 - (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + b\*c\*Sqrt[d]\*Pi\*Log[1 + (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - 2\*b\*c\*Sqrt[d]\*ArcCsc[c\*x]\*Log[1 + (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + b\*c\*Sqrt[d]\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] - b\*c\*Sqrt[d]\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] - (4\*I)\*b\*Sqrt[e]\*Log[Cos[ArcCsc[c\*x]/2]] + (4\*I)\*b\*Sqrt[e]\*Log[Sin[ArcCsc[c\*x]/2]] + (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, -((Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x])))] + (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))]))/(c\*e^(3/2))

**Maple [C]** time = 1.534, size = 407, normalized size = 0.7

$$\frac{ax}{e} - \frac{ad}{e} \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{bx \operatorname{arccsc}(cx)}{e} + \frac{bcd}{8e^2} \sum_{_R1=\operatorname{RootOf}(c^2d_Z^4+(-2c^2d-4e)_Z^2+c^2d)} \frac{-R1^2c^2d+4_R1^2e-c^2d}{-R1(-R1^2c^2d-c^2d-2e)} \left( i \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d), x)

[Out] a\*x/e-a\*d/e/(d\*e)^(1/2)\*arctan(e\*x/(d\*e)^(1/2))+b\*arccsc(c\*x)/e\*x+1/8\*c\*b/e^2\*d\*sum((\_R1^2\*c^2\*d+4\*\_R1^2\*e-c^2\*d)/\_R1/(\_R1^2\*c^2\*d-c^2\*d-2\*e)\*(I\*arccsc(c\*x)\*ln((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)+dilog((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)),\_R1=RootOf(c^2\*d\*\_Z^4+(-2\*c^2\*d-4\*e)\*\_Z^2+c^2\*d))+1/c\*b/e\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/c\*b/e\*ln(I/c/x+(1-1/c^2/x^2)^(1/2))-1/8\*c\*b/e^2\*d\*sum((\_R1^2\*c^2\*d-c^2\*d-4\*e)/\_R1/(\_R1^2\*c^2\*d-c^2\*d-2\*e)\*(I\*arccsc(c\*x)\*ln((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)+dilog((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)),\_R1=RootOf(c^2\*d\*\_Z^4+(-2\*c^2\*d-4\*e)\*\_Z^2+c^2\*d))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arccsc}(cx) + ax^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(e*x^2 + d), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{acsc}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d), x)
```

$$3.99 \quad \int \frac{x(a+b \csc^{-1}(cx))}{d+ex^2} dx$$

**Optimal.** Leaf size=507

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}$$

[Out] ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e) + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e) - ((a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/e - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e + ((I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/e

**Rubi [A]** time = 1.13856, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {5241, 4733, 4625, 3717, 2190, 2279, 2391, 4741, 4519}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2),x]

[Out] ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e) + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e) - ((a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/e - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e + ((I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/e

#### Rule 5241

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSin[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rule 4733

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.)^(n\_.)\*((f\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*ArcSin[c\*x])^n, (

$f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$

#### Rule 4625

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n/(x), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[a + b*x]^n/\text{Tan}[x], x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{IGtQ}[n, 0]$

#### Rule 3717

$\text{Int}[(c + (d*x)^m*\tan[e + \text{Pi}*k + (f*x)], x\_Symbol] \rightarrow \text{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] - \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*I*k*\text{Pi})*\text{E}^{(2*I*(e + f*x))}}/(1 + \text{E}^{(2*I*k*\text{Pi})*\text{E}^{(2*I*(e + f*x))}}), x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

#### Rule 2190

$\text{Int}[(F^{(g*(e + f*x))})^n*(c + (d*x)^m)/((a + b*(F^{(g*(e + f*x))})^n), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

#### Rule 2279

$\text{Int}[\text{Log}[a + (b*(F^{(e*(c + d*x))})^n)], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c + (d + (e*x)^n)]/(x), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

#### Rule 4741

$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n/((d + (e*x)), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n*\text{Cos}[x]/(c*d + e*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[n, 0]$

#### Rule 4519

$\text{Int}[(\text{Cos}[c + (d*x)]*(e + f*x)^m)/((a + b*\text{Sin}[c + (d*x)]), x\_Symbol] \rightarrow -\text{Simp}[(I*(e + f*x)^{m+1})/(b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(I*(c + d*x))}/(a - \text{Rt}[a^2 - b^2, 2] - I*b*\text{E}^{(I*(c + d*x))}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(I*(c + d*x))}/(a + \text{Rt}[a^2 - b^2, 2] - I*b*\text{E}^{(I*(c + d*x))}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \& \text{PosQ}[a^2 - b^2]$

#### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \csc^{-1}(cx))}{d + ex^2} dx &= -\text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{x(e + dx^2)} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{ex} - \frac{dx(a + b \sin^{-1}\left(\frac{x}{c}\right))}{e(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x} \right)}{e} + \frac{d \text{Subst} \left( \int \frac{x(a + b \sin^{-1}\left(\frac{x}{c}\right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
&= -\frac{\text{Subst} \left( \int (a + bx) \cot(x) dx, x, \csc^{-1}(cx) \right)}{e} + \frac{d \text{Subst} \left( \int \left( -\frac{\sqrt{-d}(a + b \sin^{-1}\left(\frac{x}{c}\right))}{2d(\sqrt{e} - \sqrt{-dx})} + \frac{\sqrt{-d}(a + b \sin^{-1}\left(\frac{x}{c}\right))}{2d(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{i(a + b \csc^{-1}(cx))^2}{2be} + \frac{(2i) \text{Subst} \left( \int \frac{e^{2ix}(a + bx)}{1 - e^{2ix}} dx, x, \csc^{-1}(cx) \right)}{e} - \frac{\sqrt{-d} \text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{i(a + b \csc^{-1}(cx))^2}{2be} - \frac{(a + b \csc^{-1}(cx)) \log(1 - e^{2i \csc^{-1}(cx)})}{e} + \frac{b \text{Subst} \left( \int \log(1 - e^{2ix}) dx, x, \csc^{-1}(cx) \right)}{e} \\
&= -\frac{(a + b \csc^{-1}(cx)) \log(1 - e^{2i \csc^{-1}(cx)})}{e} - \frac{(ib) \text{Subst} \left( \int \frac{\log(1-x)}{x} dx, x, e^{2i \csc^{-1}(cx)} \right)}{2e} + \frac{\sqrt{-d} \text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{(a + b \csc^{-1}(cx)) \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \csc^{-1}(cx)) \log\left(1 + \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \csc^{-1}(cx)) \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} \\
&= \frac{(a + b \csc^{-1}(cx)) \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \csc^{-1}(cx)) \log\left(1 + \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \csc^{-1}(cx)) \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} \\
&= \frac{(a + b \csc^{-1}(cx)) \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \csc^{-1}(cx)) \log\left(1 + \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \csc^{-1}(cx)) \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e}
\end{aligned}$$

**Mathematica [B]** time = 0.366883, size = 1123, normalized size = 2.21

$$8ib \csc^{-1}(cx)^2 + 4b \log\left(\frac{e^{-i \csc^{-1}(cx)}(\sqrt{e} - \sqrt{dc^2 + e})}{c\sqrt{d}} + 1\right) \csc^{-1}(cx) + 4b \log\left(\frac{e^{-i \csc^{-1}(cx)}(\sqrt{dc^2 + e} - \sqrt{e})}{c\sqrt{d}} + 1\right) \csc^{-1}(cx) + 4b \log\left(1 - \frac{(\sqrt{e} - \sqrt{dc^2 + e})}{c\sqrt{d}}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

[Out] (I\*b\*Pi^2 - (4\*I)\*b\*Pi\*ArcCsc[c\*x] + (8\*I)\*b\*ArcCsc[c\*x]^2 - (16\*I)\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[(((I)\*c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + 2\*ArcCsc[c\*x])/4])/Sqrt[c^2\*d + e]] - (16\*I)\*b\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*ArcTan[(((I)\*c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + 2\*ArcCsc[c\*x])/4])/Sqrt[c^2\*d + e]] - 2\*b\*Pi\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d])\*E^(I\*ArcCsc[c\*x])] + 4\*b\*ArcCsc[c\*x]\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d])\*E^(I\*ArcCsc[c\*x])] - 8\*b\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])]]/Sqrt[2]]\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d])\*E^(I\*ArcCsc[c\*x])] - 2\*b\*Pi\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d])\*E^(I\*ArcCsc[c\*x])] + 4\*b\*ArcCsc[c\*x]\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d])\*E^(I\*ArcCsc[c\*x])])

$$\begin{aligned}
& + e)/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] - 8*b*\text{ArcSin}[\sqrt{1 + (I\sqrt{e})/(c\sqrt{d})}]/\sqrt{2}] \\
& * \text{Log}[1 + (-\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] - 2*b*\text{Pi}*\text{Log}[1 - (\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] \\
& + 4*b*\text{ArcCsc}[c*x]*\text{Log}[1 - (\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] + 8*b*\text{ArcSin}[\sqrt{1 + (I\sqrt{e})/(c\sqrt{d})}]/\sqrt{2}] \\
& * \text{Log}[1 - (\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] - 2*b*\text{Pi}*\text{Log}[1 + (\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] \\
& + 4*b*\text{ArcCsc}[c*x]*\text{Log}[1 + (\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] + 8*b*\text{ArcSin}[\sqrt{1 - (I\sqrt{e})/(c\sqrt{d})}]/\sqrt{2}] \\
& * \text{Log}[1 + (\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] - 8*b*\text{ArcCsc}[c*x]*\text{Log}[1 - E^{((2*I)\text{ArcCsc}[c*x])}] \\
& + 2*b*\text{Pi}*\text{Log}[\sqrt{e} - (I\sqrt{d})/x] + 2*b*\text{Pi}*\text{Log}[\sqrt{e} + (I\sqrt{d})/x] + 4*a*\text{Log}[d + e*x^2] + (4*I)*b*\text{PolyLog}[2, (\sqrt{e} - \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] \\
& + (4*I)*b*\text{PolyLog}[2, (-\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] + (4*I)*b*\text{PolyLog}[2, -((\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] \\
& + (4*I)*b*\text{PolyLog}[2, (\sqrt{e} + \sqrt{c^2*d + e})/(c\sqrt{d}E^{(I\text{ArcCsc}[c*x])})] + (4*I)*b*\text{PolyLog}[2, E^{((2*I)\text{ArcCsc}[c*x])}]/(8*e)
\end{aligned}$$

**Maple [C]** time = 0.422, size = 401, normalized size = 0.8

$$\frac{a \ln(c^2 e x^2 + c^2 d)}{2 e} - \frac{i b}{e} \sum_{\substack{\_R1=\text{RootOf}(c^2 d\_Z^4+(-2 c^2 d-4 e)\_Z^2+c^2 d)}} \frac{-R1^2 c^2 d - c^2 d - 4 e}{-R1^2 c^2 d - c^2 d - 2 e} \left( \text{arccsc}(c x) \ln\left(\frac{1}{-R1} \left(-R1 - \frac{i}{c x} - \sqrt{\dots}\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d),x)

[Out] 1/2\*a/e\*ln(c^2\*e\*x^2+c^2\*d)-1/4\*I\*b/e\*sum((\\_R1^2\*c^2\*d-c^2\*d-4\*e)/(\\_R1^2\*c^2\*d-c^2\*d-2\*e)\*(I\*arccsc(c\*x)\*ln((\\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\\_R1)+dilog((\\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\\_R1)),\\_R1=RootOf(c^2\*d\*\\_Z^4+(-2\*c^2\*d-4\*e)\*\\_Z^2+c^2\*d))-I\*b/e\*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-b/e\*arccsc(c\*x)\*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I\*b/e\*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4\*I\*c^2\*b\*d/e\*sum((\\_R1^2-1)/(\\_R1^2\*c^2\*d-c^2\*d-2\*e)\*(I\*arccsc(c\*x)\*ln((\\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\\_R1)+dilog((\\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\\_R1)),\\_R1=RootOf(c^2\*d\*\\_Z^4+(-2\*c^2\*d-4\*e)\*\\_Z^2+c^2\*d))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b \int \frac{x \arctan\left(1, \sqrt{c x + 1} \sqrt{c x - 1}\right)}{e x^2 + d} dx + \frac{a \log\left(e x^2 + d\right)}{2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d),x, algorithm="maxima")

[Out] b\*integrate(x\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e\*x^2 + d), x) + 1/2\*a\*log(e\*x^2 + d)/e

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b x \text{arccsc}(c x) + a x}{e x^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)/(e*x^2 + d), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(a + b \operatorname{arccsc}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d), x)
```

### 3.100 $\int \frac{a+b \csc^{-1}(cx)}{d+ex^2} dx$

**Optimal.** Leaf size=529

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] -((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

**Rubi [A]** time = 0.870293, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {5231, 4667, 4741, 4519, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2), x]
```

```
[Out] -((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

**Rule 5231**

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^(2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

**Rule 4667**

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x]
```

```
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

#### Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_/((d_) + (e_.)*(x_)), x_Symbol]
:> Subst[Int[((a + b*x)^n*cos[x])/(c*d + e*sin[x]), x], x, ArcSin[c*x]] /;
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*(e_.) + (f_.)*(x_))^(m_)]/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^n_)*((c_.) + (d_.)*(x_))^(m_)]/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^n_), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^n_], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^n)]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{d + ex^2} dx &= -\text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} - \frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} \\
&= -\frac{\text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{e}} - \frac{\text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{e}} \\
&= -\frac{\text{Subst} \left( \int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2d+e}}{c} - i\sqrt{-d}e^{ix}} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{e}} - \frac{\text{Subst} \left( \int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c} + \frac{\sqrt{c^2d+e}}{c} - i\sqrt{-d}e^{ix}} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{e}} \\
&= -\frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a + b \csc^{-1}(cx)) \log \left( 1 + \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} \\
&= -\frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a + b \csc^{-1}(cx)) \log \left( 1 + \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} \\
&= -\frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a + b \csc^{-1}(cx)) \log \left( 1 + \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}$$

**Mathematica [B]** time = 0.380951, size = 1068, normalized size = 2.02

$$i \left( 4ia \tan^{-1} \left( \frac{\sqrt{ex}}{\sqrt{d}} \right) + 8ib \sin^{-1} \left( \frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}} \right) \tan^{-1} \left( \frac{(\sqrt{e} - ic\sqrt{d}) \cot \left( \frac{1}{4} (2 \csc^{-1}(cx) + \pi) \right)}{\sqrt{dc^2 + e}} \right) - 8ib \sin^{-1} \left( \frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}} \right) \tan^{-1} \left( \frac{(i\sqrt{dc} + \sqrt{e}) \cot \left( \frac{1}{4} (2 \csc^{-1}(cx) + \pi) \right)}{\sqrt{dc^2 + e}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x^2), x]

[Out]  $((-I/4)*((4*I)*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])$

$$+ 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - b*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (2*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, -(Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (2*I)*b*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/(Sqrt[d]*Sqrt[e])$$

**Maple [C]** time = 0.94, size = 272, normalized size = 0.5

$$a \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{cb}{2} \sum_{_R1=\text{RootOf}(c^2d_Z^4+(-2c^2d-4e)_Z^2+c^2d)} \frac{1}{_R1(-_R1^2c^2d-c^2d-2e)} \left( i \arccsc(cx) \ln\left(\frac{1}{_R1}\left(-_R1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/(e\*x^2+d),x)

[Out] a/(d\*e)^(1/2)\*arctan(e\*x/(d\*e)^(1/2))-1/2\*c\*b\*sum(1/\_R1/(\_R1^2\*c^2\*d-c^2\*d-2\*e)\*(I\*arccsc(c\*x)\*ln((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)+dilog((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)),\_R1=RootOf(c^2\*d\*\_Z^4+(-2\*c^2\*d-4\*e)\*\_Z^2+c^2\*d))-1/2\*c\*b\*sum(\_R1/(\_R1^2\*c^2\*d-c^2\*d-2\*e)\*(I\*arccsc(c\*x)\*ln((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)+dilog((\_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/\_R1)),\_R1=RootOf(c^2\*d\*\_Z^4+(-2\*c^2\*d-4\*e)\*\_Z^2+c^2\*d))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arccsc(cx) + a}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/(e\*x^2 + d), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x\*\*2+d), x)

[Out] Integral((a + b\*acsc(c\*x))/(d + e\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d), x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(e\*x^2 + d), x)

### 3.101 $\int \frac{a+b \csc^{-1}(cx)}{x(d+ex^2)} dx$

**Optimal.** Leaf size=479

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}$$

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcC
sc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*Pol
yLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/d
+ ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/d + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d
```

**Rubi [A]** time = 0.900407, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5241, 4733, 4741, 4519, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcC
sc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*Pol
yLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/d
+ ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/d + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d
```

#### Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

#### Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
```

$e, 0]$  && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 4741

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^n\_./((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> Subst[Int[(a + b\*x)^n\*cos[x]/(c\*d + e\*sin[x]), x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 4519

Int[(Cos[(c\_.) + (d\_.)\*(x\_)]\*(e\_.) + (f\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> -Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (Int[(e + f\*x)^m\*E^(I\*(c + d\*x))/(a - Rt[a^2 - b^2, 2] - I\*b\*E^(I\*(c + d\*x))), x] + Int[(e + f\*x)^m\*E^(I\*(c + d\*x))/(a + Rt[a^2 - b^2, 2] - I\*b\*E^(I\*(c + d\*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^n\_.], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^n\_.)]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x(d + ex^2)} dx &= -\text{Subst} \left( \int \frac{x(a + b \sin^{-1}(\frac{x}{c}))}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{\sqrt{-d}(a + b \sin^{-1}(\frac{x}{c}))}{2d(\sqrt{e} - \sqrt{-dx})} + \frac{\sqrt{-d}(a + b \sin^{-1}(\frac{x}{c}))}{2d(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} + \frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} \\
&= -\frac{\text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{-d}} + \frac{\text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{-d}} \\
&= \frac{i(a + b \csc^{-1}(cx))^2}{2bd} + \frac{\text{Subst} \left( \int \frac{e^{ix(a+bx)}}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2d+e}}{c} - i\sqrt{-d}e^{ix}} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{-d}} + \frac{\text{Subst} \left( \int \frac{e^{ix(a+bx)}}{\frac{\sqrt{e}}{c} + \frac{\sqrt{c^2d+e}}{c} - i\sqrt{-d}e^{ix}} dx, x, \csc^{-1}(cx) \right)}{2\sqrt{-d}} \\
&= \frac{i(a + b \csc^{-1}(cx))^2}{2bd} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 + \frac{ic\sqrt{-d}}{\sqrt{e}} \right)}{2d} \\
&= \frac{i(a + b \csc^{-1}(cx))^2}{2bd} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 + \frac{ic\sqrt{-d}}{\sqrt{e}} \right)}{2d} \\
&= \frac{i(a + b \csc^{-1}(cx))^2}{2bd} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \csc^{-1}(cx)) \log \left( 1 + \frac{ic\sqrt{-d}}{\sqrt{e}} \right)}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.856946, size = 407, normalized size = 0.85

$$ib \left( \text{PolyLog} \left( 2, \frac{(-2\sqrt{e(c^2d+e)+c^2d+2e})e^{2i \csc^{-1}(cx)}}{c^2d} \right) + \text{PolyLog} \left( 2, \frac{(2(\sqrt{e(c^2d+e)+e})+c^2d)e^{2i \csc^{-1}(cx)}}{c^2d} \right) + 4 \sin^{-1} \left( \sqrt{-\frac{e}{c^2d}} \right) \tan^{-1} \left( \frac{\sqrt{e(c^2d+e)}}{cex\sqrt{1}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)), x]

[Out] (4\*a\*Log[x] - 2\*a\*Log[d + e\*x^2] + I\*b\*(2\*ArcCsc[c\*x]^2 + 4\*ArcSin[Sqrt[-(e/(c^2\*d))]]\*ArcTan[Sqrt[e\*(c^2\*d + e)]/(c\*e\*Sqrt[1 - 1/(c^2\*x^2)]]\*x] + (2\*I)\*ArcCsc[c\*x]\*Log[1 - ((c^2\*d + 2\*e - 2\*Sqrt[e\*(c^2\*d + e)])\*E^((2\*I)\*ArcCsc[c\*x]))/(c^2\*d)] - (2\*I)\*ArcSin[Sqrt[-(e/(c^2\*d))]]\*Log[1 - ((c^2\*d + 2\*e - 2\*Sqrt[e\*(c^2\*d + e)])\*E^((2\*I)\*ArcCsc[c\*x]))/(c^2\*d)] + (2\*I)\*ArcCsc[c\*x]\*Log[1 - ((c^2\*d + 2\*(e + Sqrt[e\*(c^2\*d + e)]))\*E^((2\*I)\*ArcCsc[c\*x]))/(c^2\*d)] + (2\*I)\*ArcSin[Sqrt[-(e/(c^2\*d))]]\*Log[1 - ((c^2\*d + 2\*(e + Sqrt[e\*(c^2\*d + e)]))\*E^((2\*I)\*ArcCsc[c\*x]))/(c^2\*d)] + PolyLog[2, ((c^2\*d + 2\*e - 2\*Sqrt[e\*(c^2\*d + e)])\*E^((2\*I)\*ArcCsc[c\*x]))/(c^2\*d)] + PolyLog[2, ((c^2\*d + 2\*(e + Sqrt[e\*(c^2\*d + e)]))\*E^((2\*I)\*ArcCsc[c\*x]))/(c^2\*d)))/(4\*d)

**Maple [C]** time = 0.57, size = 2875, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\arccsc(cx))/x/(e*x^2+d), x)$

[Out] 
$$\begin{aligned} & -2*b/c^4*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)*e/d^3*(e*(c^2*d+e))^{1/2}+2*b/c^4*e^3*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/d^3/(c^2*d+e)-I*b/c^4*e^3*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^3/(c^2*d+e)-1/8*I*b*c^2*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/e/(c^2*d+e)*(e*(c^2*d+e))^{1/2}-2*I*b/c^2*e^2*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^2/(c^2*d+e)-4*I*b/c^2*\arccsc(cx)^2/(c^2*d+e)/d^2*e^2-2*I*b/c^4*e^3*\arccsc(cx)^2/d^3/(c^2*d+e)+2*I*b/c^4*\arccsc(cx)^2*e/d^3*(e*(c^2*d+e))^{1/2}+3*b/c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/(c^2*d+e)/d^2*(e*(c^2*d+e))^{1/2}*e-3/2*I*b/c^2*e*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^2/(c^2*d+e)*(e*(c^2*d+e))^{1/2}-2*I*b/c^4*e^2*\arccsc(cx)^2/d^3/(c^2*d+e)*(e*(c^2*d+e))^{1/2}+1/4*I*b*(e*(c^2*d+e))^{1/2}/d/(c^2*d+e)*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))-I*b*(e*(c^2*d+e))^{1/2}/d/(c^2*d+e)*\arccsc(cx)^2+3/2*b*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/(c^2*d+e)/d*(e*(c^2*d+e))^{1/2}+5/2*b*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/(c^2*d+e)/d*e+I*b/c^2*\arccsc(cx)^2/d^2*(e*(c^2*d+e))^{1/2}-b/c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/d^2*(e*(c^2*d+e))^{1/2}-2*b/c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/d^2*e+a/d*\ln(cx)-I*b/c^4*e^2*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^3/(c^2*d+e)*(e*(c^2*d+e))^{1/2}-3*I*b/c^2*\arccsc(cx)^2/(c^2*d+e)/d^2*(e*(c^2*d+e))^{1/2}*e+2*b/c^4*e^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/d^3/(c^2*d+e)*(e*(c^2*d+e))^{1/2}+1/8*I*b*c^2*(e*(c^2*d+e))^{1/2}/e/(c^2*d+e)*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))-1/4*b*c^2*(e*(c^2*d+e))^{1/2}/e/(c^2*d+e)*\arccsc(cx)*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))-1/2*I*b*c^2*\arccsc(cx)^2/(c^2*d+e)-1/4*I*b*c^2*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/(c^2*d+e)+I*b/c^4*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*e/d^3*(e*(c^2*d+e))^{1/2}+1/4*b*c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/e/(c^2*d+e)*(e*(c^2*d+e))^{1/2}+4*b/c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/(c^2*d+e)/d^2*e^2+I*b/c^2*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^2*e-2*b/c^4*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)*e^2/d^3+I*b/c^4*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*e^2/d^3-1/2*b*(e*(c^2*d+e))^{1/2}/d/(c^2*d+e)*\arccsc(cx)*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))+1/2*I*b/c^2*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^2*(e*(c^2*d+e))^{1/2}-3/4*I*b*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d/(c^2*d+e)*(e*(c^2*d+e))^{1/2}-5/2*I*b*\arccsc(cx)^2*e/d/(c^2*d+e)-5/4*I*b*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*e/d/(c^2*d+e)+2*I*b/c^2*\arccsc(cx)^2/d^2*e+2*I*b/c^4*\arccsc(cx)^2*e^2/d^3+I*b/d*\arccsc(cx)^2-1/2*b*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*\arccsc(cx)/d+1/4*I*b*\text{polylog}(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d+1/2*I*b/d*\text{sum}((\_R1^2*c^2*d-2*c^2*d-4*e)/(\_R1^2*c^2*d-c^2*d-2*e)*(I*\arccsc(cx)*\ln(\_R1-I/c/x-(1-1/c^2/x^2)^{1/2}))/\_R1)+\text{dilog}((\_R1-I/c/x-(1-1/c^2/x^2)^{1/2}))/$$

`_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/2*a/d*ln(c^2*e*x^2+c^2*d)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{\log(ex^2+d)}{d}-\frac{2\log(x)}{d}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)}{ex^3+dx}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="maxima")`

[Out] `-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^3 + d*x), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arccsc(cx)+a}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="fricas")`

[Out] `integral((b*arccsc(c*x) + a)/(e*x^3 + d*x), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\arccsc(cx)}{x(d+ex^2)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x**2+d),x)`

[Out] `Integral((a + b*arccsc(c*x))/(x*(d + e*x**2)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{b\arccsc(cx)+a}{(ex^2+d)x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/((e*x^2 + d)*x), x)`



### 3.102 $\int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex^2)} dx$

**Optimal.** Leaf size=572

$$\frac{ib\sqrt{e}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}$$

[Out]  $-\left(\frac{b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]}{d}\right) - \frac{a}{d*x} - \frac{b*\text{ArcCsc}[c*x]}{d*x} - \left(\frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} + \frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} - \frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} + \frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} - \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}} + \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}} - \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}} + \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}}$

**Rubi [A]** time = 1.09982, antiderivative size = 572, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5241, 4733, 4619, 261, 4667, 4741, 4519, 2190, 2279, 2391}

$$\frac{ib\sqrt{e}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\text{PolyLog}\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\text{PolyLog}\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCsc}[c*x])/(x^2*(d + e*x^2)), x]$

[Out]  $-\left(\frac{b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]}{d}\right) - \frac{a}{d*x} - \frac{b*\text{ArcCsc}[c*x]}{d*x} - \left(\frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} + \frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} - \frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} + \frac{\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])*\text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{2*(-d)^{(3/2)}} - \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}} + \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}} - \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}} + \left(\frac{I}{2}\right)*\frac{b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]}{(-d)^{(3/2)}}$

**Rule 5241**

$\text{Int}[(a + \text{ArcCsc}[c*x])*(b*x)^n*(d + e*x^2)^m, x] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcSin}[x/c])^n/x^{m+2*(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[p]$

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || IGtQ[n, 0])
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2] - I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^2(d + ex^2)} dx &= -\text{Subst} \left( \int \frac{x^2 \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{d} - \frac{e \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \text{Subst} \left( \int \sin^{-1} \left( \frac{x}{c} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \csc^{-1}(cx)}{dx} + \frac{b \text{Subst} \left( \int \frac{x}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cd} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2d} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2d} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \csc^{-1}(cx)}{dx} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2d} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2d} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \csc^{-1}(cx)}{dx} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c} - \sqrt{-d}e^{-i\sqrt{-d}e^{ix}}} dx, x, \csc^{-1}(cx) \right)}{2d} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c} + \sqrt{-d}e^{i\sqrt{-d}e^{ix}}} dx, x, \csc^{-1}(cx) \right)}{2d} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \csc^{-1}(cx)}{dx} - \frac{\sqrt{e} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2(-d)^{3/2}} + \frac{\sqrt{e} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d + e}} \right)}{2(-d)^{3/2}} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \csc^{-1}(cx)}{dx} - \frac{\sqrt{e} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2(-d)^{3/2}} + \frac{\sqrt{e} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d + e}} \right)}{2(-d)^{3/2}} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \csc^{-1}(cx)}{dx} - \frac{\sqrt{e} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}} \right)}{2(-d)^{3/2}} + \frac{\sqrt{e} (a + b \csc^{-1}(cx)) \log \left( 1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d + e}} \right)}{2(-d)^{3/2}}
\end{aligned}$$

**Mathematica [B]** time = 1.61997, size = 1241, normalized size = 2.17

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x^2)),x]

[Out]  $-(a/(d*x)) - (a*\text{Sqrt}[e]*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/d^{(3/2)} + b*(-((c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x + \text{ArcCsc}[c*x])/(d*x)) + (\text{Sqrt}[e]*(\text{Pi}^2 - 4*\text{Pi}*\text{ArcCsc}[c*x] + 8*\text{ArcCsc}[c*x]^2 - 32*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTan}[( (-I)*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Cot}[(\text{Pi} + 2*\text{ArcCsc}[c*x])/4]]/\text{Sqrt}[c^2*d + e]) + (4*I)*\text{Pi}*\text{Log}[1 + (\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])/(c*\text{Sqrt}[d]*E^{(I*\text{ArcCsc}[c*x])})] - (8*I)*\text{ArcCsc}[c*x]*\text{Log}[1 + (\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])/(c*\text{Sqrt}[d]*E^{(I*\text{ArcCsc}[c*x])})] + (16*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]*\text{Log}[1 + (\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])/(c*\text{Sqrt}[d]*E^{(I*\text{ArcCsc}[c*x])})] + (4*I)*\text{Pi}*\text{Log}[1 + (\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])/(c*\text{Sqrt}[d]*E^{(I*\text{ArcCsc}[c*x])})])$

$$\begin{aligned}
& - (8I) \operatorname{ArcCsc}[c*x] \operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] - (16I) \operatorname{ArcSin}\left[\frac{\sqrt{1 - (I*\sqrt{e})/(c*\sqrt{d})}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + (8I) \operatorname{ArcCsc}[c*x] \operatorname{Log}\left[1 - E^{((2I)*\operatorname{ArcCsc}[c*x])}\right] - (4I) \operatorname{Pi} \operatorname{Log}\left[\frac{\sqrt{e} + (I*\sqrt{d})}{x}\right] + 8 \operatorname{PolyLog}\left[2, \frac{-\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + 8 \operatorname{PolyLog}\left[2, -\frac{(\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + 4 \operatorname{PolyLog}\left[2, E^{((2I)*\operatorname{ArcCsc}[c*x])}\right] / (16*d^{(3/2)}) - \frac{\sqrt{e}*(\operatorname{Pi}^2 - 4*\operatorname{Pi}*\operatorname{ArcCsc}[c*x] + 8*\operatorname{ArcCsc}[c*x]^2 - 32*\operatorname{ArcSin}\left[\frac{\sqrt{1 + (I*\sqrt{e})/(c*\sqrt{d})}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(I*c*\sqrt{d} + \sqrt{e})*\operatorname{Cot}\left[\frac{\operatorname{Pi} + 2*\operatorname{ArcCsc}[c*x]}{4}\right]}{\sqrt{c^2*d + e}}\right] + (4I)*\operatorname{Pi} \operatorname{Log}\left[1 + \frac{(-\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] - (8I) \operatorname{ArcCsc}[c*x] \operatorname{Log}\left[1 + \frac{(-\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + (16I) \operatorname{ArcSin}\left[\frac{\sqrt{1 + (I*\sqrt{e})/(c*\sqrt{d})}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{(-\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + (4I) \operatorname{Pi} \operatorname{Log}\left[1 - \frac{(\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] - (8I) \operatorname{ArcCsc}[c*x] \operatorname{Log}\left[1 - \frac{(\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] - (16I) \operatorname{ArcSin}\left[\frac{\sqrt{1 + (I*\sqrt{e})/(c*\sqrt{d})}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{(\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + (8I) \operatorname{ArcCsc}[c*x] \operatorname{Log}\left[1 - E^{((2I)*\operatorname{ArcCsc}[c*x])}\right] - (4I) \operatorname{Pi} \operatorname{Log}\left[\frac{\sqrt{e} - (I*\sqrt{d})}{x}\right] + 8 \operatorname{PolyLog}\left[2, \frac{(\sqrt{e} - \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + 8 \operatorname{PolyLog}\left[2, \frac{(\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}} E^{(I*\operatorname{ArcCsc}[c*x])}\right] + 4 \operatorname{PolyLog}\left[2, E^{((2I)*\operatorname{ArcCsc}[c*x])}\right] / (16*d^{(3/2)})
\end{aligned}$$

**Maple [C]** time = 1.537, size = 332, normalized size = 0.6

$$-\frac{ae}{d} \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{a}{dx} - \frac{cb}{d} \sqrt{\frac{c^2x^2 - 1}{c^2x^2}} - \frac{\operatorname{arccsc}(cx)}{dx} + \frac{cbe}{2d} \sum_{_R1=\operatorname{RootOf}(c^2d_Z^4+(-2c^2d-4e)_Z^2+c^2d)} \frac{1}{_R1} \frac{1}{(-R1^2c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d),x)

[Out]  $-a*e/d/(d*e)^{(1/2)}*\arctan(e*x/(d*e)^{(1/2)})-a/d/x-c*b/d*((c^2*x^2-1)/c^2/x^2)^{(1/2)}-b*\operatorname{arccsc}(c*x)/d/x+1/2*c*b*e/d*\sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\operatorname{arccsc}(c*x)*\ln((\_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((\_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*c*b*e/d*\sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\operatorname{arccsc}(c*x)*\ln((\_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((\_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),\_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/(e\*x^4 + d\*x^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2 (d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*2/(e\*x\*\*2+d),x)

[Out] Integral((a + b\*acsc(c\*x))/(x\*\*2\*(d + e\*x\*\*2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/((e\*x^2 + d)\*x^2), x)

$$3.103 \quad \int \frac{x^5 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx$$

**Optimal.** Leaf size=628

$$\frac{ibdPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibdPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibdPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3} + \frac{ibdPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}$$

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsc[c*x]))/(2*e^2*(e +
d/x^2)) + (x^2*(a + b*ArcCsc[c*x]))/(2*e^2) - (b*d*ArcTan[Sqrt[c^2*d + e]/
(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)]/(2*e^(5/2)*Sqrt[c^2*d + e]) - (d*(a +
b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 - (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a +
b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^
2*d + e])])/e^3 + (2*d*(a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/
e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[
2, E^((2*I)*ArcCsc[c*x])])/e^3
```

**Rubi [A]** time = 1.32061, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {5241, 4733, 4627, 264, 4625, 3717, 2190, 2279, 2391, 4729, 377, 205, 4741, 4519}

$$\frac{ibdPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibdPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibdPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3} + \frac{ibdPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\operatorname{csc}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsc[c*x]))/(2*e^2*(e +
d/x^2)) + (x^2*(a + b*ArcCsc[c*x]))/(2*e^2) - (b*d*ArcTan[Sqrt[c^2*d + e]/
(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)]/(2*e^(5/2)*Sqrt[c^2*d + e]) - (d*(a +
b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 - (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a +
b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^
2*d + e])])/e^3 + (2*d*(a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/
e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[
```

2,  $E^{((2*I)*ArcCsc[c*x])}/e^3$

#### Rule 5241

$\text{Int}[(a + \text{ArcCsc}[c \cdot x] \cdot b)^n \cdot x^m \cdot (d + e \cdot x)^{p-2}]$ , x\_Symbol]  $\rightarrow$   $-\text{Subst}[\text{Int}[(e + d \cdot x^2)^p \cdot (a + b \cdot \text{ArcSin}[x/c])^n] / x^{m+2(p+1)}, x], x, 1/x]$  /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rule 4733

$\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p]$ , x\_Symbol]  $\rightarrow$   $\text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSin}[c \cdot x])^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x], x]$  /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2 \cdot d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 4627

$\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n \cdot (d \cdot x)^m]$ , x\_Symbol]  $\rightarrow$   $\text{Simp}[(d \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n / (d \cdot (m+1)), x] - \text{Dist}[(b \cdot c \cdot n) / (d \cdot (m+1)), \text{Int}[(d \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{n-1}] / \text{Sqrt}[1 - c^2 \cdot x^2], x], x]$  /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 264

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^{p-1}]$ , x\_Symbol]  $\rightarrow$   $\text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^n)^{p+1} / (a \cdot c \cdot (m+1)), x]$  /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 4625

$\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n / (x)]$ , x\_Symbol]  $\rightarrow$   $\text{Subst}[\text{Int}[(a + b \cdot x)^n / \text{Tan}[x], x], x, \text{ArcSin}[c \cdot x]]$  /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 3717

$\text{Int}[(c + d \cdot x)^m \cdot \tan[e + \text{Pi} \cdot k + f \cdot x]]$ , x\_Symbol]  $\rightarrow$   $\text{Simp}[(I \cdot (c + d \cdot x)^{m+1}) / (d \cdot (m+1)), x] - \text{Dist}[2 \cdot I, \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}] / (1 + E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}), x], x]$  /; FreeQ[{c, d, e, f}, x] && IntegerQ[4 \cdot k] && IGtQ[m, 0]

#### Rule 2190

$\text{Int}[(F)^{(g \cdot (e + f \cdot x))} \cdot (c + d \cdot x)^m / ((a + b \cdot (F)^{(g \cdot (e + f \cdot x))})^n)]$ , x\_Symbol]  $\rightarrow$   $\text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F)^{(g \cdot (e + f \cdot x))})^n] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + (b \cdot (F)^{(g \cdot (e + f \cdot x))})^n] / a], x], x]$  /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

$\text{Int}[\text{Log}[a + b \cdot (F)^{(e \cdot (c + d \cdot x))}]^n]$ , x\_Symbol]  $\rightarrow$   $\text{Dist}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F)^{(e \cdot (c + d \cdot x))}]^n]$ , x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

$\text{Int}[\text{Log}[(d + e \cdot x)^n] / (x)]$ , x\_Symbol]  $\rightarrow$   $-\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x]$  /; FreeQ[{c, d, e, n}, x] && EqQ[c \cdot d, 1]

Rule 4729

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))\*(x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c)/(2\*e\*(p + 1)), Int[(d + e\*x^2)^(p + 1)/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2\*d + e, 0] && NeQ[p, -1]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 4741

Int[((a\_.) + ArcSin[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Subst[Int[((a + b\*x)^n\*cos[x])/(c\*d + e\*sin[x]), x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 4519

Int[(Cos[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_) + (b\_.)\*Sin[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> -Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a - Rt[a^2 - b^2, 2] - I\*b\*E^(I\*(c + d\*x))), x] + Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a + Rt[a^2 - b^2, 2] - I\*b\*E^(I\*(c + d\*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]

Rubi steps



$$\begin{aligned}
\int \frac{x^5 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x^3 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e^2 x^3} - \frac{2d (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^3 x} + \frac{d^2 x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)^2} + \frac{2d^2 x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^3 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{(2d) \operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^3} - \frac{(2d^2) \operatorname{Subst} \left( \int \frac{x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} - \operatorname{Subst} \left( \int \frac{d^2 x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)^2} dx, x, \frac{1}{x} \right) - \frac{2d^2 x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^3 (e + dx^2)} \\
&= \frac{d (a + b \operatorname{csc}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{2e^2} + \frac{(2d) \operatorname{Subst} \left( \int (a + bx) \cot(x) dx, x, \operatorname{csc}^{-1}(cx) \right)}{e^3} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csc}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{2e^2} - \frac{id (a + b \operatorname{csc}^{-1}(cx))^2}{be^3} - \frac{(-d)^5}{e^3} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csc}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{2e^2} - \frac{id (a + b \operatorname{csc}^{-1}(cx))^2}{be^3} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x} \right)}{2e^{5/2} \sqrt{c^2 d + e}} + \frac{2d^2 x (a + b \operatorname{csc}^{-1} \left( \frac{x}{c} \right))}{e^3 (e + dx^2)} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csc}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x} \right)}{2e^{5/2} \sqrt{c^2 d + e}} - \frac{d (a + b \operatorname{csc}^{-1}(cx))^2}{be^3} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csc}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x} \right)}{2e^{5/2} \sqrt{c^2 d + e}} - \frac{d (a + b \operatorname{csc}^{-1}(cx))^2}{be^3} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csc}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x} \right)}{2e^{5/2} \sqrt{c^2 d + e}} - \frac{d (a + b \operatorname{csc}^{-1}(cx))^2}{be^3} \\
&= \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csc}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x} \right)}{2e^{5/2} \sqrt{c^2 d + e}} - \frac{d (a + b \operatorname{csc}^{-1}(cx))^2}{be^3}
\end{aligned}$$

**Mathematica [B]** time = 5.61441, size = 1480, normalized size = 2.36

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2,x]

```
[Out] -(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(I*d*Pi^2 -
(2*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsc[c*x] - 2*e*x^2*ArcCsc[
c*x] + (d^(3/2)*ArcCsc[c*x])/(Sqrt[d - I*Sqrt[e]*x) + (d^(3/2)*ArcCsc[c*x]
)/(Sqrt[d + I*Sqrt[e]*x) + (8*I)*d*ArcCsc[c*x]^2 - 2*d*ArcSin[1/(c*x)] - (
16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*S
qrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*d*
ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((I*c*Sqrt[d] + Sq
rt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*d*Pi*Log[1 + (Sqrt
[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log
[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (-Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - 2*d*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc
[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*
Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*Ar
cCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 2*d*Pi*Log[Sqrt[e] - (I*Sqrt[d])
/x] + 2*d*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[
e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e])*Sqrt[1 - 1/(c^2*x^2)]
*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (d
*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])*Sqrt[1 - 1/(c^2*x^2)]*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d - I*Sqrt[e]
*x))])/Sqrt[-(c^2*d) - e] + (4*I)*d*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, -((Sqrt[e] + Sqrt
[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*d*PolyLog[2, (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, E^((
2*I)*ArcCsc[c*x]))]/(4*e^3)
```

---

**Maple [C]** time = 0.734, size = 729, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*a*x^2/e^2-a/e^3*d*ln(c^2*e*x^2+c^2*d)-1/2*c^2*a/e^3*d^2/(c^2*e*x^2+c^2*
d)+1/2*c^2*b/(c^2*e*x^2+c^2*d)/e*arccsc(c*x)*x^4+c^2*b/(c^2*e*x^2+c^2*d)/e^
2*arccsc(c*x)*d*x^2+1/2*c*b/(c^2*e*x^2+c^2*d)/e*((c^2*x^2-1)/c^2/x^2)^(1/2)
*x^3+1/2*c*b/(c^2*e*x^2+c^2*d)/e^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d-1/2*I*b/
(c^2*e*x^2+c^2*d)/e*x^2-1/2*I*b/(c^2*e*x^2+c^2*d)/e^2*d+1/2*I*b*(e*(c^2*d+e
))^(1/2)/(c^2*d+e)/e^3*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2
*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))*d+1/2*I*b/e^3*d*sum((_R1^2*c^2*d-c^2*d-4*e
)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2)
)/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2
*c^2*d-4*e)*_Z^2+c^2*d))+2*I*b/e^3*d*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))+2*b/e
^3*d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*I*b/e^3*d*dilog(1+I/c/x+
(1-1/c^2/x^2)^(1/2))+1/2*I*c^2*b/e^3*d^2*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2
*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x
```

$-(1-1/c^2/x^2)^{(1/2)}/_R1))$ ,  $_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d$   
))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} a \left( \frac{d^2}{e^4 x^2 + d e^3} - \frac{x^2}{e^2} + \frac{2 d \log(ex^2 + d)}{e^3} \right) + b \int \frac{x^5 \arctan(1, \sqrt{cx + 1} \sqrt{cx - 1})}{e^2 x^4 + 2 dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2\*a\*(d^2/(e^4\*x^2 + d\*e^3) - x^2/e^2 + 2\*d\*log(e\*x^2 + d)/e^3) + b\*integrate(x^5\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{bx^5 \operatorname{arccsc}(cx) + ax^5}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^5\*arccsc(c\*x) + a\*x^5)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^5/(e\*x^2 + d)^2, x)

$$3.104 \quad \int \frac{x^3(a+b \csc^{-1}(cx))}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=590

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2e^2}$$

[Out]  $-(a + b \operatorname{ArcCsc}[c*x])/(2*e*(e + d/x^2)) + (b \operatorname{ArcTan}[\operatorname{Sqrt}[c^2*d + e]/(c*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x)]/(2*e^{3/2}*\operatorname{Sqrt}[c^2*d + e]) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 - (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 + (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 - (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 + (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) - ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcCsc}[c*x])}])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/e^2 + ((I/2)*b*PolyLog[2, E^{((2*I)*\operatorname{ArcCsc}[c*x])}])/e^2$

**Rubi [A]** time = 1.23845, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5241, 4733, 4625, 3717, 2190, 2279, 2391, 4729, 377, 205, 4741, 4519}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2e^2} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2e^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3*(a + b \operatorname{ArcCsc}[c*x]))/(d + e*x^2)^2, x]$

[Out]  $-(a + b \operatorname{ArcCsc}[c*x])/(2*e*(e + d/x^2)) + (b \operatorname{ArcTan}[\operatorname{Sqrt}[c^2*d + e]/(c*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - 1/(c^2*x^2)]*x)]/(2*e^{3/2}*\operatorname{Sqrt}[c^2*d + e]) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 - (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 + (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 - (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) + ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 + (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/(2*e^2) - ((a + b \operatorname{ArcCsc}[c*x])* \operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcCsc}[c*x])}])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*\operatorname{Sqrt}[-d]*E^{(I*\operatorname{ArcCsc}[c*x])})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])])/e^2 + ((I/2)*b*PolyLog[2, E^{((2*I)*\operatorname{ArcCsc}[c*x])}])/e^2$

**Rule 5241**

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSin[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m] && IntegerQ[p]

#### Rule 4733

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcSin[c\*x])^n, (f\*x)^m\*(d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2\*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 4625

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tan[x], x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 3717

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] - Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x)))/(1 + E^(2\*I\*k\*Pi)\*E^(2\*I\*(e + f\*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4729

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)]\*(b\_.))\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSin[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c)/(2\*e\*(p + 1)), Int[(d + e\*x^2)^(p + 1)/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2\*d + e, 0] && NeQ[p, -1]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_/((d_) + (e_.)*(x_)), x_Symbol]
:= Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e^2 x} - \frac{dx (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{dx (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \frac{x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \frac{x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left( \int (a + bx) \cot(x) dx, x, \operatorname{csc}^{-1}(cx) \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \left( -\frac{\sqrt{-d} (a + b \sin^{-1} \left( \frac{x}{c} \right))}{2d (\sqrt{e} - \sqrt{e - dx^2})} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \operatorname{csc}^{-1}(cx))^2}{2be^2} + \frac{(2i) \operatorname{Subst} \left( \int \frac{e^{2ix} (a + bx)}{1 - e^{2ix}} dx, x, \operatorname{csc}^{-1}(cx) \right)}{e^2} - \frac{\sqrt{-d} (a + b \operatorname{csc}^{-1}(cx))}{e^2} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \operatorname{csc}^{-1}(cx))^2}{2be^2} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} - \frac{(a + b \operatorname{csc}^{-1}(cx)) \log \left( 1 - \frac{ic \sqrt{-d} e^{i \operatorname{csc}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{e^2} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} - \frac{(a + b \operatorname{csc}^{-1}(cx)) \log \left( 1 - e^{2i \operatorname{csc}^{-1}(cx)} \right)}{e^2} - \frac{(ib) \sqrt{-d} (a + b \operatorname{csc}^{-1}(cx))}{e^2} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} + \frac{(a + b \operatorname{csc}^{-1}(cx)) \log \left( 1 - \frac{ic \sqrt{-d} e^{i \operatorname{csc}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^2} + \frac{(ib) \sqrt{-d} (a + b \operatorname{csc}^{-1}(cx))}{e^2} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} + \frac{(a + b \operatorname{csc}^{-1}(cx)) \log \left( 1 - \frac{ic \sqrt{-d} e^{i \operatorname{csc}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^2} + \frac{(ib) \sqrt{-d} (a + b \operatorname{csc}^{-1}(cx))}{e^2} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{3/2} \sqrt{c^2 d + e}} + \frac{(a + b \operatorname{csc}^{-1}(cx)) \log \left( 1 - \frac{ic \sqrt{-d} e^{i \operatorname{csc}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2e^2} + \frac{(ib) \sqrt{-d} (a + b \operatorname{csc}^{-1}(cx))}{e^2}
\end{aligned}$$

**Mathematica [B]** time = 2.04494, size = 1442, normalized size = 2.44

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2,x]

```
[Out] (I*b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsc[c*x] + (2*b*Sqrt[d]*Arc
Csc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsc[c*x])/(Sqrt[d] + I*
Sqrt[e]*x) + (8*I)*b*ArcCsc[c*x]^2 - 4*b*ArcSin[1/(c*x)] - (16*I)*b*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((( -I)*c*Sqrt[d] + Sqrt[e
])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
+ 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] -
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*P
i*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*
ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]
))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcCsc[c*x]*Log[1
- E^((2*I)*ArcCsc[c*x])] + 2*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 2*b*Pi*Lo
g[Sqrt[e] + (I*Sqrt[d])/x] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e])*Sqrt[1 - 1/(c^2*x^2)])*x])/Sqrt[-(
c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e] + (2*b*Sqrt[e]*Lo
g[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])*Sqr
t[1 - 1/(c^2*x^2)])*x])/Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt
[-(c^2*d) - e] + 4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, (-Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, -(Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2,
(Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyL
og[2, E^((2*I)*ArcCsc[c*x])]/(8*e^2)
```

---

**Maple [C]** time = 0.536, size = 541, normalized size = 0.9

$$\frac{c^2 ad}{2e^2(c^2 ex^2 + c^2 d)} + \frac{a \ln(c^2 ex^2 + c^2 d)}{2e^2} - \frac{c^2 b x^2 \operatorname{arccsc}(cx)}{2e(c^2 ex^2 + c^2 d)} - \frac{\frac{i}{2} b}{(c^2 d + e)e^2} \sqrt{e(c^2 d + e)} \operatorname{Artanh}\left(\frac{1}{4} \left(2c^2 d \left(\frac{i}{cx} + \sqrt{1 - \frac{1}{c^2 x^2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a/e^2*d/(c^2*e*x^2+c^2*d)+1/2*a/e^2*ln(c^2*e*x^2+c^2*d)-1/2*c^2*b*x
^2*arccsc(c*x)/e/(c^2*e*x^2+c^2*d)-1/2*I*b*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^
2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2)))^2-2*c^2*d-4*e)/(c^2*d*e+
e^2)^(1/2))-1/4*I*b/e^2*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)
*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-
I*b/e^2*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-b/e^2*arccsc(c*x)*ln(1+I/c/x+(1-1/
c^2/x^2)^(1/2))+I*b/e^2*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*c^2*b/e^2*
d*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(
c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a \left( \frac{d}{e^3 x^2 + d e^2} + \frac{\log(e x^2 + d)}{e^2} \right) + b \int \frac{x^3 \arctan(1, \sqrt{c x + 1} \sqrt{c x - 1})}{e^2 x^4 + 2 d e x^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(d/(e^3\*x^2 + d\*e^2) + log(e\*x^2 + d)/e^2) + b\*integrate(x^3\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b x^3 \operatorname{arccsc}(c x) + a x^3}{e^2 x^4 + 2 d e x^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^3\*arccsc(c\*x) + a\*x^3)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(c x) + a) x^3}{(e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^3/(e\*x^2 + d)^2, x)

$$3.105 \quad \int \frac{x(a+b \csc^{-1}(cx))}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=131

$$-\frac{a+b \csc^{-1}(cx)}{2e(d+ex^2)} - \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

[Out]  $-(a + b \operatorname{ArcCsc}[c*x])/(2*e*(d + e*x^2)) - (b*c*x*\operatorname{ArcTan}[\operatorname{Sqrt}[-1 + c^2*x^2]])/(2*d*e*\operatorname{Sqrt}[c^2*x^2]) + (b*c*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 + c^2*x^2])/\operatorname{Sqrt}[c^2*d + e]])/(2*d*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[c^2*d + e]*\operatorname{Sqrt}[c^2*x^2])$

**Rubi [A]** time = 0.121013, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5237, 446, 86, 63, 205}

$$-\frac{a+b \csc^{-1}(cx)}{2e(d+ex^2)} - \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x*(a + b*\operatorname{ArcCsc}[c*x]))/(d + e*x^2)^2, x]$

[Out]  $-(a + b \operatorname{ArcCsc}[c*x])/(2*e*(d + e*x^2)) - (b*c*x*\operatorname{ArcTan}[\operatorname{Sqrt}[-1 + c^2*x^2]])/(2*d*e*\operatorname{Sqrt}[c^2*x^2]) + (b*c*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 + c^2*x^2])/\operatorname{Sqrt}[c^2*d + e]])/(2*d*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[c^2*d + e]*\operatorname{Sqrt}[c^2*x^2])$

#### Rule 5237

$\operatorname{Int}[(a + b \operatorname{ArcCsc}[c*x])/(d + e*x^2)^2, x]$   
 $\operatorname{Int}[(a + b \operatorname{ArcCsc}[c*x])/(d + e*x^2)^2, x] \rightarrow \operatorname{Simp}[(d + e*x^2)^{p+1}*(a + b \operatorname{ArcCsc}[c*x])/(2*e*(p+1)), x] + \operatorname{Dist}[(b*c*x)/(2*e*(p+1)*\operatorname{Sqrt}[c^2*x^2]), \operatorname{Int}[(d + e*x^2)^{p+1}/(x*\operatorname{Sqrt}[c^2*x^2 - 1]), x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 446

$\operatorname{Int}[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x]$   
 $\operatorname{Int}[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(m+1)/n - 1}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m+1)/n]]

#### Rule 86

$\operatorname{Int}[(e + f*x)^p/((a + b*x)*(c + d*x)), x]$   
 $\operatorname{Int}[(e + f*x)^p/((a + b*x)*(c + d*x)), x] \rightarrow \operatorname{Dist}[b/(b*c - a*d), \operatorname{Int}[(e + f*x)^p/(a + b*x), x], x] - \operatorname{Dist}[d/(b*c - a*d), \operatorname{Int}[(e + f*x)^p/(c + d*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

#### Rule 63

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x]$   
 $\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p*(m+1) - 1}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{1/p}], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ

[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned} \int \frac{x(a + b \csc^{-1}(cx))}{(d + ex^2)^2} dx &= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex^2)} - \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)} dx}{2e\sqrt{c^2x^2}} \\ &= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex^2)} - \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}(d+ex)} dx, x, x^2\right)}{4e\sqrt{c^2x^2}} \\ &= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex^2)} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}(d+ex)} dx, x, x^2\right)}{4d\sqrt{c^2x^2}} - \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{4de\sqrt{c^2x^2}} \\ &= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex^2)} + \frac{(bx) \text{Subst}\left(\int \frac{1}{d+\frac{e}{2}+\frac{ex^2}{2}} dx, x, \sqrt{-1+c^2x^2}\right)}{2cd\sqrt{c^2x^2}} - \frac{(bx) \text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^2}{2}} dx, x, \sqrt{-1+c^2x^2}\right)}{2cde\sqrt{c^2x^2}} \\ &= -\frac{a + b \csc^{-1}(cx)}{2e(d + ex^2)} - \frac{bcx \tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx \tan^{-1}\left(\frac{\sqrt{e\sqrt{-1+c^2x^2}}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [C]** time = 0.725949, size = 286, normalized size = 2.18

$$\frac{\frac{2a}{d+ex^2} + \frac{b\sqrt{e} \log\left(\frac{4ide-4cd\sqrt{ex}\left(c\sqrt{d+i}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)}{b\sqrt{c^2(-d)-e}\left(\sqrt{d-i}\sqrt{ex}\right)}\right)}{d\sqrt{c^2(-d)-e}} + \frac{b\sqrt{e} \log\left(\frac{4i\left(-de+cd\sqrt{ex}\left(\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e+ic\sqrt{d}}\right)\right)}{b\sqrt{c^2(-d)-e}\left(\sqrt{d+i}\sqrt{ex}\right)}\right)}{d\sqrt{c^2(-d)-e}} + \frac{2b \csc^{-1}(cx)}{d+ex^2} - \frac{2b \sin^{-1}\left(\frac{1}{cx}\right)}{d}}{4e}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2, x]

[Out] -((2\*a)/(d + e\*x^2) + (2\*b\*ArcCsc[c\*x])/(d + e\*x^2) - (2\*b\*ArcSin[1/(c\*x)])/d + (b\*Sqrt[e]\*Log[((4\*I)\*d\*e - 4\*c\*d\*Sqrt[e]\*(c\*Sqrt[d] + I\*Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x]/(b\*Sqrt[-(c^2\*d) - e]\*(Sqrt[d] - I\*Sqrt[e]\*x)))/(d\*Sqrt[-(c^2\*d) - e]) + (b\*Sqrt[e]\*Log[((4\*I)\*(-d\*e) + c\*d\*Sqrt[e]\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x]/(b\*Sqrt[-(c^2\*d) - e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/(d\*Sqrt[-(c^2\*d) - e]))/(4\*e)

**Maple [B]** time = 0.276, size = 354, normalized size = 2.7

$$-\frac{c^2a}{2e(c^2ex^2 + c^2d)} - \frac{bc^2 \operatorname{arccsc}(cx)}{2e(c^2ex^2 + c^2d)} + \frac{b}{2ecx d} \sqrt{c^2x^2 - 1} \arctan\left(\frac{1}{\sqrt{c^2x^2 - 1}}\right) \frac{1}{\sqrt{\frac{c^2x^2 - 1}{c^2x^2}}} - \frac{b}{4ecx d} \sqrt{c^2x^2 - 1} \ln\left(2 \frac{1}{ecx + \sqrt{c^2x^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)`

[Out] 
$$-1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arccsc(c*x)+1/2/c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d*arctan(1/(c^2*x^2-1)^{(1/2)})-1/4/c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(-c^2*d+e)/e)^{(1/2)}*\ln(2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e-(-c^2*e*d)^{(1/2)}*c*x-e)/(e*c*x+(-c^2*e*d)^{(1/2)}))-1/4/c*b/e*(c^2*x^2-1)^{(1/2)}/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x/d/(-c^2*d+e)/e)^{(1/2)}*\ln(-2*((-c^2*d+e)/e)^{(1/2)}*(c^2*x^2-1)^{(1/2)}*e+(-c^2*e*d)^{(1/2)}*c*x-e)/(-e*c*x+(-c^2*e*d)^{(1/2)}))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\left( (c^2 e^2 x^2 + c^2 d e) \int \frac{x e^{\left(\frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1)\right)}}{c^2 e^2 x^4 + (c^2 d e - e^2) x^2 - d e} dx + \arctan\left(1, \sqrt{cx+1} \sqrt{cx-1}\right) \right) b}{2 (e^2 x^2 + d e)} - \frac{a}{2 (e^2 x^2 + d e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

[Out] 
$$-1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*integrate(1/2*x*e^{(1/2*\log(c*x + 1) + 1/2*\log(c*x - 1))}/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)*e^{(\log(c*x + 1) + \log(c*x - 1))}), x) + \arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)$$

**Fricas [A]** time = 3.46322, size = 833, normalized size = 6.36

$$\frac{\left[ 2 a c^2 d^2 + 2 a d e + \sqrt{-c^2 d e - e^2} (b e x^2 + b d) \log\left(\frac{c^2 e x^2 - c^2 d - 2 \sqrt{-c^2 d e - e^2} \sqrt{c^2 x^2 - 1} - 2 e}{e x^2 + d}\right) + 2 (b c^2 d^2 + b d e) \operatorname{arccsc}(c x) + 4 (b c^2 d^2 - b d e) \operatorname{arctan}\left(\frac{\sqrt{-c^2 d e - e^2} \sqrt{c^2 x^2 - 1}}{c^2 d + e}\right) \right]}{4 (c^2 d^3 e + d^2 e^2 + (c^2 d^2 e^2 + d e^3) x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")`

[Out] 
$$[-1/4*(2*a*c^2*d^2 + 2*a*d*e + \sqrt{-c^2*d*e - e^2})*(b*e*x^2 + b*d)*\log((c^2*e*x^2 - c^2*d - 2*\sqrt{-c^2*d*e - e^2})*\sqrt{c^2*x^2 - 1} - 2*e)/(e*x^2 + d)) + 2*(b*c^2*d^2 + b*d*e)*\operatorname{arccsc}(c*x) + 4*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*\operatorname{arctan}(-c*x + \sqrt{c^2*x^2 - 1})/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e - \sqrt{c^2*d*e + e^2})*(b*e*x^2 + b*d)*\operatorname{arctan}(\sqrt{c^2*d*e + e^2}*\sqrt{c^2*x^2 - 1}/(c^2*d + e)) + (b*c^2*d^2 + b*d*e)*\operatorname{arccsc}(c*x) + 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*\operatorname{arctan}(-c*x + \sqrt{c^2*x^2 - 1})/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2)]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^2, x)
```

$$3.106 \quad \int \frac{a+b \csc^{-1}(cx)}{x(d+ex^2)^2} dx$$

**Optimal.** Leaf size=566

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}$$

```
[Out] -(e*(a + b*ArcCsc[c*x]))/(2*d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/x)]/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

**Rubi [A]** time = 1.16131, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5241, 4733, 4729, 377, 205, 4741, 4519, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2), x]
```

```
[Out] -(e*(a + b*ArcCsc[c*x]))/(2*d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/x)]/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

**Rule 5241**

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^n_.*(x_)^m_.*((d_.) + (e_.)*(x_)^2)^p_.], x_Symbol] :> -Subst[Int[(e + d*x^2)^p*(a + b*ArcSin[x/c])^n]/x^
```

$(m + 2*(p + 1)), x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{IGtQ}[n, 0]$   
 $\&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p]$

#### Rule 4733

$\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)]^{(n_.)}*((f_.)*(x_.))^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$

#### Rule 4729

$\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)]*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p + 1)}*(a + b*\text{ArcSin}[c*x])]/(2*e*(p + 1)), x] - \text{Dist}[(b*c)/(2*e*(p + 1)), \text{Int}[(d + e*x^2)^{(p + 1)}/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{NeQ}[p, -1]$

#### Rule 377

$\text{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}/((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*p + 1, 0] \&\& \text{IntegerQ}[n]$

#### Rule 205

$\text{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$

#### Rule 4741

$\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)]^{(n_.)}/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n*\text{Cos}[x]/(c*d + e*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[n, 0]$

#### Rule 4519

$\text{Int}[(\text{Cos}[c_.] + (d_.)*(x_.))*((e_.) + (f_.)*(x_.))^{(m_.)}]/((a_.) + (b_.)*\text{Sin}[c_.] + (d_.)*(x_.)), x\_Symbol] \rightarrow -\text{Simp}[(I*(e + f*x)^{(m + 1)})/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(I*(c + d*x))}/(a - \text{Rt}[a^2 - b^2, 2] - I*b*\text{E}^{(I*(c + d*x))}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(I*(c + d*x))}/(a + \text{Rt}[a^2 - b^2, 2] - I*b*\text{E}^{(I*(c + d*x))}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{PosQ}[a^2 - b^2]$

#### Rule 2190

$\text{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}}/((a_.) + (b_.)*(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n}/a)]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n}/a)], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

#### Rule 2279

$\text{Int}[\text{Log}[(a_.) + (b_.)*((F_.)^{((e_.)*((c_.) + (d_.)*(x_.)))^{(n_.)}})], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)))^n}], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \csc^{-1}(cx)}{x(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{x^3 \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
 &= -\text{Subst} \left( \int \left( -\frac{ex \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)^2} + \frac{x \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
 &= -\frac{\text{Subst} \left( \int \frac{x^{a+b \sin^{-1} \left( \frac{x}{c} \right)}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{x^{a+b \sin^{-1} \left( \frac{x}{c} \right)}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
 &= -\frac{e \left( a + b \csc^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} - \frac{\text{Subst} \left( \int \left( -\frac{\sqrt{-d} \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e}-\sqrt{-dx})} + \frac{\sqrt{-d} \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e}+\sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{(be) \text{Subst} \left( \int \frac{x^{a+b \sin^{-1} \left( \frac{x}{c} \right)}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
 &= -\frac{e \left( a + b \csc^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{\text{Subst} \left( \int \frac{a+b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e}-\sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} - \frac{\text{Subst} \left( \int \frac{a+b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e}+\sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} + \frac{(be) \text{Subst} \left( \int \frac{x^{a+b \sin^{-1} \left( \frac{x}{c} \right)}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
 &= -\frac{e \left( a + b \csc^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2d^2 \sqrt{c^2 d + e}} + \frac{\text{Subst} \left( \int \frac{(a+bx) \cos(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sin(x)} dx, x, \csc^{-1}(cx) \right)}{2(-d)^{3/2}} - \frac{\text{Subst} \left( \int \frac{e^{ix} (a+bx)}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2 d + e}}{c} - i\sqrt{-d}} dx, x, \csc^{-1}(cx) \right)}{2(-d)^{3/2}} \\
 &= -\frac{e \left( a + b \csc^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i \left( a + b \csc^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2d^2 \sqrt{c^2 d + e}} - \frac{\text{Subst} \left( \int \frac{e^{ix} (a+bx)}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2 d + e}}{c} - i\sqrt{-d}} dx, x, \csc^{-1}(cx) \right)}{2(-d)^{3/2}} \\
 &= -\frac{e \left( a + b \csc^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i \left( a + b \csc^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2d^2 \sqrt{c^2 d + e}} - \frac{\left( a + b \csc^{-1}(cx) \right) \log \left( 1 - \frac{\sqrt{c^2 d + e}}{c} \right)}{2d^2} \\
 &= -\frac{e \left( a + b \csc^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i \left( a + b \csc^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2d^2 \sqrt{c^2 d + e}} - \frac{\left( a + b \csc^{-1}(cx) \right) \log \left( 1 - \frac{\sqrt{c^2 d + e}}{c} \right)}{2d^2} \\
 &= -\frac{e \left( a + b \csc^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{i \left( a + b \csc^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{2d^2 \sqrt{c^2 d + e}} - \frac{\left( a + b \csc^{-1}(cx) \right) \log \left( 1 - \frac{\sqrt{c^2 d + e}}{c} \right)}{2d^2}
 \end{aligned}$$

**Mathematica [F]** time = 41.9371, size = 0, normalized size = 0.

$$\int \frac{a + b \csc^{-1}(cx)}{x(d + ex^2)^2} dx$$



Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^2), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^2), x]

**Maple [C]** time = 0.839, size = 3036, normalized size = 5.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^2, x)

[Out] 
$$-1/8*I*b*c^2*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d/e/(c^2*d+e)*(e*(c^2*d+e))^{1/2}-I*b/c^4*e^2*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/(c^2*d+e)/d^4*(e*(c^2*d+e))^{1/2}-3*I*b/c^2*arccsc(c*x)^2/d^3/(c^2*d+e)*(e*(c^2*d+e))^{1/2}*e^{-3/2}*I*b/c^2*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^3/(c^2*d+e)*(e*(c^2*d+e))^{1/2}*e^{-1/4}*b*c^2*(e*(c^2*d+e))^{1/2}/e/(c^2*d+e)/d*arccsc(c*x)*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))+3*b/c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/d^3/(c^2*d+e)*(e*(c^2*d+e))^{1/2}*e+2*b/c^4*e^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/(c^2*d+e)/d^4*(e*(c^2*d+e))^{1/2}+1/4*b*c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/d/e/(c^2*d+e)*(e*(c^2*d+e))^{1/2}-2*I*b/c^4*e^2*arccsc(c*x)^2/(c^2*d+e)/d^4*(e*(c^2*d+e))^{1/2}+a/d^2*\ln(c*x)+1/8*I*b*c^2*(e*(c^2*d+e))^{1/2}/e/(c^2*d+e)/d*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))+I*b*arccsc(c*x)^2/d^2-1/2*b*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/d^2+1/2*I*b/d^2*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{1/2})/_R1))+dilog((_R1-I/c/x-(1-1/c^2/x^2)^{1/2})/_R1), _R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/4*I*b*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^2+1/2*a*c^2/d/(c^2*e*x^2+c^2*d)-3/4*I*b*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^2/(c^2*d+e)*(e*(c^2*d+e))^{1/2}-5/4*I*b*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d^2/(c^2*d+e)*e-5/2*I*b*arccsc(c*x)^2/d^2/(c^2*d+e)*e-I*b*(e*(c^2*d+e))^{1/2}/d^2/(c^2*d+e)*arccsc(c*x)^2-1/2*I*b*(e*(c^2*d+e))^{1/2}/d^2/(c^2*d+e)*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+e)+e^2)^{1/2}))+1/4*I*b*(e*(c^2*d+e))^{1/2}/d^2/(c^2*d+e)*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))-1/2*b*(e*(c^2*d+e))^{1/2}/d^2/(c^2*d+e)*arccsc(c*x)*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e))+5/2*b*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/d^2/(c^2*d+e)*e-1/2*I*b*c^2*arccsc(c*x)^2/d/(c^2*d+e)-1/4*I*b*c^2*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))/d/(c^2*d+e)+2*I*b/c^2*arccsc(c*x)^2*e/d^3+2*I*b/c^4*a*arccsc(c*x)^2*e^2/d^4+I*b/c^2*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*e/d^3-2*b/c^4*e^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/d^4-b/c^2*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/d^3*(e*(c^2*d+e))^{1/2}+I*b/c^2*arccsc(c*x)^2/d^3*(e*(c^2*d+e))^{1/2}+3/2*b*\ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e))*arccsc(c*x)/d^2/(c^2*d+e)*(e*(c^2*d+e))^{1/2}+1/2*I*b/c^2*polylog(2, c^2*d*(I/c/x+(1-1/c^2/x^2)^{1/2})^2/(c^2*d-2*(e*(c^2*d+e))^{1/2}+2$$

$$\begin{aligned}
 & *e)) / d^3 * (e^{c^2 d + e})^{1/2} - 1/2 * a / d^2 * \ln(c^2 e x^2 + c^2 d) - 2 * b / c^4 * e * \ln(1 - c \\
 & ^2 * d * (I/c/x + (1 - 1/c^2/x^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 * e)) * \arccsc \\
 & c(c * x) / d^4 * (e^{c^2 * d + e})^{1/2} + I * b / c^4 * \text{polylog}(2, c^2 * d * (I/c/x + (1 - 1/c^2/x^2) \\
 & ^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 * e)) * e / d^4 * (e^{c^2 * d + e})^{1/2} + 4 * b / \\
 & c^2 * \ln(1 - c^2 * d * (I/c/x + (1 - 1/c^2/x^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 \\
 & * e)) * \arccsc(c * x) / d^3 / (c^2 * d + e) * e^2 + 2 * b / c^4 * e^3 * \ln(1 - c^2 * d * (I/c/x + (1 - 1/c^2/x \\
 & ^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 * e)) * \arccsc(c * x) / (c^2 * d + e) / d^4 - 2 \\
 & * I * b / c^2 * \text{polylog}(2, c^2 * d * (I/c/x + (1 - 1/c^2/x^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e} \\
 & ))^{1/2} + 2 * e)) / d^3 / (c^2 * d + e) * e^2 + 2 * I * b / c^4 * \arccsc(c * x)^2 * e / d^4 * (e^{c^2 * d + e} \\
 & )^{1/2} - 4 * I * b / c^2 * \arccsc(c * x)^2 / d^3 / (c^2 * d + e) * e^2 - 1/2 * b * c^2 * x^2 * \arccsc(c * x) \\
 & * e / (c^2 * e * x^2 + c^2 * d) / d^2 - 2 * I * b / c^4 * e^3 * \arccsc(c * x)^2 / (c^2 * d + e) / d^4 - I * b / c^4 * \\
 & e^3 * \text{polylog}(2, c^2 * d * (I/c/x + (1 - 1/c^2/x^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 * e)) / (c^2 * d + e) / d^4 + I * b / c^4 * \text{polylog}(2, c^2 * d * (I/c/x + (1 - 1/c^2/x^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 * e)) * e^2 / d^4 + 1/2 * b * c^2 * \ln(1 - c^2 * d * (I/c/x + (1 - 1/c^2/x^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 * e)) * \arccsc(c * x) / d / (c^2 * d + e) - 2 * b / c^2 * e * \ln(1 - c^2 * d * (I/c/x + (1 - 1/c^2/x^2)^{1/2})^{1/2} / (c^2 * d - 2 * (e^{c^2 * d + e})^{1/2} + 2 * e)) * \arccsc(c * x) / d^3
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a \left( \frac{1}{d e x^2 + d^2} - \frac{\log(e x^2 + d)}{d^2} + \frac{2 \log(x)}{d^2} \right) + b \int \frac{\arctan(1, \sqrt{c x + 1} \sqrt{c x - 1})}{e^2 x^5 + 2 d e x^3 + d^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(1/(d\*e\*x^2 + d^2) - log(e\*x^2 + d)/d^2 + 2\*log(x)/d^2) + b\*integrate(arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \arccsc(cx) + a}{e^2 x^5 + 2 d e x^3 + d^2 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^2*x), x)
```

$$3.107 \quad \int \frac{x^4 (a + b \csc^{-1}(cx))}{(d + ex^2)^2} dx$$

**Optimal.** Leaf size=803

$$\frac{x(a + b \csc^{-1}(cx))}{e^2} - \frac{3\sqrt{-d} \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e - \sqrt{dc^2 + e}}}\right)(a + b \csc^{-1}(cx))}{4e^{5/2}} + \frac{3\sqrt{-d} \log\left(\frac{i\sqrt{-d}e^{i \csc^{-1}(cx)}c}{\sqrt{e - \sqrt{dc^2 + e}}} + 1\right)(a + b \csc^{-1}(cx))}{4e^{5/2}}$$

[Out]  $-(d*(a + b*\text{ArcCsc}[c*x]))/(4*e^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] - d/x)) + (d*(a + b*\text{ArcCs}$   
 $c[c*x]))/(4*e^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] + d/x)) + (x*(a + b*\text{ArcCsc}[c*x]))/e^2 + ($   
 $b*\text{ArcTanh}[\text{Sqrt}[1 - 1/(c^2*x^2)]])/(c*e^2) + (b*\text{Sqrt}[d]*\text{ArcTanh}[(c^2*d - (\text{S}$   
 $\text{qrt}[-d]*\text{Sqrt}[e])/x]/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*e$   
 $^2*\text{Sqrt}[c^2*d + e]) + (b*\text{Sqrt}[d]*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x]/(c*$   
 $\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*e^2*\text{Sqrt}[c^2*d + e]) -$   
 $(3*\text{Sqrt}[-d]*(a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]))]/(\text{S}$   
 $\text{qrt}[e] - \text{Sqrt}[c^2*d + e]))/(4*e^(5/2)) + (3*\text{Sqrt}[-d]*(a + b*\text{ArcCsc}[c*x])*L$   
 $og[1 + (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]))]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(4*e$   
 $^(5/2)) - (3*\text{Sqrt}[-d]*(a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[$   
 $c*x]))/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(4*e^(5/2)) + (3*\text{Sqrt}[-d]*(a + b*\text{ArcCs}$   
 $c[c*x])*Log[1 + (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]))/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e$   
 $]))/(4*e^(5/2)) - (((3*I)/4)*b*\text{Sqrt}[-d]*PolyLog[2, ((-I)*c*\text{Sqrt}[-d]*E^(I*Ar$   
 $cCsc[c*x]))/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/e^(5/2) + (((3*I)/4)*b*\text{Sqrt}[-d]*P$   
 $olyLog[2, (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]))/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/e$   
 $^(5/2) - (((3*I)/4)*b*\text{Sqrt}[-d]*PolyLog[2, ((-I)*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]$   
 $))/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/e^(5/2) + (((3*I)/4)*b*\text{Sqrt}[-d]*PolyLog[2,$   
 $(I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]))/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/e^(5/2)$

**Rubi [A]** time = 2.40039, antiderivative size = 803, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5241, 4733, 4627, 266, 63, 208, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$\frac{x(a + b \csc^{-1}(cx))}{e^2} - \frac{3\sqrt{-d} \log\left(1 - \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e - \sqrt{dc^2 + e}}}\right)(a + b \csc^{-1}(cx))}{4e^{5/2}} + \frac{3\sqrt{-d} \log\left(\frac{i\sqrt{-d}e^{i \csc^{-1}(cx)}c}{\sqrt{e - \sqrt{dc^2 + e}}} + 1\right)(a + b \csc^{-1}(cx))}{4e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2,x]

[Out]  $-(d*(a + b*\text{ArcCsc}[c*x]))/(4*e^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] - d/x)) + (d*(a + b*\text{ArcCs}$   
 $c[c*x]))/(4*e^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] + d/x)) + (x*(a + b*\text{ArcCsc}[c*x]))/e^2 + ($   
 $b*\text{ArcTanh}[\text{Sqrt}[1 - 1/(c^2*x^2)]])/(c*e^2) + (b*\text{Sqrt}[d]*\text{ArcTanh}[(c^2*d - (\text{S}$   
 $\text{qrt}[-d]*\text{Sqrt}[e])/x]/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*e$   
 $^2*\text{Sqrt}[c^2*d + e]) + (b*\text{Sqrt}[d]*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x]/(c*$   
 $\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*e^2*\text{Sqrt}[c^2*d + e]) -$   
 $(3*\text{Sqrt}[-d]*(a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]))]/(\text{S}$   
 $\text{qrt}[e] - \text{Sqrt}[c^2*d + e]))/(4*e^(5/2)) + (3*\text{Sqrt}[-d]*(a + b*\text{ArcCsc}[c*x])*L$   
 $og[1 + (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[c*x]))/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(4*e$   
 $^(5/2)) - (3*\text{Sqrt}[-d]*(a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^(I*\text{ArcCsc}[$   
 $c*x]))/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(4*e^(5/2)) + (3*\text{Sqrt}[-d]*(a + b*\text{ArcCs}$

$$c[c*x]) * \log\left[1 + \frac{(I*c*\sqrt{-d}*E^{(I*ArcCsc[c*x])})}{(\sqrt{e} + \sqrt{c^2*d + e})}\right] / (4*e^{(5/2)}) - \left(\frac{(3*I)}{4}\right) * b * \sqrt{-d} * \text{PolyLog}\left[2, \frac{((-I)*c*\sqrt{-d}*E^{(I*ArcCsc[c*x])})}{(\sqrt{e} - \sqrt{c^2*d + e})}\right] / e^{(5/2)} + \left(\frac{(3*I)}{4}\right) * b * \sqrt{-d} * \text{PolyLog}\left[2, \frac{(I*c*\sqrt{-d}*E^{(I*ArcCsc[c*x])})}{(\sqrt{e} - \sqrt{c^2*d + e})}\right] / e^{(5/2)} - \left(\frac{(3*I)}{4}\right) * b * \sqrt{-d} * \text{PolyLog}\left[2, \frac{((-I)*c*\sqrt{-d}*E^{(I*ArcCsc[c*x])})}{(\sqrt{e} + \sqrt{c^2*d + e})}\right] / e^{(5/2)} + \left(\frac{(3*I)}{4}\right) * b * \sqrt{-d} * \text{PolyLog}\left[2, \frac{(I*c*\sqrt{-d}*E^{(I*ArcCsc[c*x])})}{(\sqrt{e} + \sqrt{c^2*d + e})}\right] / e^{(5/2)}$$

#### Rule 5241

$$\text{Int}[(a_.) + \text{ArcCsc}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcSin}[x/c])^n/x^{(m + 2*(p + 1))}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p]$$

#### Rule 4733

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((f_.)*(x_.))^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$$

#### Rule 4627

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)}*(a + b*\text{ArcSin}[c*x])^n / (d*(m + 1)), x] - \text{Dist}[(b*c*n) / (d*(m + 1)), \text{Int}[(d*x)^{(m + 1)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)} / \sqrt{1 - c^2*x^2}, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$$

#### Rule 266

$$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$$

#### Rule 63

$$\text{Int}[(a_.) + (b_.)*(x_.)]^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$$

#### Rule 208

$$\text{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$$

#### Rule 4667

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[p] \&\& (\text{GtQ}[p, 0] \parallel \text{IGtQ}[n, 0])$$

#### Rule 4743

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.) + (e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{ArcSin}[c*x])^n / (e*(m + 1)), x] -$$

```
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1)))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

### Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 4741

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/((d_) + (e_)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

### Rule 4519

```
Int[(Cos[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_))/((a_) + (b_)*Sin[(c_) + (d_)*(x_)]), x_Symbol] := -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2] - I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^4 (a + b \csc^{-1}(cx))}{(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x^2 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e^2 x^2} - \frac{d (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{d (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)} dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{x (a + b \csc^{-1}(cx))}{e^2} - \frac{b \text{Subst} \left( \int \frac{1}{x \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{ce^2} + \frac{d \text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{d \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{5/2}} + \frac{d \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{5/2}} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{d \text{Subst} \left( \int \frac{(a + bx) \cos \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^5} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce^2} \\
&= -\frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \csc^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \csc^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{1 - \frac{1}{c^2 x^2}} \right)}{ce^2}
\end{aligned}$$

**Mathematica [B]** time = 6.15333, size = 1634, normalized size = 2.03

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2,x]

[Out] (a\*x)/e^2 + (a\*d\*x)/(2\*e^2\*(d + e\*x^2)) - (3\*a\*Sqrt[d]\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*e^(5/2)) + b\*(-(d\*(-(ArcCsc[c\*x])/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x)) + (I\*(ArcSin[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(-I)\*c\*Sqrt[d] - Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x])/(Sqrt[-(c^2\*d) - e])\*(Sqrt[d] + I\*Sqrt[e]\*x))/Sqrt[-(c^2\*d) - e]))/Sqrt[d]))/(4\*e^2) - (d\*(-(ArcCsc[c\*x]/(I\*Sqrt[d]\*Sqrt[e] + e\*x)) - (I\*(ArcSin[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(-Sqrt[e] + c\*(-I)\*c\*Sqrt[d] + Sqrt[-(c^2\*d) - e]\*Sqrt[1 - 1/(c^2\*x^2)])\*x])/(Sqrt[-(c^2\*d) - e])\*(Sqrt[d] - I\*Sqrt[e]\*x))/Sqrt[-(c^2\*d) - e]))/Sqrt[d]))/(4\*e^2) + (3\*Sqrt[d]\*(Pi^2 - 4\*Pi\*ArcCsc[c\*x] + 8\*ArcCsc[c\*x]^2 - 32\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*ArcTan[((-I)\*c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + 2\*ArcCsc[c\*x])/4])/Sqrt[c^2\*d + e]] + (4\*I)\*Pi\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (8\*I)\*ArcCsc[c\*x]\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + (16\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 + (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + (4\*I)\*Pi\*Log[1 + (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (8\*I)\*ArcCsc[c\*x]\*Log[1 + (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (16\*I)\*ArcSin[Sqrt[1 - (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 + (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + (8\*I)\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] - (4\*I)\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] + 8\*PolyLog[2, (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + 8\*PolyLog[2, -((Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x])))] + 4\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/(32\*e^(5/2)) - (3\*Sqrt[d]\*(Pi^2 - 4\*Pi\*ArcCsc[c\*x] + 8\*ArcCsc[c\*x]^2 - 32\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*ArcTan[(I\*c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + 2\*ArcCsc[c\*x])/4])/Sqrt[c^2\*d + e]] + (4\*I)\*Pi\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (8\*I)\*ArcCsc[c\*x]\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + (16\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 + (-Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + (4\*I)\*Pi\*Log[1 - (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (8\*I)\*ArcCsc[c\*x]\*Log[1 - (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] - (16\*I)\*ArcSin[Sqrt[1 + (I\*Sqrt[e])/(c\*Sqrt[d])])/Sqrt[2]]\*Log[1 - (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + (8\*I)\*ArcCsc[c\*x]\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])] - (4\*I)\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] + 8\*PolyLog[2, (Sqrt[e] - Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + 8\*PolyLog[2, (Sqrt[e] + Sqrt[c^2\*d + e])/(c\*Sqrt[d]\*E^(I\*ArcCsc[c\*x]))] + 4\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/(32\*e^(5/2)) + ((ArcCsc[c\*x]\*Cot[ArcCsc[c\*x]/2])/2 + Log[Cos[ArcCsc[c\*x]/2]] - Log[Sin[ArcCsc[c\*x]/2]] + (ArcCsc[c\*x]\*Tan[ArcCsc[c\*x]/2])/2)/(c\*e^2)

**Maple [C]** time = 10.047, size = 1888, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x)



```
[Out] a*x/e^2+1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d)-3/2*a/e^2*d/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2))+c^2*b*x^3*arccsc(c*x)/e/(c^2*e*x^2+c^2*d)+3/2*c^2*b*x*arcc
sc(c*x)/e^2/(c^2*e*x^2+c^2*d)*d+1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))
^(1/2)-2*e)*d)^(1/2))/e^2/d+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(
1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)
-2*e)*d)^(1/2))/e^2/d^2*(e*(c^2*d+e))^(1/2)+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e
*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/d^2-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2
*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d*(e*(c^2*d+e))^(1/2)-1/c^2*b*(-(
c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(
1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d-1/c^4*b*(
-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c
^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(
d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2
))/e^2/d-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(d*c
*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e
^2/d^2*(e*(c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2))/e/d^2+1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*a
rctanh(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2))/e^2/(c^2*d+e)/d*(e*(c^2*d+e))^(1/2)-1/c^2*b*((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d+1/c^4*b*((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-1/c^4*
b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(d*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2-1/c*b
/e^2*ln(I/c/x+(1-1/c^2/x^2)^(1/2)-1)+1/c*b/e^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2)
)-3/16*c*b/e^3*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*
(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-
1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+3
/16*c*b/e^3*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^2*d-2*e)
*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arccsc}(cx) + ax^4}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^4\*arccsc(c\*x) + a\*x^4)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^4}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^4/(e\*x^2 + d)^2, x)

$$3.108 \quad \int \frac{x^2(a+b \csc^{-1}(cx))}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=765

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}$$

```
[Out] (a + b*ArcCsc[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsc[c*x])/(4
*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c
*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]/(4*Sqrt[d]*e*Sqrt[c^2*d +
e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])]/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*
x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/
(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcC
sc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcC
sc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e
])]/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(4*Sqrt[-d]*e^(3/2)) - ((I/4)*
b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)]/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]
))/(Sqrt[e] - Sqrt[c^2*d + e])]/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (
(-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(Sqrt[-d]*
e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])]/(Sqrt[-d]*e^(3/2))
```

**Rubi [A]** time = 1.25474, antiderivative size = 765, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5241, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i \csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

```
[Out] (a + b*ArcCsc[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsc[c*x])/(4
*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c
*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]/(4*Sqrt[d]*e*Sqrt[c^2*d +
e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])]/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*
x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/
(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcC
sc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcC
sc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e
])]/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(4*Sqrt[-d]*e^(3/2)) - ((I/4)*
b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)]/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]
))/(Sqrt[e] - Sqrt[c^2*d + e])]/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (
(-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]/(Sqrt[-d]*
e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])]/(Sqrt[-d]*e^(3/2))
```

$$\frac{1}{\sqrt{e} - \sqrt{c^2 d + e}} \frac{1}{\sqrt{-d} e^{3/2}} - \left( \frac{I}{4} b \text{PolyLog}[2, (-I) c \sqrt{-d} E^{(I \text{ArcCsc}[c x])}] \right) \frac{1}{\sqrt{e} + \sqrt{c^2 d + e}} \frac{1}{\sqrt{-d} e^{3/2}} + \left( \frac{I}{4} b \text{PolyLog}[2, (I c \sqrt{-d} E^{(I \text{ArcCsc}[c x])})] \right) \frac{1}{\sqrt{e} + \sqrt{c^2 d + e}} \frac{1}{\sqrt{-d} e^{3/2}}$$

#### Rule 5241

$$\text{Int}[(a + \text{ArcCsc}[c x] b)^n (d + e x)^m, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d x^2)^p (a + b \text{ArcSin}[x/c])^n / x^{m+2(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p]$$

#### Rule 4667

$$\text{Int}[(a + \text{ArcSin}[c x] b)^n (d + e x^2)^p, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \text{ArcSin}[c x])^n (d + e x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{NeQ}[c^2 d + e, 0] \&\& \text{IntegerQ}[p] \&\& (\text{GtQ}[p, 0] \parallel \text{IGtQ}[n, 0])$$

#### Rule 4743

$$\text{Int}[(a + \text{ArcSin}[c x] b)^n (d + e x)^m, x\_Symbol] \rightarrow \text{Simp}[(d + e x)^{m+1} (a + b \text{ArcSin}[c x])^n / (e(m+1)), x] - \text{Dist}[(b c n) / (e(m+1)), \text{Int}[(d + e x)^{m+1} (a + b \text{ArcSin}[c x])^{n-1}] / \sqrt{1 - c^2 x^2}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$$

#### Rule 725

$$\text{Int}[1 / ((d + e x) \sqrt{a + c x^2}), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1 / (c d^2 + a e^2 - x^2), x], x, (a e - c d x) / \sqrt{a + c x^2}] /; \text{FreeQ}\{a, c, d, e, x\}$$

#### Rule 206

$$\text{Int}[(a + b x^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$

#### Rule 4741

$$\text{Int}[(a + \text{ArcSin}[c x] b)^n / (d + e x), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b x)^n \text{Cos}[x] / (c d + e \text{Sin}[x]), x], x, \text{ArcSin}[c x]] /; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{IGtQ}[n, 0]$$

#### Rule 4519

$$\text{Int}[(\text{Cos}[c x] + d x) (e + f x)^m / ((a + b \text{Sin}[c x] + d x)), x\_Symbol] \rightarrow -\text{Simp}[(I (e + f x)^{m+1}) / (b f (m+1)), x] + (\text{Int}[(e + f x)^m E^{(I(c + d x))} / (a - \text{Rt}[a^2 - b^2, 2] - I b E^{(I(c + d x))}), x] + \text{Int}[(e + f x)^m E^{(I(c + d x))} / (a + \text{Rt}[a^2 - b^2, 2] - I b E^{(I(c + d x))}), x]) /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{PosQ}[a^2 - b^2]$$

#### Rule 2190

$$\text{Int}[(F^{(g x)} (e + f x))^n (c + d x)^m / ((a + b F^{(g x)} (e + f x))^n), x\_Symbol] \rightarrow \text{Simp}[(c + d x)^m \text{Log}[1 + (b F^{(g x)} (e + f x))^n / a] / (b f g^n \text{Log}[F]), x] - \text{Di}$$

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( -\frac{d(a + b \sin^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \sin^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} - \frac{d(a + b \sin^{-1} \left( \frac{x}{c} \right))}{2e(-de - d^2x^2)} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{d \operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{-de - d^2x^2} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \operatorname{Subst} \left( \int \frac{1}{(\sqrt{-d}\sqrt{e} - dx) \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4ce} + \frac{b \operatorname{Subst} \left( \int \frac{1}{(\sqrt{-d}\sqrt{e} + dx) \sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4ce} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{\operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} - \frac{\operatorname{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} \\
&= \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csc}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}} - \frac{b \tanh^{-1} \left( \frac{c^2d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1 - \frac{1}{c^2x^2}}} \right)}{4\sqrt{de}\sqrt{c^2d+e}}
\end{aligned}$$

**Mathematica [A]** time = 2.33828, size = 1482, normalized size = 1.94

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2,x]

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
b*((2*ArcCsc[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsc[c*x])/(I*Sqrt[d] +
Sqrt[e]*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*ArcTan[(((
(-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sqr
t[d] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*ArcTan[(((I*c*S
qrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sqrt[d] -
(I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/S
qrt[d] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d]
)])/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]
))])/Sqrt[d] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((4*I)*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + (I*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 - (Sqr
t[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Log[1 + (Sqrt[e] +
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((2*I)*ArcCsc[c
*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqr
t[d] + ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])])/Sqrt[2]]*Log[1 + (Sqr
t[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Lo
g[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[d] + (I*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x])/S
qrt[d] - ((2*I)*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d]
- Sqrt[-(c^2*d) - e])*Sqrt[1 - 1/(c^2*x^2)])*x])/(Sqrt[-(c^2*d) - e]*(Sqrt[
d] + I*Sqrt[e]*x)))/(Sqrt[d]*Sqrt[-(c^2*d) - e]) + ((2*I)*Sqrt[e]*Log[(2*S
qrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])*Sqrt[1 -
1/(c^2*x^2)])*x])/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x)))/(Sqrt[d]*S
qrt[-(c^2*d) - e]) + (2*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, -(Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + (2*PolyLog[2, (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d]))/(8*e^(3/2))
```

---

**Maple [C]** time = 1.572, size = 1722, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)-1/2*c^2*b*arccsc(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2)))/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/d^2-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2)))/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/d^3*(e*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2)))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^3+1/2/c^2*b*(-(c^2*d-2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^(1/2)))/((-c^2*d
+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+1
/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-1/
c^2/x^2)^(1/2)))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d
+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(d*c*(I/c/x+(1-
1/c^2/x^2)^(1/2)))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d
```

$$\begin{aligned} &^3*(e*(c^2*d+e))^{(1/2)}+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \\ &*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e \\ &*d)^{(1/2)})*e/(c^2*d+e)/d^3-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \\ &*(1/2)*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)} \\ &+2*e)*d)^{(1/2)})/e/d^2+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*a \\ &rctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d \\ &)^{(1/2)})/e/d^3*(e*(c^2*d+e))^{(1/2)}-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2* \\ &e)*d)^{(1/2)}*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e)) \\ &)^{(1/2)}+2*e)*d)^{(1/2)})/d^3-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \\ &*(1/2)*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+ \\ &2*e)*d)^{(1/2)})/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^{(1/2)}+1/c^2*b*((c^2*d+2*(e*(c^ \\ &2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d \\ &+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/(c^2*d+e)/d^2-1/c^4*b*((c^2*d+2*(e*(c \\ &^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2* \\ &d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/(c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)}+1/ \\ &c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*arctanh(d*c*(I/c/x+(1-1/c \\ &^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})*e/(c^2*d+e)/d^3 \\ &-1/4*c*b/e*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-( \\ &1-1/c^2/x^2)^{(1/2)})/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=Ro \\ &otOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/4*c*b/e*sum(_R1/(_R1^2*c^2*d- \\ &c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+dilog((_R \\ &1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^ \\ &2+c^2*d)) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arccsc}(cx) + ax^2}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsc(c\*x) + a\*x^2)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*2,x)



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^2/(e\*x^2 + d)^2, x)

$$3.109 \quad \int \frac{a+b \csc^{-1}(cx)}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=759

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

[Out]  $-(a + b*\text{ArcCsc}[c*x])/(4*d*(\text{Sqrt}[-d]*\text{Sqrt}[e] - d/x)) + (a + b*\text{ArcCsc}[c*x])/(4*d*(\text{Sqrt}[-d]*\text{Sqrt}[e] + d/x)) + (b*\text{ArcTanh}[(c^2*d - (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*d^{(3/2)}*\text{Sqrt}[c^2*d + e]) + (b*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*d^{(3/2)}*\text{Sqrt}[c^2*d + e]) + ((a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((a + b*\text{ArcCsc}[c*x])*Log[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) + ((a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((a + b*\text{ArcCsc}[c*x])*Log[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) + ((I/4)*b*PolyLog[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((I/4)*b*PolyLog[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) + ((I/4)*b*PolyLog[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((I/4)*b*PolyLog[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e])$

**Rubi [A]** time = 2.28816, antiderivative size = 759, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5231, 4733, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/(d + e\*x^2)^2, x]

[Out]  $-(a + b*\text{ArcCsc}[c*x])/(4*d*(\text{Sqrt}[-d]*\text{Sqrt}[e] - d/x)) + (a + b*\text{ArcCsc}[c*x])/(4*d*(\text{Sqrt}[-d]*\text{Sqrt}[e] + d/x)) + (b*\text{ArcTanh}[(c^2*d - (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*d^{(3/2)}*\text{Sqrt}[c^2*d + e]) + (b*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(4*d^{(3/2)}*\text{Sqrt}[c^2*d + e]) + ((a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((a + b*\text{ArcCsc}[c*x])*Log[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) + ((a + b*\text{ArcCsc}[c*x])*Log[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((a + b*\text{ArcCsc}[c*x])*Log[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) + ((I/4)*b*PolyLog[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((I/4)*b*PolyLog[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) + ((I/4)*b*PolyLog[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e]) - ((I/4)*b*PolyLog[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{(3/2)}*\text{Sqrt}[e])$

$$\text{yLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/((-d)^{(3/2)}*\text{Sqrt}[e]) - ((I/4)*b*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/((-d)^{(3/2)}*\text{Sqrt}[e])$$
Rule 5231

$$\text{Int}[(a + \text{ArcCsc}[c*x]*b)^n * (d + e*x^2)^p, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p * (a + b*\text{ArcSin}[x/c])^n / x^{2*(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$$
Rule 4733

$$\text{Int}[(a + \text{ArcSin}[c*x]*b)^n * (f*x)^m * (d + e*x^2)^p, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n * (f*x)^m * (d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$
Rule 4667

$$\text{Int}[(a + \text{ArcSin}[c*x]*b)^n * (d + e*x^2)^p, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n * (d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (\text{IntegerQ}[p, 0] \ || \ \text{IGtQ}[n, 0])$$
Rule 4743

$$\text{Int}[(a + \text{ArcSin}[c*x]*b)^n * (d + e*x^2)^m, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1} * (a + b*\text{ArcSin}[c*x])^n / (e*(m+1)), x] - \text{Dist}[(b*c*n)/(e*(m+1)), \text{Int}[(d + e*x)^{m+1} * (a + b*\text{ArcSin}[c*x])^{n-1}]/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$$
Rule 725

$$\text{Int}[1/((d + e*x)*\text{Sqrt}[a + c*x^2]), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}\{a, c, d, e\}, x]$$
Rule 206

$$\text{Int}[(a + b*x^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{IntegerQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$
Rule 4741

$$\text{Int}[(a + \text{ArcSin}[c*x]*b)^n / (d + e*x), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n * \text{Cos}[x] / (c*d + e*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{IGtQ}[n, 0]$$
Rule 4519

$$\text{Int}[(\text{Cos}[c*x] + d*x) * (e + f*x)^m / (a + b*\text{Sin}[c*x] + d*x), x\_Symbol] \rightarrow -\text{Simp}[(I*(e + f*x)^{m+1}) / (b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m * E^{(I*(c + d*x))} / (a - \text{Rt}[a^2 - b^2, 2] - I*b*E^{(I*(c + d*x))}), x] + \text{Int}[(e + f*x)^m * E^{(I*(c + d*x))} / (a + \text{Rt}[a^2 - b^2, 2] - I*b*E^{(I*(c + d*x))}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{PosQ}[a^2 - b^2]$$

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{x^2 (a + b \sin^{-1}(\frac{x}{c}))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{e(a + b \sin^{-1}(\frac{x}{c}))}{d(e + dx^2)^2} + \frac{a + b \sin^{-1}(\frac{x}{c})}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{\text{Subst} \left( \int \left( \frac{a + b \sin^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \sin^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \left( -\frac{d(a + b \sin^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \sin^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} \right) dx, x, \frac{1}{x} \right)}{d} \\
&= -\left( \frac{1}{4} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right) \right) - \frac{1}{4} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right) - \frac{1}{2} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{-d}\sqrt{e} - dx} dx, x, \frac{1}{x} \right) - \frac{1}{2} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{-d}\sqrt{e} + dx} dx, x, \frac{1}{x} \right) \\
&= -\frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{1}{2} \text{Subst} \left( \int \left( -\frac{a + b \sin^{-1}(\frac{x}{c})}{2d\sqrt{e}(\sqrt{e} - \sqrt{-dx})} - \frac{a + b \sin^{-1}(\frac{x}{c})}{2d\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{b \text{Subst} \left( \int \frac{1}{d^2 + \frac{de}{c^2} - x^2} dx, x, \frac{-d + \sqrt{-d}\sqrt{e}}{c^2 x} \right)}{4cd} + \frac{b \text{Subst} \left( \int \frac{1}{d^2 + \frac{de}{c^2} - x^2} dx, x, \frac{-d + \sqrt{-d}\sqrt{e}}{c^2 x} \right)}{4cd} \\
&= -\frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2 d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} + \frac{b \tanh^{-1} \left( \frac{c^2 d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} \\
&= -\frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2 d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} + \frac{b \tanh^{-1} \left( \frac{c^2 d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} \\
&= -\frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2 d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} + \frac{b \tanh^{-1} \left( \frac{c^2 d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} \\
&= -\frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2 d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} + \frac{b \tanh^{-1} \left( \frac{c^2 d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} \\
&= -\frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \csc^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tanh^{-1} \left( \frac{c^2 d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}} + \frac{b \tanh^{-1} \left( \frac{c^2 d + \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d + e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{4d^{3/2}\sqrt{c^2 d + e}}
\end{aligned}$$

**Mathematica [A]** time = 2.55897, size = 1477, normalized size = 1.95

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x^2)^2,x]

[Out] 
$$\begin{aligned} & \left( \frac{a*x}{d^2 + d*e*x^2} + \frac{a*\text{ArcTan}\left[\frac{\sqrt{e}*x}{\sqrt{d}}\right]}{d^{3/2}*\sqrt{e}} \right) \\ & + \left( \frac{b*((2*\sqrt{d})*\text{ArcCsc}[c*x])}{((-1)*\sqrt{d}*\sqrt{e} + e*x) + (2*\sqrt{d})*\text{ArcCsc}[c*x]} \right) \\ & + \left( \frac{8*\text{ArcSin}\left[\frac{\sqrt{1 - (I*\sqrt{e})}}{c*\sqrt{d}}\right]}{\sqrt{2}} \right) \\ & + \left( \frac{\text{ArcTan}\left[\frac{((-1)*c*\sqrt{d} + \sqrt{e})*\text{Cot}\left[\frac{\text{Pi} + 2*\text{ArcCsc}[c*x]}{4}\right]}{\sqrt{c^2*d + e}}\right]}{\sqrt{e}} \right) \\ & - \left( \frac{8*\text{ArcSin}\left[\frac{\sqrt{1 + (I*\sqrt{e})}}{c*\sqrt{d}}\right]}{\sqrt{2}} \right) \\ & + \left( \frac{\text{ArcTan}\left[\frac{(I*c*\sqrt{d} + \sqrt{e})*\text{Cot}\left[\frac{\text{Pi} + 2*\text{ArcCsc}[c*x]}{4}\right]}{\sqrt{c^2*d + e}}\right]}{\sqrt{e}} \right) \\ & - \left( \frac{I*\text{Pi}*\text{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & + \left( \frac{(2*I)*\text{ArcCsc}[c*x]*\text{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & - \left( \frac{(4*I)*\text{ArcSin}\left[\frac{\sqrt{1 - (I*\sqrt{e})}}{c*\sqrt{d}}\right]}{\sqrt{2}} \right) \\ & + \left( \frac{I*\text{Pi}*\text{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & + \left( \frac{I*\text{Pi}*\text{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & - \left( \frac{(2*I)*\text{ArcCsc}[c*x]*\text{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & + \left( \frac{(4*I)*\text{ArcSin}\left[\frac{\sqrt{1 + (I*\sqrt{e})}}{c*\sqrt{d}}\right]}{\sqrt{2}} \right) \\ & + \left( \frac{I*\text{Pi}*\text{Log}\left[1 - \frac{\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & - \left( \frac{(2*I)*\text{ArcCsc}[c*x]*\text{Log}\left[1 - \frac{\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & + \left( \frac{(4*I)*\text{ArcSin}\left[\frac{\sqrt{1 + (I*\sqrt{e})}}{c*\sqrt{d}}\right]}{\sqrt{2}} \right) \\ & + \left( \frac{I*\text{Pi}*\text{Log}\left[1 - \frac{\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & - \left( \frac{(2*I)*\text{ArcCsc}[c*x]*\text{Log}\left[1 - \frac{\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & + \left( \frac{(4*I)*\text{ArcSin}\left[\frac{\sqrt{1 + (I*\sqrt{e})}}{c*\sqrt{d}}\right]}{\sqrt{2}} \right) \\ & + \left( \frac{I*\text{Pi}*\text{Log}\left[\frac{\sqrt{e} - (I*\sqrt{d})}{x}\right]}{\sqrt{e}} \right) \\ & + \left( \frac{I*\text{Pi}*\text{Log}\left[\frac{\sqrt{e} + (I*\sqrt{d})}{x}\right]}{\sqrt{e}} \right) \\ & + \left( \frac{(2*I)*\text{Log}\left[\frac{(2*\sqrt{d})*\sqrt{e}*(\sqrt{e} + c*((-1)*c*\sqrt{d} - \sqrt{-(c^2*d - e)}*\sqrt{1 - 1/(c^2*x^2)})*x)}{\sqrt{-(c^2*d - e)}*(\sqrt{d} + I*\sqrt{e}*x)}\right]}{\sqrt{-(c^2*d - e)}} \right) \\ & - \left( \frac{(2*I)*\text{Log}\left[\frac{(2*\sqrt{d})*\sqrt{e}*(-\sqrt{e} + c*((-1)*c*\sqrt{d} + \sqrt{-(c^2*d - e)}*\sqrt{1 - 1/(c^2*x^2)})*x)}{\sqrt{-(c^2*d - e)}*(\sqrt{d} - I*\sqrt{e}*x)}\right]}{\sqrt{-(c^2*d - e)}} \right) \\ & + \left( \frac{2*\text{PolyLog}\left[2, \frac{\sqrt{e} - \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & - \left( \frac{2*\text{PolyLog}\left[2, \frac{-\sqrt{e} + \sqrt{c^2*d + e}}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & - \left( \frac{2*\text{PolyLog}\left[2, -\frac{(\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \\ & + \left( \frac{2*\text{PolyLog}\left[2, \frac{(\sqrt{e} + \sqrt{c^2*d + e})}{c*\sqrt{d}}\right]*E^{(I*\text{ArcCsc}[c*x])}}{\sqrt{e}} \right) \end{aligned}$$

**Maple [C]** time = 1.395, size = 1716, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x)

[Out] 
$$\begin{aligned} & \frac{1}{2}*c^2*a*x/d/(c^2*e*x^2+c^2*d) + \frac{1}{2}*a/d/(d*e)^{(1/2)}*\arctan(e*x/(d*e)^{(1/2)}) \\ & + \frac{1}{2}*c^2*b*\arccsc(c*x)*x/d/(c^2*e*x^2+c^2*d) - \frac{1}{4}*c*b/d*\sum\left(\frac{1}{_R1}/(c^2*d - c^2*d - 2*e)\right) \\ & * \left( I*\arccsc(c*x)*\ln\left(\frac{_R1 - I/c/x - (1 - 1/c^2/x^2)^{(1/2)}}{_R1}\right) + \text{dilog}\left(\frac{_R1 - I/c/x - (1 - 1/c^2/x^2)^{(1/2)}}{_R1}\right), \right. \\ & \left. _R1 = \text{RootOf}(c^2*d*_Z^4 + (-2*c^2*d - 4*e)*_Z^2 + c^2*d) \right) \\ & - \frac{1}{4}*c*b/d*\sum\left(\frac{1}{_R1}/(c^2*d - c^2*d - 2*e)\right) \\ & * \left( I*\arccsc(c*x)*\ln\left(\frac{_R1 - I/c/x - (1 - 1/c^2/x^2)^{(1/2)}}{_R1}\right) + \text{dilog}\left(\frac{_R1 - I/c/x - (1 - 1/c^2/x^2)^{(1/2)}}{_R1}\right), \right. \\ & \left. _R1 = \text{RootOf}(c^2*d*_Z^4 + (-2*c^2*d - 4*e)*_Z^2 + c^2*d) \right) \end{aligned}$$

$$R1 - I/c/x - (1 - 1/c^2/x^2)^{1/2} / \_R1) + \text{dilog}((\_R1 - I/c/x - (1 - 1/c^2/x^2)^{1/2}) / \_R1), \_R1 = \text{RootOf}(c^2*d*_Z^4 + (-2*c^2*d - 4*e)*_Z^2 + c^2*d)) + 1/2/c^2*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctan(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((-c^2*d + 2*(e*(c^2*d+e))^{1/2} - 2*e)*d)^{1/2} / d^3 + 1/c^4*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctan(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((-c^2*d + 2*(e*(c^2*d+e))^{1/2} - 2*e)*d)^{1/2} / d^4 * (e*(c^2*d+e))^{1/2} + 1/c^4*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctan(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((-c^2*d + 2*(e*(c^2*d+e))^{1/2} - 2*e)*d)^{1/2} / d^4 * e - 1/2/c^2*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctan(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((-c^2*d + 2*(e*(c^2*d+e))^{1/2} - 2*e)*d)^{1/2} / (c^2*d+e) / d^3 * (e*(c^2*d+e))^{1/2} - 1/c^2*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctan(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((-c^2*d + 2*(e*(c^2*d+e))^{1/2} - 2*e)*d)^{1/2} * e / (c^2*d+e) / d^3 - 1/c^4*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctan(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((-c^2*d + 2*(e*(c^2*d+e))^{1/2} - 2*e)*d)^{1/2} / d^4 / (c^2*d+e) * (e*(c^2*d+e))^{1/2} * e - 1/c^4*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctan(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((-c^2*d + 2*(e*(c^2*d+e))^{1/2} - 2*e)*d)^{1/2} / d^4 / (c^2*d+e) * (e*(c^2*d+e))^{1/2} * e - 1/c^4*b*(-(c^2*d - 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctanh(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} / d^3 - 1/c^4*b*((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctanh(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} / d^4 * (e*(c^2*d+e))^{1/2} + 1/c^4*b*((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctanh(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} / d^4 * e + 1/2/c^2*b*((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctanh(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} / (c^2*d+e) / d^3 * (e*(c^2*d+e))^{1/2} - 1/c^2*b*((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctanh(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * e / (c^2*d+e) / d^3 + 1/c^4*b*((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctanh(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} / d^4 / (c^2*d+e) * (e*(c^2*d+e))^{1/2} * e - 1/c^4*b*((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} * \arctanh(d*c*(I/c/x + (1 - 1/c^2/x^2)^{1/2})) / ((c^2*d + 2*(e*(c^2*d+e))^{1/2} + 2*e)*d)^{1/2} / d^4 / (c^2*d+e) * e^2$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b \operatorname{arccsc}(cx) + a}{e^2 x^4 + 2 d e x^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(e\*x^2 + d)^2, x)



$$3.110 \quad \int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex^2)^2} dx$$

**Optimal.** Leaf size=806

$$\frac{a}{d^2x} - \frac{b \csc^{-1}(cx)}{d^2x} + \frac{e(a+b \csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{e(a+b \csc^{-1}(cx))}{4d^2\left(\frac{d}{x} + \sqrt{-d}\sqrt{e}\right)} - \frac{be \tanh^{-1}\left(\frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}} - \frac{be \tanh^{-1}\left(\frac{dc^2+e}{c\sqrt{d}\sqrt{dc^2+e}}\right)}{4d^{5/2}\sqrt{dc^2+e}}$$

[Out]  $-\left(\frac{b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]}{d^2}\right) - a/(d^2*x) - (b*\text{ArcCsc}[c*x])/(d^2*x) + (e*(a + b*\text{ArcCsc}[c*x]))/(4*d^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] - d/x)) - (e*(a + b*\text{ArcCsc}[c*x]))/(4*d^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] + d/x)) - (b*e*\text{ArcTanh}[(c^2*d - (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])])/(4*d^{5/2}* \text{Sqrt}[c^2*d + e]) - (b*e*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])])/(4*d^{5/2}* \text{Sqrt}[c^2*d + e]) + (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) - (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) + (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) - (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) + (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(d^{5/2}) - (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(d^{5/2}) + (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(d^{5/2}) - (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(d^{5/2})$

**Rubi [A]** time = 2.36407, antiderivative size = 806, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {5241, 4733, 4619, 261, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$\frac{a}{d^2x} - \frac{b \csc^{-1}(cx)}{d^2x} + \frac{e(a+b \csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{e(a+b \csc^{-1}(cx))}{4d^2\left(\frac{d}{x} + \sqrt{-d}\sqrt{e}\right)} - \frac{be \tanh^{-1}\left(\frac{c^2d - \frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}} - \frac{be \tanh^{-1}\left(\frac{dc^2+e}{c\sqrt{d}\sqrt{dc^2+e}}\right)}{4d^{5/2}\sqrt{dc^2+e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x^2)^2), x]

[Out]  $-\left(\frac{b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]}{d^2}\right) - a/(d^2*x) - (b*\text{ArcCsc}[c*x])/(d^2*x) + (e*(a + b*\text{ArcCsc}[c*x]))/(4*d^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] - d/x)) - (e*(a + b*\text{ArcCsc}[c*x]))/(4*d^2*(\text{Sqrt}[-d]*\text{Sqrt}[e] + d/x)) - (b*e*\text{ArcTanh}[(c^2*d - (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])])/(4*d^{5/2}* \text{Sqrt}[c^2*d + e]) - (b*e*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])])/(4*d^{5/2}* \text{Sqrt}[c^2*d + e]) + (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) - (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) + (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) - (3*\text{Sqrt}[e]*(a + b*\text{ArcCsc}[c*x])* \text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(4*(-d)^{5/2}) + (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(d^{5/2}) - (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(d^{5/2}) + (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(d^{5/2}) - (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(d^{5/2})$

$$c[c*x])*\text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(4*(-d)^{(5/2)}) + (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]/(-d)^{(5/2)} - (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]/(-d)^{(5/2)} + (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(-d)^{(5/2)} - (((3*I)/4)*b*\text{Sqrt}[e]*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(-d)^{(5/2)})$$
Rule 5241

$$\text{Int}[(a_.) + \text{ArcCsc}[(c_.)*(x_)]*(b_.)^{(n_.)}*(x_)^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcSin}[x/c])^n/x^{(m + 2*(p + 1))}, x], x, 1/x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p]$$
Rule 4733

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)^{(n_.)}*((f_.)*(x_))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$$
Rule 4619

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSin}[c*x])^n, x] - \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcSin}[c*x])^{(n - 1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{GtQ}[n, 0]$$
Rule 261

$$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p + 1)}/(b*n*(p + 1)), x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{EqQ}[m, n - 1] \&\& \text{NeQ}[p, -1]$$
Rule 4667

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)^{(n_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (d + e*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[p] \&\& (\text{GtQ}[p, 0] \parallel \text{IGtQ}[n, 0])$$
Rule 4743

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)^{(n_.)}*((d_.) + (e_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{ArcSin}[c*x])^n/(e*(m + 1)), x] - \text{Dist}[(b*c*n)/(e*(m + 1)), \text{Int}[(d + e*x)^{(m + 1)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)})/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$$
Rule 725

$$\text{Int}[1/(((d_.) + (e_.)*(x_))*\text{Sqrt}[(a_.) + (c_.)*(x_)^2]), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}[\{a, c, d, e\}, x]$$
Rule 206

$$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/$$

$\text{Rt}[a, 2]]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

#### Rule 4741

$\text{Int}[(a + \text{ArcSin}[c*(x)]*(b))^n/(d + (e)*(x)), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n*\text{Cos}[x]/(c*d + e*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{IGtQ}[n, 0]$

#### Rule 4519

$\text{Int}[(\text{Cos}[c + (d)*(x)]*(e + (f)*(x))^m)/(a + (b)*\text{Sin}[c + (d)*(x)]), x\_Symbol] \rightarrow -\text{Simp}[(I*(e + f*x)^{m+1})/(b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{I*(c + d*x)})/(a - \text{Rt}[a^2 - b^2, 2] - I*b*\text{E}^{I*(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{I*(c + d*x)})/(a + \text{Rt}[a^2 - b^2, 2] - I*b*\text{E}^{I*(c + d*x)}), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \& \& \text{PosQ}[a^2 - b^2]$

#### Rule 2190

$\text{Int}[(F^{(g)*(e + (f)*(x))})^n*(c + (d)*(x))^m/(a + (b)*(F^{(g)*(e + (f)*(x))})^n), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g)*(e + f*x)})^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{(g)*(e + f*x)})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

#### Rule 2279

$\text{Int}[\text{Log}[a + (b)*(F^{(e)*(c + (d)*(x))})^n], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n)], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

#### Rule 2391

$\text{Int}[\text{Log}[c + (d + (e)*(x)^n)]/(x), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^2 (d + ex^2)^2} dx &= -\text{Subst} \left( \int \frac{x^4 (a + b \sin^{-1}(\frac{x}{c}))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \sin^{-1}(\frac{x}{c})}{d^2} + \frac{e^2 (a + b \sin^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)^2} - \frac{2e (a + b \sin^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{\text{Subst} \left( \int (a + b \sin^{-1}(\frac{x}{c})) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{a}{d^2 x} - \frac{b \text{Subst} \left( \int \sin^{-1}(\frac{x}{c}) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \left( \frac{a + b \sin^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \sin^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{b \text{Subst} \left( \int \frac{x}{\sqrt{1 - \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cd^2} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{d^2} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{\sqrt{e} \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{be \tanh^{-1} \left( \frac{c}{c\sqrt{d}\sqrt{e} - d} \right)}{4d^{5/2}\sqrt{c^2}} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{be \tanh^{-1} \left( \frac{c}{c\sqrt{d}\sqrt{e} - d} \right)}{4d^{5/2}\sqrt{c^2}} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{be \tanh^{-1} \left( \frac{c}{c\sqrt{d}\sqrt{e} - d} \right)}{4d^{5/2}\sqrt{c^2}} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{be \tanh^{-1} \left( \frac{c}{c\sqrt{d}\sqrt{e} - d} \right)}{4d^{5/2}\sqrt{c^2}} \\
&= \frac{bc\sqrt{1 - \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \csc^{-1}(cx)}{d^2 x} + \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \csc^{-1}(cx))}{4d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{be \tanh^{-1} \left( \frac{c}{c\sqrt{d}\sqrt{e} - d} \right)}{4d^{5/2}\sqrt{c^2}}
\end{aligned}$$

**Mathematica [A]** time = 2.46789, size = 1525, normalized size = 1.89

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x^2)^2), x]

[Out] 
$$\begin{aligned} & \left( \frac{-8a\sqrt{d}}{x} - \frac{4a\sqrt{d}e}{d + ex^2} - 12a\sqrt{e}\operatorname{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}} + b\frac{-8c\sqrt{d}\sqrt{1 - 1/(c^2x^2)}}{d + ex^2} - \frac{8\sqrt{d}\operatorname{ArcCsc}[c*x]}{x} - \frac{2\sqrt{d}e\operatorname{ArcCsc}[c*x]}{(-I)\sqrt{d}\sqrt{e} + ex} - \frac{2\sqrt{d}e\operatorname{ArcCsc}[c*x]}{I\sqrt{d}\sqrt{e} + ex} - 24\sqrt{e}\operatorname{ArcSin}\left[\frac{\sqrt{1 - (I\sqrt{e})/(c\sqrt{d})}}{\sqrt{2}}\right]\operatorname{ArcTan}\left[\frac{((-I)c\sqrt{d} + \sqrt{e})\operatorname{Cot}\left[\frac{\pi + 2\operatorname{ArcCsc}[c*x]}{4}\right]}{\sqrt{c^2d + e}}\right] + 24\sqrt{e}\operatorname{ArcSin}\left[\frac{\sqrt{1 + (I\sqrt{e})/(c\sqrt{d})}}{\sqrt{2}}\right]\operatorname{ArcTan}\left[\frac{(Ic\sqrt{d} + \sqrt{e})\operatorname{Cot}\left[\frac{\pi + 2\operatorname{ArcCsc}[c*x]}{4}\right]}{\sqrt{c^2d + e}}\right] + (3I)\sqrt{e}\pi\operatorname{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] - (6I)\sqrt{e}\operatorname{ArcCsc}[c*x]\operatorname{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + (12I)\sqrt{e}\operatorname{ArcSin}\left[\frac{\sqrt{1 - (I\sqrt{e})/(c\sqrt{d})}}{\sqrt{2}}\right]\operatorname{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] - (3I)\sqrt{e}\pi\operatorname{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + (6I)\sqrt{e}\operatorname{ArcCsc}[c*x]\operatorname{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] - (12I)\sqrt{e}\operatorname{ArcSin}\left[\frac{\sqrt{1 + (I\sqrt{e})/(c\sqrt{d})}}{\sqrt{2}}\right]\operatorname{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] - (3I)\sqrt{e}\pi\operatorname{Log}\left[1 - \frac{\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + (6I)\sqrt{e}\operatorname{ArcCsc}[c*x]\operatorname{Log}\left[1 - \frac{\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + (12I)\sqrt{e}\operatorname{ArcSin}\left[\frac{\sqrt{1 + (I\sqrt{e})/(c\sqrt{d})}}{\sqrt{2}}\right]\operatorname{Log}\left[1 - \frac{\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + (3I)\sqrt{e}\pi\operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] - (6I)\sqrt{e}\operatorname{ArcCsc}[c*x]\operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] - (12I)\sqrt{e}\operatorname{ArcSin}\left[\frac{\sqrt{1 - (I\sqrt{e})/(c\sqrt{d})}}{\sqrt{2}}\right]\operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + (3I)\sqrt{e}\pi\operatorname{Log}\left[\frac{\sqrt{e} - (I\sqrt{d})}{x}\right] - (3I)\sqrt{e}\pi\operatorname{Log}\left[\frac{\sqrt{e} + (I\sqrt{d})}{x}\right] - ((2I)e\operatorname{Log}\left[\frac{2\sqrt{d}\sqrt{e}(\sqrt{e} + c((-I)c\sqrt{d} - \sqrt{-(c^2d - e)}\sqrt{1 - 1/(c^2x^2)})x)}{\sqrt{-(c^2d - e)}(\sqrt{d} + I\sqrt{e}x)}\right] + ((2I)e\operatorname{Log}\left[\frac{2\sqrt{d}\sqrt{e}(-\sqrt{e} + c((-I)c\sqrt{d} + \sqrt{-(c^2d - e)}\sqrt{1 - 1/(c^2x^2)})x)}{\sqrt{-(c^2d - e)}(\sqrt{d} - I\sqrt{e}x)}\right]) - 6\sqrt{e}\operatorname{PolyLog}\left[2, \frac{\sqrt{e} - \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + 6\sqrt{e}\operatorname{PolyLog}\left[2, \frac{-\sqrt{e} + \sqrt{c^2d + e}}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] + 6\sqrt{e}\operatorname{PolyLog}\left[2, -\frac{(\sqrt{e} + \sqrt{c^2d + e})}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] - 6\sqrt{e}\operatorname{PolyLog}\left[2, \frac{(\sqrt{e} + \sqrt{c^2d + e})}{c\sqrt{d}E^{(I\operatorname{ArcCsc}[c*x])}}\right] \right) / (8d^{5/2}) \end{aligned}$$

**Maple [C]** time = 10.007, size = 1784, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^2, x)

[Out] 
$$\begin{aligned} & -\frac{1}{2}a/d^2e/c^2x/(c^2e*x^2+c^2d) - \frac{3}{2}a/d^2e/(d*e)^{1/2}*\arctan(e*x/(d*e)^{1/2}) - a/d^2/x - b*\arccsc(c*x)/d^2/x - \frac{1}{2}b/c^2*(-(c^2d-2*(e*(c^2d+e))^{1/2}+2*e)*d)^{1/2}*\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{1/2})/((-c^2d+2*(e*(c^2d+e))^{1/2}-2*e)*d)^{1/2})/d^4*e - b/c^4*(-(c^2d-2*(e*(c^2d+e))^{1/2}+2*e) \end{aligned}$$

$$\begin{aligned}
& *d)^{(1/2)} * e^{2*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^5-1/2*b/c^2*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^4 * e-b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^{2*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^5+3/4*c*b/d^2 * e * sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)), _R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+3/4*c*b/d^2 * e * sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)), _R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)} * e+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^{2*arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^5/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}-1/2*b/c^2*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)} * e-b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^{2*arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^5/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}-b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e * arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^5*(e*(c^2*d+e))^{(1/2)}+b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4/(c^2*d+e) * e^2+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^3 * arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^5/(c^2*d+e)-c*b/d^2*((c^2*x^2-1)/c^2/x^2)^{(1/2)}-1/2*b*c^2*x*e*arccsc(c*x)/(c^2*e*x^2+c^2*d)/d^2+b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e * arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^5*(e*(c^2*d+e))^{(1/2)}+b/c^2*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^4/(c^2*d+e) * e^2+b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} * e^3 * arctanh(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^5/(c^2*d+e)
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{e^2 x^6 + 2 d e x^4 + d^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] `integral((b*arccsc(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**2,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^2*x^2), x)`

$$3.111 \quad \int \frac{x^5 (a + b \csc^{-1}(cx))}{(d + ex^2)^3} dx$$

**Optimal.** Leaf size=727

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}$$

[Out] (b\*c\*d\*Sqrt[1 - 1/(c^2\*x^2)])/(8\*e^2\*(c^2\*d + e)\*(e + d/x^2)\*x) - (a + b\*ArcCsc[c\*x])/(4\*e\*(e + d/x^2)^2) - (a + b\*ArcCsc[c\*x])/(2\*e^2\*(e + d/x^2)) + (b\*ArcTan[Sqrt[c^2\*d + e]/(c\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)]/(2\*e^(5/2)\*Sqrt[c^2\*d + e]) + (b\*(c^2\*d + 2\*e)\*ArcTan[Sqrt[c^2\*d + e]/(c\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)]/(8\*e^(5/2)\*(c^2\*d + e)^(3/2)) + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e^3) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e^3) + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e^3) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e^3) - ((a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/e^3 - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e^3 - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e^3 - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e^3 - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e^3 + ((I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/e^3

**Rubi [A]** time = 1.44199, antiderivative size = 727, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {5241, 4733, 4625, 3717, 2190, 2279, 2391, 4729, 382, 377, 205, 4741, 4519}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} - \frac{ibPolyLog\left(2, \frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^3, x]

[Out] (b\*c\*d\*Sqrt[1 - 1/(c^2\*x^2)])/(8\*e^2\*(c^2\*d + e)\*(e + d/x^2)\*x) - (a + b\*ArcCsc[c\*x])/(4\*e\*(e + d/x^2)^2) - (a + b\*ArcCsc[c\*x])/(2\*e^2\*(e + d/x^2)) + (b\*ArcTan[Sqrt[c^2\*d + e]/(c\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)]/(2\*e^(5/2)\*Sqrt[c^2\*d + e]) + (b\*(c^2\*d + 2\*e)\*ArcTan[Sqrt[c^2\*d + e]/(c\*Sqrt[e]\*Sqrt[1 - 1/(c^2\*x^2)]\*x)]/(8\*e^(5/2)\*(c^2\*d + e)^(3/2)) + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e^3) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/(2\*e^3) + ((a + b\*ArcCsc[c\*x])\*Log[1 - (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e^3) + ((a + b\*ArcCsc[c\*x])\*Log[1 + (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/(2\*e^3) - ((a + b\*ArcCsc[c\*x])\*Log[1 - E^((2\*I)\*ArcCsc[c\*x])])/e^3 - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e^3 - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] - Sqrt[c^2\*d + e])])/e^3 - ((I/2)\*b\*PolyLog[2, ((-I)\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e^3 - ((I/2)\*b\*PolyLog[2, (I\*c\*Sqrt[-d]\*E^(I\*ArcCsc[c\*x]))/(Sqrt[e] + Sqrt[c^2\*d + e])])/e^3 + ((I/2)\*b\*PolyLog[2, E^((2\*I)\*ArcCsc[c\*x])])/e^3



$$\begin{aligned} & e]])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] \\ & - Sqrt[c^2*d + e]))/e^3 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc \\ & [c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[- \\ & d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))/e^3 + ((I/2)*b*PolyLog[ \\ & 2, E^((2*I)*ArcCsc[c*x]))/e^3 \end{aligned}$$
Rule 5241

$$\text{Int}[(a + \text{ArcCsc}[c \cdot x] \cdot (b \cdot x)^{n_1}) \cdot (d + (e \cdot x)^2)^{p_1}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d \cdot x^2)^p \cdot (a + b \cdot \text{ArcSin}[x/c])^n] / x^{m+2(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IntegerQ}[m] \ \&\& \text{IntegerQ}[p]$$
Rule 4733

$$\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot (b \cdot x)^{n_1}) \cdot (f \cdot x)^{m_1} \cdot (d + (e \cdot x)^2)^{p_1}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSin}[c \cdot x])^n, (f \cdot x)^m \cdot (d + e \cdot x^2)^p], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{NeQ}[c^2 \cdot d + e, 0] \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IntegerQ}[p] \ \&\& \text{IntegerQ}[m]$$
Rule 4625

$$\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot (b \cdot x)^{n_1}) / (x), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b \cdot x)^n / \text{Tan}[x], x], x, \text{ArcSin}[c \cdot x]] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \text{IGtQ}[n, 0]$$
Rule 3717

$$\text{Int}[(c + (d \cdot x)^{m_1}) \cdot \tan[(e + \text{Pi} \cdot (k \cdot x) + (f \cdot x)^{n_1})], x\_Symbol] \rightarrow \text{Simp}[(I \cdot (c + d \cdot x)^{m+1}) / (d \cdot (m+1)), x] - \text{Dist}[2 \cdot I, \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}] / (1 + E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}), x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \text{IntegerQ}[4 \cdot k] \ \&\& \text{IGtQ}[m, 0]$$
Rule 2190

$$\text{Int}[(F)^{(g \cdot (e + (f \cdot x)^{n_1}))^{n_1}} \cdot (c + (d \cdot x)^{m_1}) / ((a + (b \cdot (F)^{(g \cdot (e + (f \cdot x)^{n_1}))^{n_1}}))^{n_1}), x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F)^{(g \cdot (e + f \cdot x))})^n] / a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + (b \cdot (F)^{(g \cdot (e + f \cdot x))})^n] / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \text{IGtQ}[m, 0]$$
Rule 2279

$$\text{Int}[\text{Log}[a + (b \cdot (F)^{(e \cdot (c + (d \cdot x)^{n_1}))^{n_1}})], x\_Symbol] \rightarrow \text{Dist}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F)^{(e \cdot (c + d \cdot x))} \cdot n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \text{GtQ}[a, 0]$$
Rule 2391

$$\text{Int}[\text{Log}[(c + (d + (e \cdot x)^{n_1})) / (x)], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x \ \&\& \text{EqQ}[c \cdot d, 1]$$
Rule 4729

$$\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot (b \cdot x)^{n_1}) \cdot (d + (e \cdot x)^2)^{p_1}, x\_Symbol] \rightarrow \text{Simp}[(d + e \cdot x^2)^{p+1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x]) / (2 \cdot e \cdot (p+1)), x] - \text{Dist}[(b \cdot c) / (2 \cdot e \cdot (p+1)), \text{Int}[(d + e \cdot x^2)^{p+1} / \text{Sqrt}[1 - c^2 \cdot x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \text{NeQ}[c^2 \cdot d + e, 0] \ \&\& \text{NeQ}[p, -1]$$
Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

### Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*SIN[x]), x], x, ArcSin[c*x]] /;
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

### Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_))^(m_.)/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \csc^{-1}(cx))}{(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x (e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e^3 x} - \frac{dx (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^3} - \frac{dx (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)^2} - \frac{dx (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e^3 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \text{Subst} \left( \int \frac{x (a + b \sin^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \text{Subst} \left( \int \frac{x^2 (a + b \sin^{-1} \left( \frac{x}{c} \right))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{a + b \csc^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \csc^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} - \frac{\text{Subst} \left( \int (a + bx) \cot(x) dx, x, \csc^{-1}(cx) \right)}{e^3} + \frac{d \text{Subst} \left( \int \frac{x (a + b \csc^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= \frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \csc^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \csc^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \csc^{-1}(cx))^2}{2be^3} + \frac{d \text{Subst} \left( \int \frac{x (a + b \csc^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} \quad (2i) \\
&= \frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \csc^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \csc^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \csc^{-1}(cx))^2}{2be^3} + \frac{d \text{Subst} \left( \int \frac{x (a + b \csc^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} \quad b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right) \\
&= \frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \csc^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \csc^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} + \frac{d \text{Subst} \left( \int \frac{x (a + b \csc^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= \frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \csc^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \csc^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} + \frac{d \text{Subst} \left( \int \frac{x (a + b \csc^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= \frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \csc^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \csc^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} + \frac{d \text{Subst} \left( \int \frac{x (a + b \csc^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= \frac{bcd \sqrt{1 - \frac{1}{c^2 x^2}}}{8e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \csc^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \csc^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2} x}} \right)}{2e^{5/2} \sqrt{c^2 d + e}} + \frac{d \text{Subst} \left( \int \frac{x (a + b \csc^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3}
\end{aligned}$$

**Mathematica [B]** time = 7.56537, size = 2053, normalized size = 2.82

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^3,x]

```
[Out] -(a*d^2)/(4*e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x^2
])/ (2*e^3) + b*(((7*I)/16)*Sqrt[d]*(-ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e] +
e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(
(-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*
d) - e]*(Sqrt[d] + I*Sqrt[e]*x)))/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) -
(((7*I)/16)*Sqrt[d]*(-ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[
1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + S
qrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] -
I*Sqrt[e]*x)))/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*((I*c*Sqrt[e]*S
qrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) -
ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c
*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x)))/((2*c^2*d + e)*((-I)*
Sqrt[d] + Sqrt[e]*x)))/(d*(c^2*d + e)^(3/2)))/(16*e^(5/2)) - (d*(((I)*c*
Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*
x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*
Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e]
+ c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x)))/((2*c^2*d + e)
*(I*Sqrt[d] + Sqrt[e]*x)))/(d*(c^2*d + e)^(3/2)))/(16*e^(5/2)) + ((I/16)*
(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCs
c[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt
[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Lo
g[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*S
qrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x]))])/e^3 +
((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*
ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt
[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*
I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e
])/ (c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e
])/ (c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x]))])/e^
3)
```

---

**Maple [C]** time = 0.842, size = 1498, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x)

```
[Out] 1/2*a/e^3*ln(c^2*e*x^2+c^2*d)+c^2*a/e^3*d/(c^2*e*x^2+c^2*d)-1/4*c^4*a*d^2/e^3/(c^2*e*x^2+c^2*d)^2-3/4*c^6*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)*d*x^4-1/2*c^6*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)*d^2*x^2+1/8*c^5*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)*x^3*d+1/8*c^5*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d^2+I*c^2*b/e^3/(c^2*d+e)*d*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/8*I*c^4*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d^2-1/4*I*c^4*b/e^3/(c^2*d+e)*d^2*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/4*c^4*b/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)*x^4-1/2*c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)*d*x^2+I*b/e^2/(c^2*d+e)*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*x^2*d-3/4*I*b*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))-b/e^2/(c^2*d+e)*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*b/e^2/(c^2*d+e)*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-I*c^2*b/e^3/(c^2*d+e)*d*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-5/8*I*c^2*b*(e*(c^2*d+e))^(1/2)/e^3/(c^2*d+e)^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))*d-I*b/e^2/(c^2*d+e)*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*c^2*b/e^3/(c^2*d+e)*d*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-c^2*b/e^3/(c^2*d+e)*d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/8*I*c^4*b/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*x^4-1/4*I*c^2*b/e^2/(c^2*d+e)*d*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} a \left( \frac{4 d e x^2 + 3 d^2}{e^5 x^4 + 2 d e^4 x^2 + d^2 e^3} + \frac{2 \log(e x^2 + d)}{e^3} \right) + b \int \frac{x^5 \arctan(1, \sqrt{c x + 1} \sqrt{c x - 1})}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2 + d)/e^3) + b*integrate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b x^5 \operatorname{arccsc}(c x) + a x^5}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsc(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^5/(e\*x^2 + d)^3, x)

$$3.112 \quad \int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex^2)^3} dx$$

**Optimal.** Leaf size=157

$$\frac{x^4 (a + b \csc^{-1}(cx))}{4d (d + ex^2)^2} + \frac{bcx (c^2 d + 2e) \tan^{-1} \left( \frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{\sqrt{c^2 d + e}} \right)}{8de^{3/2} \sqrt{c^2 x^2} (c^2 d + e)^{3/2}} - \frac{bcx \sqrt{c^2 x^2 - 1}}{8e \sqrt{c^2 x^2} (c^2 d + e) (d + ex^2)}$$

[Out]  $-(b*c*x*\text{Sqrt}[-1 + c^2*x^2])/(8*e*(c^2*d + e)*\text{Sqrt}[c^2*x^2]*(d + e*x^2)) + (x^4*(a + b*\text{ArcCsc}[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d + 2*e)*x*\text{ArcTan}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/\text{Sqrt}[c^2*d + e]])/(8*d*e^{(3/2)}*(c^2*d + e)^{(3/2)}*\text{Sqrt}[c^2*x^2])$

**Rubi [A]** time = 0.182081, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {264, 5239, 12, 446, 78, 63, 205}

$$\frac{x^4 (a + b \csc^{-1}(cx))}{4d (d + ex^2)^2} + \frac{bcx (c^2 d + 2e) \tan^{-1} \left( \frac{\sqrt{e} \sqrt{c^2 x^2 - 1}}{\sqrt{c^2 d + e}} \right)}{8de^{3/2} \sqrt{c^2 x^2} (c^2 d + e)^{3/2}} - \frac{bcx \sqrt{c^2 x^2 - 1}}{8e \sqrt{c^2 x^2} (c^2 d + e) (d + ex^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(a + b*\text{ArcCsc}[c*x]))/(d + e*x^2)^3, x]$

[Out]  $-(b*c*x*\text{Sqrt}[-1 + c^2*x^2])/(8*e*(c^2*d + e)*\text{Sqrt}[c^2*x^2]*(d + e*x^2)) + (x^4*(a + b*\text{ArcCsc}[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d + 2*e)*x*\text{ArcTan}[(\text{Sqrt}[e]*\text{Sqrt}[-1 + c^2*x^2])/\text{Sqrt}[c^2*d + e]])/(8*d*e^{(3/2)}*(c^2*d + e)^{(3/2)}*\text{Sqrt}[c^2*x^2])$

#### Rule 264

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(c_*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x] \ \&\& \ \text{EqQ}[(m+1)/n + p + 1, 0] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 5239

$\text{Int}[(a_*) + \text{ArcCsc}[(c_*)*(x_*)*(b_*)]*((f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(p_*)}, x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcCsc}[c*x], u, x] + \text{Dist}[(b*c*x)/\text{Sqrt}[c^2*x^2], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[c^2*x^2 - 1])], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x] \ \&\& \ ((\text{IGtQ}[p, 0] \ \&\& \ !(\text{ILtQ}[(m-1)/2, 0]) \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{IGtQ}[(m+1)/2, 0] \ \&\& \ !(\text{ILtQ}[p, 0]) \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{ILtQ}[(m+2*p+1)/2, 0] \ \&\& \ !\text{ILtQ}[(m-1)/2, 0]))$

#### Rule 12

$\text{Int}[(a_*)*(u_*), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_*) /; \text{FreeQ}[b, x]]$

#### Rule 446

$\text{Int}[(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})^{(q_*)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)}*(a + b*x)^p$

$*(c + d*x)^q, x], x, x^n], x] /; FreeQ[\{a, b, c, d, m, n, p, q\}, x] \&\& NeQ[b*c - a*d, 0] \&\& IntegerQ[Simplify[(m + 1)/n]]$

**Rule 78**

$Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x\_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[\{a, b, c, d, e, f, n\}, x] \&\& LtQ[p, -1] \&\& (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || (EqQ[e, 0] || (EqQ[c, 0] || LtQ[p, n])))$

**Rule 63**

$Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x\_Symbol] := With[\{p = Denominator[m]\}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[\{a, b, c, d\}, x] \&\& NeQ[b*c - a*d, 0] \&\& LtQ[-1, m, 0] \&\& LeQ[-1, n, 0] \&\& LeQ[Denominator[n], Denominator[m]] \&\& IntLinearQ[a, b, c, d, m, n, x]$

**Rule 205**

$Int[((a_) + (b_.)*(x_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[\{a, b\}, x] \&\& PosQ[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex^2)^3} dx &= \frac{x^4 (a + b \csc^{-1}(cx))}{4d (d + ex^2)^2} + \frac{(bcx) \int \frac{x^3}{4d\sqrt{-1+c^2x^2}(d+ex^2)^2} dx}{\sqrt{c^2x^2}} \\ &= \frac{x^4 (a + b \csc^{-1}(cx))}{4d (d + ex^2)^2} + \frac{(bcx) \int \frac{x^3}{\sqrt{-1+c^2x^2}(d+ex^2)^2} dx}{4d\sqrt{c^2x^2}} \\ &= \frac{x^4 (a + b \csc^{-1}(cx))}{4d (d + ex^2)^2} + \frac{(bcx) \text{Subst}\left(\int \frac{x}{\sqrt{-1+c^2x}(d+ex)^2} dx, x, x^2\right)}{8d\sqrt{c^2x^2}} \\ &= -\frac{bcx\sqrt{-1+c^2x^2}}{8e(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} + \frac{x^4(a+b\csc^{-1}(cx))}{4d(d+ex^2)^2} + \frac{(bc(c^2d+2e)x)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}(d+ex)} dx, x, x^2\right)}{16de(c^2d+e)\sqrt{c^2x^2}} \\ &= -\frac{bcx\sqrt{-1+c^2x^2}}{8e(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} + \frac{x^4(a+b\csc^{-1}(cx))}{4d(d+ex^2)^2} + \frac{(b(c^2d+2e)x)\text{Subst}\left(\int \frac{1}{d+\frac{e}{2}+\frac{ex^2}{2}} dx, x, x^2\right)}{8cde(c^2d+e)\sqrt{c^2x^2}} \\ &= -\frac{bcx\sqrt{-1+c^2x^2}}{8e(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} + \frac{x^4(a+b\csc^{-1}(cx))}{4d(d+ex^2)^2} + \frac{bc(c^2d+2e)x \tan^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}(c^2d+e)^{3/2}\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [C]** time = 1.05786, size = 390, normalized size = 2.48

$$-\frac{8a}{d+ex^2} + \frac{4ad}{(d+ex^2)^2} + \frac{b\sqrt{e}(c^2d+2e) \log\left(\frac{16de^{3/2}\sqrt{c^2(-d)-e}\left(cx\left(c\sqrt{d-i}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}+i\sqrt{e}\right)\right)}{b(c^2d+2e)(\sqrt{d+i}\sqrt{ex})}\right)}{d(c^2(-d)-e)^{3/2}} + \frac{b\sqrt{e}(c^2d+2e) \log\left(-\frac{16de^{3/2}\sqrt{c^2(-d)-e}\left(-\sqrt{e}+cx\left(\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)\right)}{b(c^2d+2e)(\sqrt{ex+i}\sqrt{d})}\right)}{d(c^2(-d)-e)^{3/2}}$$



Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^3,x]

[Out] 
$$\begin{aligned} & \left( \frac{4ad}{(d+ex^2)^2} - \frac{8a}{d+ex^2} - \frac{2bce\sqrt{1-1/(c^2x^2)}}{(c^2d+e)(d+ex^2)} - \frac{4b(d+2ex^2)\text{ArcCsc}[cx]}{(d+ex^2)^2} \right. \\ & + \frac{4b\text{ArcSin}[1/(cx)]}{d} + \frac{b\sqrt{e}(c^2d+2e)\text{Log}[(16d\sqrt{-(c^2d-e)}e^{3/2}(I\sqrt{e}+c(c\sqrt{d}-I\sqrt{-(c^2d-e)}\sqrt{1-1/(c^2x^2)}))x]}{b(c^2d+2e)(\sqrt{d}+I\sqrt{e}x)}}{d(-(c^2d-e)^{3/2})} \\ & + \frac{b\sqrt{e}(c^2d+2e)\text{Log}[-16d\sqrt{-(c^2d-e)}e^{3/2}(-\sqrt{e}+c(-I)c\sqrt{d}+\sqrt{-(c^2d-e)}\sqrt{1-1/(c^2x^2)})x]}{b(c^2d+2e)(I\sqrt{d}+\sqrt{e}x)}}{d(-(c^2d-e)^{3/2})} \Big) / (16e^2) \end{aligned}$$

**Maple [B]** time = 0.29, size = 1870, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x)

[Out] 
$$\begin{aligned} & -1/2c^2a/e^2/(c^2ex^2+c^2d)+1/4c^4a/e^2d/(c^2ex^2+c^2d)^2-1/2c^2b\text{arccsc}(cx)/e^2/(c^2ex^2+c^2d)+1/4c^4b\text{arccsc}(cx)/e^2d/(c^2ex^2+c^2d)^2-1/4c^3b(c^2x^2-1)^{1/2}/((c^2x^2-1)/c^2/x^2)^{1/2}x/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2})\text{arctan}(1/(c^2x^2-1)^{1/2}) \\ & -1/4c^3b(c^2x^2-1)^{1/2}/e/((c^2x^2-1)/c^2/x^2)^{1/2}/xd/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2})\text{arctan}(1/(c^2x^2-1)^{1/2}) \\ & +1/16c^3b(c^2x^2-1)^{1/2}/((c^2x^2-1)/c^2/x^2)^{1/2}x/(-c^2d+e)/e^{1/2}/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2}) \\ & \ln(2*((-c^2d+e)/e)^{1/2}(c^2x^2-1)^{1/2}e^{-(c^2e)d)^{1/2}cx-e)/(ex^2+(-c^2e)d)^{1/2}) \\ & +1/16c^3b(c^2x^2-1)^{1/2}/e/((c^2x^2-1)/c^2/x^2)^{1/2}/xd/(-c^2d+e)/e^{1/2}/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2}) \\ & \ln(2*((-c^2d+e)/e)^{1/2}(c^2x^2-1)^{1/2}e^{-(c^2e)d)^{1/2}cx-e)/(ex^2+(-c^2e)d)^{1/2}) \\ & +1/16c^3b(c^2x^2-1)^{1/2}/((c^2x^2-1)/c^2/x^2)^{1/2}x/(-c^2d+e)/e^{1/2}/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2}) \\ & \ln(-2*((-c^2d+e)/e)^{1/2}(c^2x^2-1)^{1/2}e^{-(c^2e)d)^{1/2}cx-e}/(-ex^2+(-c^2e)d)^{1/2}) \\ & +1/16c^3b(c^2x^2-1)^{1/2}/e/((c^2x^2-1)/c^2/x^2)^{1/2}/xd/(-c^2d+e)/e^{1/2}/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2}) \\ & \ln(-2*((-c^2d+e)/e)^{1/2}(c^2x^2-1)^{1/2}e^{-(c^2e)d)^{1/2}cx-e}/(-ex^2+(-c^2e)d)^{1/2}) \\ & +1/8c^3b/((c^2x^2-1)/c^2/x^2)^{1/2}x/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2}) \\ & \ln(2*((-c^2d+e)/e)^{1/2}(c^2x^2-1)^{1/2}e^{-(c^2e)d)^{1/2}cx-e}/(-ex^2+(-c^2e)d)^{1/2}) \\ & +1/8c^3b(c^2x^2-1)^{1/2}/((c^2x^2-1)/c^2/x^2)^{1/2}x/(-c^2d+e)/e^{1/2}/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2}) \\ & \ln(2*((-c^2d+e)/e)^{1/2}(c^2x^2-1)^{1/2}e^{-(c^2e)d)^{1/2}cx-e}/(-ex^2+(-c^2e)d)^{1/2}) \\ & +1/8c^3b(c^2x^2-1)^{1/2}/e/((c^2x^2-1)/c^2/x^2)^{1/2}x/(-c^2d+e)/e^{1/2}/(c^2d+e)/(ex^2+(-c^2e)d)^{1/2}/(-ex^2+(-c^2e)d)^{1/2}) \\ & \ln(-2*((-c^2d+e)/e)^{1/2}(c^2x^2-1)^{1/2}e^{-(c^2e)d)^{1/2}cx-e}/(-ex^2+(-c^2e)d)^{1/2}) \end{aligned}$$

$$\frac{1}{2})) + \frac{1}{8} c b (c^2 x^2 - 1)^{1/2} / ((c^2 x^2 - 1) / c^2 / x^2)^{1/2} / x / (-c^2 d + e) / e)^{1/2} / (c^2 d + e) / (e c x + (-c^2 e d)^{1/2}) / (-e c x + (-c^2 e d)^{1/2}) * \ln(-2 * ((-c^2 d + e) / e)^{1/2} * (c^2 x^2 - 1)^{1/2} * e + (-c^2 e d)^{1/2} * c x - e) / (-e c x + (-c^2 e d)^{1/2}))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(2ex^2 + d)a}{4(e^4x^4 + 2de^3x^2 + d^2e^2)} - \frac{\left(2ex^2 \arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) + d \arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) + (e^4x^4 + 2de^3x^2 + d^2e^2)\right)}{4(e^4x^4 + 2de^3x^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out]  $-1/4*(2e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*(2e*x^2*\arctan(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}) + d*\arctan(1, \sqrt{c*x + 1}*\sqrt{c*x - 1})) + 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)*\int(1/4*(2*c^2*e*x^3 + c^2*d*x)*e^{1/2*\log(c*x + 1) + 1/2*\log(c*x - 1)})/(c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^{(\log(c*x + 1) + \log(c*x - 1))}, x)*b/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)$

**Fricas [B]** time = 6.48531, size = 2093, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out]  $[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{-c^2*d*e - e^2}*\log((c^2*e*x^2 - c^2*d - 2*\sqrt{-c^2*d*e - e^2}*\sqrt{c^2*x^2 - 1} - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arccsc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) + 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{c^2*d*e + e^2}*\arctan(\sqrt{c^2*d*e + e^2}*\sqrt{c^2*x^2 - 1}/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arccsc(c*x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) + (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^3/(e\*x^2 + d)^3, x)

$$3.113 \quad \int \frac{x(a+b \csc^{-1}(cx))}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=193

$$-\frac{a+b \csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}} + \frac{bcx(3c^2d+2e) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}(c^2d+e)^{3/2}} + \frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}(c^2d+e)(d+ex^2)}$$

[Out] (b\*c\*x\*Sqrt[-1 + c^2\*x^2])/(8\*d\*(c^2\*d + e)\*Sqrt[c^2\*x^2]\*(d + e\*x^2)) - (a + b\*ArcCsc[c\*x])/(4\*e\*(d + e\*x^2)^2) - (b\*c\*x\*ArcTan[Sqrt[-1 + c^2\*x^2]])/(4\*d^2\*e\*Sqrt[c^2\*x^2]) + (b\*c\*(3\*c^2\*d + 2\*e)\*x\*ArcTan[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/Sqrt[c^2\*d + e]])/(8\*d^2\*Sqrt[e]\*(c^2\*d + e)^(3/2)\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.191103, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5237, 446, 103, 156, 63, 205}

$$-\frac{a+b \csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{bcx \tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}} + \frac{bcx(3c^2d+2e) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}(c^2d+e)^{3/2}} + \frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}(c^2d+e)(d+ex^2)}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^3,x]

[Out] (b\*c\*x\*Sqrt[-1 + c^2\*x^2])/(8\*d\*(c^2\*d + e)\*Sqrt[c^2\*x^2]\*(d + e\*x^2)) - (a + b\*ArcCsc[c\*x])/(4\*e\*(d + e\*x^2)^2) - (b\*c\*x\*ArcTan[Sqrt[-1 + c^2\*x^2]])/(4\*d^2\*e\*Sqrt[c^2\*x^2]) + (b\*c\*(3\*c^2\*d + 2\*e)\*x\*ArcTan[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/Sqrt[c^2\*d + e]])/(8\*d^2\*Sqrt[e]\*(c^2\*d + e)^(3/2)\*Sqrt[c^2\*x^2])

#### Rule 5237

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsc[c\*x]))/(2\*e\*(p + 1)), x] + Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 103

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.))\*((e\_.) + (f\_.)\*(x\_)^(p\_.)), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

Rule 156

Int[(((e\_.) + (f\_.)\*(x\_.))^(p\_.)\*((g\_.) + (h\_.)\*(x\_.)))/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] := Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 63

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{x(a + b \csc^{-1}(cx))}{(d + ex^2)^3} dx &= -\frac{a + b \csc^{-1}(cx)}{4e(d + ex^2)^2} - \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^2} dx}{4e\sqrt{c^2x^2}} \\ &= -\frac{a + b \csc^{-1}(cx)}{4e(d + ex^2)^2} - \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}(d+ex^2)} dx, x, x^2\right)}{8e\sqrt{c^2x^2}} \\ &= \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a + b \csc^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \text{Subst}\left(\int \frac{-c^2d-e+\frac{1}{2}c^2ex}{x\sqrt{-1+c^2x}(d+ex)} dx, x, x^2\right)}{8de(c^2d+e)\sqrt{c^2x^2}} \\ &= \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a + b \csc^{-1}(cx)}{4e(d + ex^2)^2} - \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}} dx, x, x^2\right)}{8d^2e\sqrt{c^2x^2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}} dx, x, \sqrt{-1+c^2x^2}\right)}{4cd^2e\sqrt{c^2x^2}} \\ &= \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a + b \csc^{-1}(cx)}{4e(d + ex^2)^2} - \frac{bcx \tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} + \frac{bc(3c^2d+2e)}{8d^2\sqrt{e}(c^2(-d)-e)^{3/2}} \end{aligned}$$

**Mathematica [C]** time = 0.877974, size = 385, normalized size = 1.99

$$\frac{1}{16} \left[ -\frac{4a}{e(d + ex^2)^2} + \frac{b(3c^2d + 2e) \log\left(\frac{16d^2\sqrt{e}\sqrt{c^2(-d)-e}\left(cx\left(c\sqrt{d-i}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)+i\sqrt{e}\right)}{b(3c^2d+2e)(\sqrt{d+i}\sqrt{ex})}\right)}{d^2\sqrt{e}(c^2(-d)-e)^{3/2}} + \frac{b(3c^2d + 2e) \log\left(-\frac{16d^2\sqrt{e}\sqrt{c^2(-d)-e}}{d^2\sqrt{e}(c^2(-d)-e)^{3/2}}\right)}{d^2\sqrt{e}(c^2(-d)-e)^{3/2}} \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)*
(d + e*x^2)) - (4*b*ArcCsc[c*x]))/(e*(d + e*x^2)^2) + (4*b*ArcSin[1/(c*x)])/(
(d^2*e) + (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(I*Sqrt
[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x]))/(b*(3*
c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x)))/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e])
+ (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt[e] + c*
((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)))/(b*(3*c^2*d
+ 2*e)*(I*Sqrt[d] + Sqrt[e]*x)))/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e])/16
```

**Maple [B]** time = 0.283, size = 1840, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arccsc(c*x
)-1/4*c^3*b*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/(c^2*d+e)/(e*
c*x+(-c^2*e*d)^(1/2))/d/(-e*c*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2
))-1/4*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e)/(e*c
*x+(-c^2*e*d)^(1/2))/(-e*c*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))+
3/16*c^3*b*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/(c^2*d+e)/(e*c
*x+(-c^2*e*d)^(1/2))/(-c^2*d+e)/e^(1/2)/d/(-e*c*x+(-c^2*e*d)^(1/2))*ln(2*
((-c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(e*c*x+(-
c^2*e*d)^(1/2))+3/16*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
/(c^2*d+e)/(e*c*x+(-c^2*e*d)^(1/2))/(-c^2*d+e)/e^(1/2)/(-e*c*x+(-c^2*e*d)
^(1/2))*ln(2*((-c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x
-e)/(e*c*x+(-c^2*e*d)^(1/2))+3/16*c^3*b*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c
^2/x^2)^(1/2)*x/(c^2*d+e)/(e*c*x+(-c^2*e*d)^(1/2))/(-c^2*d+e)/e^(1/2)/d/(
-e*c*x+(-c^2*e*d)^(1/2))*ln(-2*((-c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-
c^2*e*d)^(1/2)*c*x-e)/(-e*c*x+(-c^2*e*d)^(1/2))+3/16*c^3*b*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e)/(e*c*x+(-c^2*e*d)^(1/2))/(-c^2*
d+e)/e^(1/2)/(-e*c*x+(-c^2*e*d)^(1/2))*ln(-2*((-c^2*d+e)/e)^(1/2)*(c^2*x^
2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-e*c*x+(-c^2*e*d)^(1/2))-1/4*c*b*(c^
2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d^2/(c^2*d+e)/(e*c*x+(-c^2*e*d
)^(1/2))/(-e*c*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))*e^2-1/4*c*b*
(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(e*c*x+(-c^2*e*d
)^(1/2))/(-e*c*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))*e-1/8*c^3*b
/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(c^2*d+e)/(e*c*x+(-c^2*e*d)^(1/2))/(-e*c*x
+(-c^2*e*d)^(1/2))*e+1/8*c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(e*c
*x+(-c^2*e*d)^(1/2))/(-e*c*x+(-c^2*e*d)^(1/2))*e+1/8*c*b*(c^2*x^2-1)^(1/2)/
((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d^2/(c^2*d+e)/(e*c*x+(-c^2*e*d)^(1/2))/(-c^2
*d+e)/e^(1/2)/(-e*c*x+(-c^2*e*d)^(1/2))*ln(2*((-c^2*d+e)/e)^(1/2)*(c^2*x^
2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(e*c*x+(-c^2*e*d)^(1/2))*e^2+1/8*c*b*
(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(e*c*x+(-c^2*e*d
)^(1/2))/(-c^2*d+e)/e^(1/2)/(-e*c*x+(-c^2*e*d)^(1/2))*ln(2*((-c^2*d+e)/
e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(e*c*x+(-c^2*e*d)^(1/2
))*e+1/8*c*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d^2/(c^2*d+e)
/(e*c*x+(-c^2*e*d)^(1/2))/(-c^2*d+e)/e^(1/2)/(-e*c*x+(-c^2*e*d)^(1/2))*ln
(-2*((-c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-e*c
*x+(-c^2*e*d)^(1/2))*e^2+1/8*c*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(
1/2)/x/d/(c^2*d+e)/(e*c*x+(-c^2*e*d)^(1/2))/(-c^2*d+e)/e^(1/2)/(-e*c*x+(-
c^2*e*d)^(1/2))*ln(-2*((-c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(
```

$(1/2)*c*x-e)/(-e*c*x+(-c^2*e*d)^(1/2)))*e$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\left( (c^2 e^3 x^4 + 2 c^2 d e^2 x^2 + c^2 d^2 e) \int \frac{x e^{\left( \frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1) \right)}}{c^2 e^3 x^6 + (2 c^2 d e^2 - e^3) x^4 + (c^2 e^3 x^6 + (2 c^2 d e^2 - e^3) x^4 - d^2 e + (c^2 d^2 e - 2 d e^2) x^2)(cx+1)(cx-1) - d^2 e + (c^2 d^2 e - 2 d e^2) x^2} dx \right)}{4 (e^3 x^4 + 2 d e^2 x^2 + d^2 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out]  $-1/4*(4*(c^2*e^3*x^4 + 2*c^2*d*e^2*x^2 + c^2*d^2*e)*integrate(1/4*x*e^{(1/2*\log(c*x + 1) + 1/2*\log(c*x - 1))}/(c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^{(\log(c*x + 1) + \log(c*x - 1))}, x) + \arctan2(1, \sqrt{c*x + 1}*\sqrt{c*x - 1}))*b/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e) - 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)$

**Fricas [B]** time = 6.34089, size = 1839, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out]  $[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{(-c^2*d*e - e^2)*\log((c^2*e*x^2 - c^2*d - 2*\sqrt{-c^2*d*e - e^2})*\sqrt{c^2*x^2 - 1} - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*\arccsc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) - 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}]/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 - (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*\sqrt{c^2*d*e + e^2}*\arctan(\sqrt{c^2*d*e + e^2}*\sqrt{c^2*x^2 - 1})/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*\arccsc(c*x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\arctan(-c*x + \sqrt{c^2*x^2 - 1}) - (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\sqrt{c^2*x^2 - 1}]/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^3, x)
```



$$3.114 \quad \int \frac{a+b \csc^{-1}(cx)}{x(d+ex^2)^3} dx$$

**Optimal.** Leaf size=704

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2d^3} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2d^3} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2d^3} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2d^3}$$

```
[Out] -(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(8*d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(a + b*ArcCsc[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsc[c*x]))/(d^3*(e + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d^3) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)]/(d^3*Sqrt[c^2*d + e]) - (b*Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)]/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

**Rubi [A]** time = 1.31769, antiderivative size = 704, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 11, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$ , Rules used = {5241, 4733, 4729, 382, 377, 205, 4741, 4519, 2190, 2279, 2391}

$$\frac{ibPolyLog\left(2, -\frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2d^3} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{2d^3} + \frac{ibPolyLog\left(2, -\frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2d^3} + \frac{ibPolyLog\left(2, \frac{ic\sqrt{-de}^i \csc^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3), x]
```

```
[Out] -(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(8*d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(a + b*ArcCsc[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsc[c*x]))/(d^3*(e + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d^3) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)]/(d^3*Sqrt[c^2*d + e]) - (b*Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)]/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

$\ast x)))/(\text{Sqrt}[e] + \text{Sqrt}[c^2d + e]))/d^3 + ((I/2)\ast b\ast \text{PolyLog}[2, (I\ast c\ast \text{Sqrt}[-d] \ast E^{I\ast \text{ArcCsc}[c\ast x]})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2d + e])))/d^3$

#### Rule 5241

$\text{Int}[(a + \text{ArcCsc}[c\ast x])\ast (b)^n\ast (x)^m\ast ((d) + (e)\ast (x)^2)^p, x\_Symbol] \text{ :> } -\text{Subst}[\text{Int}[(e + d\ast x^2)^p\ast (a + b\ast \text{ArcSin}[x/c])^n]/x^{m+2\ast(p+1)}, x], x, 1/x] \text{ ; FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[p]$

#### Rule 4733

$\text{Int}[(a + \text{ArcSin}[c\ast x])\ast (b)^n\ast ((f)\ast (x))^m\ast ((d) + (e)\ast (x)^2)^p, x\_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(a + b\ast \text{ArcSin}[c\ast x])^n, (f\ast x)^m\ast (d + e\ast x^2)^p, x], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[c^2d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 4729

$\text{Int}[(a + \text{ArcSin}[c\ast x])\ast (b)\ast (x)\ast ((d) + (e)\ast (x)^2)^p, x\_Symbol] \text{ :> } \text{Simp}[(d + e\ast x^2)^{p+1}\ast (a + b\ast \text{ArcSin}[c\ast x])]/(2\ast e\ast (p+1)), x] - \text{Dist}[(b\ast c)/(2\ast e\ast (p+1)), \text{Int}[(d + e\ast x^2)^{p+1}/\text{Sqrt}[1 - c^2\ast x^2], x], x] \text{ ; FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \ \text{NeQ}[c^2d + e, 0] \ \&\& \ \text{NeQ}[p, -1]$

#### Rule 382

$\text{Int}[(a + (b)\ast (x)^n)^p\ast ((c) + (d)\ast (x)^n)^q, x\_Symbol] \text{ :> } -\text{Simp}[(b\ast x\ast (a + b\ast x^n)^{p+1}\ast (c + d\ast x^n)^{q+1})/(a\ast n\ast (p+1)\ast (b\ast c - a\ast d)), x] + \text{Dist}[(b\ast c + n\ast (p+1)\ast (b\ast c - a\ast d))/(a\ast n\ast (p+1)\ast (b\ast c - a\ast d)), \text{Int}[(a + b\ast x^n)^{p+1}\ast (c + d\ast x^n)^q, x], x] \text{ ; FreeQ}\{a, b, c, d, n, q\}, x \ \&\& \ \text{NeQ}[b\ast c - a\ast d, 0] \ \&\& \ \text{EqQ}[n\ast (p+q+2) + 1, 0] \ \&\& \ (\text{LtQ}[p, -1] \ \|\ \text{!LtQ}[q, -1]) \ \&\& \ \text{NeQ}[p, -1]$

#### Rule 377

$\text{Int}[(a + (b)\ast (x)^n)^p/((c) + (d)\ast (x)^n), x\_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(c - (b\ast c - a\ast d)\ast x^n), x], x, x/(a + b\ast x^n)^{1/n}] \text{ ; FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b\ast c - a\ast d, 0] \ \&\& \ \text{EqQ}[n\ast p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$

#### Rule 205

$\text{Int}[(a + (b)\ast (x)^2)^{-1}, x\_Symbol] \text{ :> } \text{Simp}[(\text{Rt}[a/b, 2]\ast \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

#### Rule 4741

$\text{Int}[(a + \text{ArcSin}[c\ast x])\ast (b)^n/((d) + (e)\ast (x)), x\_Symbol] \text{ :> } \text{Subst}[\text{Int}[(a + b\ast x)^n\ast \text{Cos}[x]/(c\ast d + e\ast \text{Sin}[x]), x], x, \text{ArcSin}[c\ast x]] \text{ ; FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{IGtQ}[n, 0]$

#### Rule 4519

$\text{Int}[(\text{Cos}[(c) + (d)\ast (x)])\ast ((e) + (f)\ast (x))^m]/((a) + (b)\ast \text{Sin}[(c) + (d)\ast (x)]), x\_Symbol] \text{ :> } -\text{Simp}[(I\ast (e + f\ast x)^{m+1})/(b\ast f\ast (m+1)), x] + (\text{Int}[(e + f\ast x)^m\ast E^{I\ast (c + d\ast x)})/(a - \text{Rt}[a^2 - b^2, 2] - I\ast b\ast E^{I\ast (c + d\ast x)}), x] + \text{Int}[(e + f\ast x)^m\ast E^{I\ast (c + d\ast x)})/(a + \text{Rt}[a^2 - b^2, 2] - I\ast b\ast E^{I\ast (c + d\ast x)}), x]) \text{ ; FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{PosQ}[a^2 - b^2]$

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{x^5 (a + b \sin^{-1}(\frac{x}{c}))}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{e^2 x (a + b \sin^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)^3} - \frac{2ex (a + b \sin^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)^2} + \frac{x (a + b \sin^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{x^{(a+b \sin^{-1}(\frac{x}{c}))}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \frac{x^{(a+b \sin^{-1}(\frac{x}{c}))}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \text{Subst} \left( \int \frac{x^{(a+b \sin^{-1}(\frac{x}{c}))}}{(e+dx^2)} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{e^2 (a + b \csc^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \csc^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} - \frac{\text{Subst} \left( \int \left( -\frac{\sqrt{-d}(a+b \sin^{-1}(\frac{x}{c}))}{2d(\sqrt{e}-\sqrt{-dx})} + \frac{\sqrt{-d}(a+b \sin^{-1}(\frac{x}{c}))}{2d(\sqrt{e}+\sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \csc^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \csc^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} - \frac{\text{Subst} \left( \int \frac{a+b \sin^{-1}(\frac{x}{c})}{\sqrt{e}-\sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2(-d)^{5/2}} \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \csc^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \csc^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d + e}}{c\sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}} \right)}{d^3 \sqrt{c^2 d + e}} \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \csc^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \csc^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \csc^{-1}(cx))^2}{2bd^3} + \dots \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \csc^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \csc^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \csc^{-1}(cx))^2}{2bd^3} + \dots \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \csc^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \csc^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \csc^{-1}(cx))^2}{2bd^3} + \dots \\
&= -\frac{bce \sqrt{1 - \frac{1}{c^2 x^2}}}{8d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 (a + b \csc^{-1}(cx))}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \csc^{-1}(cx))}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{i (a + b \csc^{-1}(cx))^2}{2bd^3} + \dots
\end{aligned}$$

**Mathematica [F]** time = 60.225, size = 0, normalized size = 0.

$$\int \frac{a + b \csc^{-1}(cx)}{x(d + ex^2)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^3), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^3), x]

**Maple [C]** time = 1.979, size = 5294, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^3,x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} a \left( \frac{2ex^2 + 3d}{d^2e^2x^4 + 2d^3ex^2 + d^4} - \frac{2 \log(ex^2 + d)}{d^3} + \frac{4 \log(x)}{d^3} \right) + b \int \frac{\arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right)}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4\*a\*((2\*e\*x^2 + 3\*d)/(d^2\*e^2\*x^4 + 2\*d^3\*e\*x^2 + d^4) - 2\*log(e\*x^2 + d)/d^3 + 4\*log(x)/d^3) + b\*integrate(arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^3*x), x)
```

$$3.115 \quad \int \frac{x^4 (a + b \csc^{-1}(cx))}{(d + ex^2)^3} dx$$

**Optimal.** Leaf size=1144

result too large to display

```
[Out] -(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2))
```

**Rubi [A]** time = 1.60013, antiderivative size = 1144, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 11, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$ , Rules used = {5241, 4667, 4743, 731, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2))
```

```
t[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])))/(16*S
qrt[d]*e^2*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*
Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])))/(16*Sqrt[d]*e*(c^2*d + e)^
(3/2)) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
+ e]*Sqrt[1 - 1/(c^2*x^2)])))/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (3*(a + b*
ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d
+ e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) -
(3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 +
(I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]
*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(
5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[
e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*
Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)
)
```

### Rule 5241

```
Int[((a_) + ArcCsc[(c_)*(x_)])*(b_)^(n_)*(x_)^(m_)*((d_) + (e_)*(x_)
^2)^(p_), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

### Rule 4667

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((d_) + (e_)*(x_)^2)^(p_), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

### Rule 4743

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((d_) + (e_)*(x_)^2)^(m_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

### Rule 731

```
Int[((d_) + (e_)*(x_)^2)^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

### Rule 725

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```



Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol]
:> Subst[Int[((a + b*x)^n*cos[x])/(c*d + e*sin[x]), x], x, ArcSin[c*x]] /;
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps



```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcCsc[c*x]/((-I)*Sqrt[
d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
Sqrt[e] + c*(-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))
)/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x)))/Sqrt[-(c^2*d) - e])/Sqrt[d]
))/(16*e^2) + (5*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(
c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(-I)*c*Sqrt[d] + Sqrt
[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x])/Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*
Sqrt[e]*x)))/Sqrt[-(c^2*d) - e])/Sqrt[d]))/(16*e^2) + ((I/16)*Sqrt[d]*((I*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqr
t[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c
*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqr
t[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
+ e)*((-I)*Sqrt[d] + Sqrt[e]*x)))]/(d*(c^2*d + e)^(3/2))))/e^2 - ((I/16)*Sq
rt[d]*(((I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqr
t[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcS
in[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e
]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)))]/(d*(c^2*d + e)^(3/2))))/e^2 - (3
*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e]
)/(c*Sqrt[d]])]/Sqrt[2]]*ArcTan[(((I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcC
sc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqr
t[c^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d]])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e))/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e))/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c
*Sqrt[d]])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e))/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Lo
g[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e))/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e))/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x]))]/(128*
Sqrt[d]*e^(5/2)) + (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d]])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e]
)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e]
+ Sqrt[c^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log
[1 + (-Sqrt[e] + Sqrt[c^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*A
rcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d]])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c
^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (S
qrt[e] + Sqrt[c^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sq
rt[1 + (I*Sqrt[e])/(c*Sqrt[d]])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCs
c[c*x])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] -
Sqrt[c^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + S
qrt[c^2*d + e))/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcC
sc[c*x]))]/(128*Sqrt[d]*e^(5/2)))
```

---

**Maple [C]** time = 2.188, size = 3175, normalized size = 2.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

[Out] 
$$\begin{aligned}
& -1/8*c^5*b*x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e*((c^2*x^2-1)/c^2/x^2)^{(1/2)*} \\
& d+3/8*a/e^2/(d*e)^{(1/2)*}\arctan(e*x/(d*e)^{(1/2)})+1/c^4*b*((c^2*d+2*(e*(c^2*d \\
& +e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2* \\
& (e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)-5/4}/ \\
& c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x+(1-1/c \\
& ^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)^2/e/d \\
& ^2*(e*(c^2*d+e))^{(1/2)-3/4}/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/ \\
& 2)*}\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2* \\
& e}*d)^{(1/2)})/e^2/(c^2*d+e)/d^2*(e*(c^2*d+e))^{(1/2)+3/4}/c^2*b*((c^2*d+2*(e*( \\
& c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2 \\
& *d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/e^2/(c^2*d+e)/d^2*(e*(c^2*d+e))^{(1/ \\
& 2)-5/8*c^6*b*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e*\operatorname{arccsc}(c*x)*d-3/8*c^6*b*x/ \\
& (c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e^2*\operatorname{arccsc}(c*x)*d^2-3/8*c^4*b*x/(c^2*e*x^2+c^ \\
& 2*d)^2/(c^2*d+e)/e*\operatorname{arccsc}(c*x)*d-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e} \\
& )*d)^{(1/2)*}\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{( \\
& 1/2)-2*e}*d)^{(1/2)})/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)+5/4}/c^2*b*(-(c^2*d \\
& -2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}) \\
& /((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2/e/d^2*(e*(c^2*d+ \\
& e))^{(1/2)-5/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e*x^3-5/4/c^2*b*((c^2*d+2*(e*(c^2*d \\
& +e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2* \\
& (e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/e/(c^2*d+e)/d^2+1/c^4*b*((c^2*d+2*(e*(c^ \\
& 2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d \\
& +2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)^2*e/d^3-3/8*b*((c^2*d+2*(e* \\
& (c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^ \\
& 2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^{(1 \\
& /2)+3/8*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\arctan(d*c*(I/c/x+(1 \\
& -1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/e^2/(c^2*d \\
& +e)^2/d*(e*(c^2*d+e))^{(1/2)-5/4}/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d \\
& )^{(1/2)*}\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/ \\
& 2)-2*e}*d)^{(1/2)})/e/(c^2*d+e)/d^2+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2* \\
& e}*d)^{(1/2)*}\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e)) \\
& ^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2*e/d^3+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1 \\
& /2)+2*e}*d)^{(1/2)*}\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2 \\
& *d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^{(1/2)-1/c^4*b*((c \\
& ^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{( \\
& 1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^3*(e*(c^2* \\
& d+e))^{(1/2)-3/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e^2*d*x-1/8*c^5*b*x^4/(c^2*e*x^2+ \\
& c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^{(1/2)+7/4}/c^2*b*(-(c^2*d-2*(e*(c^2 \\
& *d+e))^{(1/2)+2*e}*d)^{(1/2)*}\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+ \\
& 2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^2-3/16*c^3*b/e^2/(c^2*d+ \\
& e)*d*\operatorname{sum}(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\operatorname{arccsc}(c*x)*\ln((_R1-I/c/x-(1-1/c^ \\
& 2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c \\
& ^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*c^3*b/e^2/(c^2*d+e)*d*\operatorname{sum}(_R1/(_ \\
& R1^2*c^2*d-c^2*d-2*e)*(I*\operatorname{arccsc}(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1 \\
& )+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2 \\
& *d-4*e)*_Z^2+c^2*d))-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{ar \\
& ctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d) \\
& ^{(1/2)})/(c^2*d+e)/d^3-5/8*c^4*b*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\operatorname{arccsc}(c* \\
& x)+7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/c/x \\
& +(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e \\
& )^2/d^2-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\arctan(d*c*(I/ \\
& c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2 \\
& *d+e)/d^3+3/4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*c*(I/ \\
& c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/e/(c^ \\
& 2*d+e)^2/d-3/8*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\arctan(d*c*(I \\
& /c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/e^2 \\
& /((c^2*d+e)/d+3/4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\arctan(d*c* \\
& (I/c/x+(1-1/c^2/x^2)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/e \\
& /((c^2*d+e)^2/d-3/8*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)*}\operatorname{arctanh}(d*
\end{aligned}$$

$$c*(I/c/x+(1-1/c^2/x^2)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}/e^2/(c^2*d+e)/d-3/16*c*b/e/(c^2*d+e)*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*c*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arccsc}(cx) + ax^4}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*x^4\*arccsc(c\*x) + a\*x^4)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^4}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^4/(e\*x^2 + d)^3, x)

$$3.116 \quad \int \frac{x^2(a+b \csc^{-1}(cx))}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=1144

result too large to display

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) - (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2))
```

**Rubi [A]** time = 3.06887, antiderivative size = 1144, normalized size of antiderivative = 1., number of steps used = 63, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5241, 4733, 4667, 4743, 731, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+16$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) - (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2))
```

$$2*d + e] + (b*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(16*d^{(3/2)}*(c^2*d + e)^{(3/2)}) - (b*\text{ArcTanh}[(c^2*d + (\text{Sqrt}[-d]*\text{Sqrt}[e])/x)/(c*\text{Sqrt}[d]*\text{Sqrt}[c^2*d + e]*\text{Sqrt}[1 - 1/(c^2*x^2)])))/(16*d^{(3/2)}*e*\text{Sqrt}[c^2*d + e]) + ((a + b*\text{ArcCsc}[c*x])*\text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(16*(-d)^{(3/2)}*e^{(3/2)}) - ((a + b*\text{ArcCsc}[c*x])*\text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(16*(-d)^{(3/2)}*e^{(3/2)}) + ((a + b*\text{ArcCsc}[c*x])*\text{Log}[1 - (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(16*(-d)^{(3/2)}*e^{(3/2)}) - ((a + b*\text{ArcCsc}[c*x])*\text{Log}[1 + (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})]/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(16*(-d)^{(3/2)}*e^{(3/2)}) + ((I/16)*b*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/( (-d)^{(3/2)}*e^{(3/2)}) - ((I/16)*b*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/( (-d)^{(3/2)}*e^{(3/2)}) + ((I/16)*b*\text{PolyLog}[2, ((-I)*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/( (-d)^{(3/2)}*e^{(3/2)}) - ((I/16)*b*\text{PolyLog}[2, (I*c*\text{Sqrt}[-d]*E^{(I*\text{ArcCsc}[c*x])})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/( (-d)^{(3/2)}*e^{(3/2)})$$
**Rule 5241**

$$\text{Int}[(a + \text{ArcCsc}[c*x])*(b*x)^n*(d + e*x^2)^p, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcSin}[x/c])^n/x^{m+2*(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[p]$$
**Rule 4733**

$$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*(f*x)^m*(d + e*x^2)^p, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$$
**Rule 4667**

$$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*(d + e*x^2)^p, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[p] \&\& (\text{GtQ}[p, 0] \parallel \text{IGtQ}[n, 0])$$
**Rule 4743**

$$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*(d + e*x^2)^m, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{ArcSin}[c*x])^n/(e*(m+1)), x] - \text{Dist}[(b*c^n)/(e*(m+1)), \text{Int}[(d + e*x)^{m+1}*(a + b*\text{ArcSin}[c*x])^{n-1}]/\text{Sqrt}[1 - c^2*x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$$
**Rule 731**

$$\text{Int}[(d + e*x)^m*(a + c*x^2)^p, x\_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{m+1}*(a + c*x^2)^{p+1})/((m+1)*(c*d^2 + a*e^2)), x] + \text{Dist}[(c*d)/(c*d^2 + a*e^2), \text{Int}[(d + e*x)^{m+1}*(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, m, p, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[m + 2*p + 3, 0]$$
**Rule 725**

$$\text{Int}[1/((d + e*x)*\text{Sqrt}[a + c*x^2]), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}\{a, c, d, e, x\}$$

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4741

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)), x_Symbol]
:= Subst[Int[((a + b*x)^n*cos[x])/(c*d + e*sin[x]), x], x, ArcSin[c*x]] /;
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4519

```
Int[(Cos[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_))/((a_) + (b_)*Sin[
(c_) + (d_)*(x_)]), x_Symbol] := -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps



$$\begin{aligned}
\int \frac{x^2 (a + b \csc^{-1}(cx))}{(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{x^2 (a + b \sin^{-1}(\frac{x}{c}))}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{e(a + b \sin^{-1}(\frac{x}{c}))}{d(e + dx^2)^3} + \frac{a + b \sin^{-1}(\frac{x}{c})}{d(e + dx^2)^2} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(e + dx^2)^3} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{\text{Subst} \left( \int \left( -\frac{d(a + b \sin^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e-dx})^2} - \frac{d(a + b \sin^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e+dx})^2} - \frac{d(a + b \sin^{-1}(\frac{x}{c}))}{2e(-de - d^2x^2)} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \text{Subst} \left( \int \left( -\frac{d(a + b \sin^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e-dx})^2} - \frac{d(a + b \sin^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e+dx})^2} - \frac{d(a + b \sin^{-1}(\frac{x}{c}))}{2e(-de - d^2x^2)} \right) dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{3 \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e-dx})^2} dx, x, \frac{1}{x} \right)}{16e} - \frac{3 \text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e+dx})^2} dx, x, \frac{1}{x} \right)}{16e} + \frac{\text{Subst} \left( \int \frac{a + b \sin^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e-dx})^2} dx, x, \frac{1}{x} \right)}{4e} \\
&= \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)^2} + \frac{a + b \csc^{-1}(cx)}{16de \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)^2} - \frac{a + b \csc^{-1}(cx)}{16de \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)}
\end{aligned}$$

**Mathematica [A]** time = 6.12799, size = 2075, normalized size = 1.81

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^3,x]

[Out] 
$$-(a*x)/(4*e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^{3/2}*e^{3/2}) + b*(-(-ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x])/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - (-ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x])/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)))]/(d*(c^2*d + e)^{3/2}))/Sqrt[d]*e) + ((I/16)*((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)))]/(d*(c^2*d + e)^{3/2}))/Sqrt[d]*e) - (Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*ArcTan[(((I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -(Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d^{3/2}*e^{3/2}) + (Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*ArcTan[(((I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d^{3/2}*e^{3/2}))$$

**Maple [C]** time = 2.568, size = 2329, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (x^2*(a+b*\arccsc(c*x))/(e*x^2+d)^3, x)$

[Out] 
$$\begin{aligned} & 1/8*c^5*b*x^4/(c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*((c^2*x^2-1)/c^2/x^2)^{(1/2)} \\ & -1/8*c^6*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e*\arccsc(c*x)*d-1/4/c^2*b*(-(c^2 \\ & *d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)} \\ & ))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}/e/(c^2*d+e)/d^3*(e*(c^2*d+ \\ & e))^{(1/2)}+1/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(d*c \\ & *(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}/e \\ & /((c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)}+1/8*c^4*b*x^3/(c^2*e*x^2+c^2*d)^2*e/(c^2 \\ & *d+e)/d*\arccsc(c*x)+1/8*c^5*b*x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1 \\ & )/c^2/x^2)^{(1/2)}-1/8*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\arctan( \\ & d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} \\ & ))/e/(c^2*d+e)/d^2-1/8*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctan} \\ & h(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \\ & ))/e/(c^2*d+e)/d^2-1/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{ar} \\ & \operatorname{rctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d \\ & )^{(1/2)}/(c^2*d+e)/d^3-1/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \\ & * \operatorname{arctan}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2* \\ & e)*d)^{(1/2)}/(c^2*d+e)/d^3+1/8*c^6*b*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\operatorname{arcc} \\ & \operatorname{sc}(c*x)-1/8*c^4*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\operatorname{arccsc}(c*x)+1/8*c^4*a/(c^ \\ & 2*e*x^2+c^2*d)^2/d*x^3-1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e*x+1/8*a/d/e/(d*e)^{(1 \\ & /2)}*\operatorname{arctan}(e*x/(d*e))^{(1/2)}+1/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d \\ & )^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)} \\ & )+2*e)*d)^{(1/2)}/(c^2*d+e)^2*e/d^3+1/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)} \\ & +2*e)*d)^{(1/2)}*\operatorname{arctan}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+ \\ & e))^{(1/2)}-2*e)*d)^{(1/2)}/(c^2*d+e)^2*e/d^3+1/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e) \\ & ))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctan}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e \\ & *(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^{(1/2)}-1/4/c^ \\ & 2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2 \\ & /x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}/(c^2*d+e)^2/d^3*( \\ & e*(c^2*d+e))^{(1/2)}+1/8*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arcta} \\ & n(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1 \\ & /2)}/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^{(1/2)}-1/8*b*((c^2*d+2*(e*(c^2*d+e))^{(1 \\ & /2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2 \\ & *d+e))^{(1/2)}+2*e)*d)^{(1/2)}/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^{(1/2)}+1/4*b*(-( \\ & c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctan}(d*c*(I/c/x+(1-1/c^2/x^2)^{( \\ & 1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)}/(c^2*d+e)^2/d^2+1/4*b*( \\ & (c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2) \\ & )^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}/(c^2*d+e)^2/d^2-1/16*c \\ & *b/d/(c^2*d+e)*\operatorname{sum}(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\operatorname{arccsc}(c*x)*\ln((_R1-I/c \\ & /x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R \\ & 1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/16*c^3*b/e/(c^2*d+e)*\operatorname{sum}( \\ & _R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\operatorname{arccsc}(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)} \\ & ))/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+( \\ & -2*c^2*d-4*e)*_Z^2+c^2*d))-1/16*c^3*b/e/(c^2*d+e)*\operatorname{sum}(1/_R1/(_R1^2*c^2*d-c^ \\ & 2*d-2*e)*(I*\operatorname{arccsc}(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1- \\ & I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+ \\ & c^2*d))-1/16*c*b/d/(c^2*d+e)*\operatorname{sum}(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\operatorname{arccsc}(c*x) \\ & )*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/ \\ & 2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arccsc}(cx) + ax^2}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsc(c\*x) + a\*x^2)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^2/(e\*x^2 + d)^3, x)

$$3.117 \quad \int \frac{a+b \csc^{-1}(cx)}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=1134

result too large to display

```
[Out] -(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

**Rubi [A]** time = 4.04886, antiderivative size = 1134, normalized size of antiderivative = 1., number of steps used = 81, number of rules used = 12, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {5231, 4733, 4667, 4743, 731, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^3, x]
```

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[Out] -(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

$$d - (\text{Sqrt}[-d] \cdot \text{Sqrt}[e])/x / (c \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[c^2 d + e] \cdot \text{Sqrt}[1 - 1/(c^2 x^2)]) \\ ] / (16 d^{5/2} \text{Sqrt}[c^2 d + e]) - (b e \cdot \text{ArcTanh}[(c^2 d + (\text{Sqrt}[-d] \cdot \text{Sqrt}[e])/x) / (c \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[c^2 d + e] \cdot \text{Sqrt}[1 - 1/(c^2 x^2)])]) / (16 d^{5/2} (c^2 d + e)^{3/2}) + (5 b \cdot \text{ArcTanh}[(c^2 d + (\text{Sqrt}[-d] \cdot \text{Sqrt}[e])/x) / (c \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[c^2 d + e] \cdot \text{Sqrt}[1 - 1/(c^2 x^2)])]) / (16 d^{5/2} \text{Sqrt}[c^2 d + e]) - (3(a + b \cdot \text{ArcCsc}[c x]) \cdot \text{Log}[1 - (I c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 d + e])]) / (16 (-d)^{5/2} \text{Sqrt}[e]) + (3(a + b \cdot \text{ArcCsc}[c x]) \cdot \text{Log}[1 + (I c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 d + e])]) / (16 (-d)^{5/2} \text{Sqrt}[e]) - (3(a + b \cdot \text{ArcCsc}[c x]) \cdot \text{Log}[1 - (I c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 d + e])]) / (16 (-d)^{5/2} \text{Sqrt}[e]) + (3(a + b \cdot \text{ArcCsc}[c x]) \cdot \text{Log}[1 + (I c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 d + e])]) / (16 (-d)^{5/2} \text{Sqrt}[e]) - (((3 I)/16) b \cdot \text{PolyLog}[2, ((-I) c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 d + e])]) / ((-d)^{5/2} \text{Sqrt}[e]) + (((3 I)/16) b \cdot \text{PolyLog}[2, (I c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 d + e])]) / ((-d)^{5/2} \text{Sqrt}[e]) - (((3 I)/16) b \cdot \text{PolyLog}[2, ((-I) c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 d + e])]) / ((-d)^{5/2} \text{Sqrt}[e]) + (((3 I)/16) b \cdot \text{PolyLog}[2, (I c \cdot \text{Sqrt}[-d] \cdot E^{(I \cdot \text{ArcCsc}[c x])}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 d + e])]) / ((-d)^{5/2} \text{Sqrt}[e])$$
Rule 5231

$$\text{Int}[(a + \text{ArcCsc}[c(x)] \cdot (b))^{(n)} \cdot ((d) + (e) \cdot (x)^2)^{(p)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d x^2)^p (a + b \cdot \text{ArcSin}[x/c])^n / x^{2(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$$
Rule 4733

$$\text{Int}[(a + \text{ArcSin}[c(x)] \cdot (b))^{(n)} \cdot ((f) \cdot (x))^m \cdot ((d) + (e) \cdot (x)^2)^{(p)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSin}[c x])^n, (f x)^m (d + e x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[c^2 d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$
Rule 4667

$$\text{Int}[(a + \text{ArcSin}[c(x)] \cdot (b))^{(n)} \cdot ((d) + (e) \cdot (x)^2)^{(p)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSin}[c x])^n, (d + e x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{NeQ}[c^2 d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (\text{GtQ}[p, 0] \ || \ \text{IGtQ}[n, 0])$$
Rule 4743

$$\text{Int}[(a + \text{ArcSin}[c(x)] \cdot (b))^{(n)} \cdot ((d) + (e) \cdot (x))^m, x\_Symbol] \rightarrow \text{Simp}[(d + e x)^{(m+1)} (a + b \cdot \text{ArcSin}[c x])^n / (e(m+1)), x] - \text{Dist}[(b c^n) / (e(m+1)), \text{Int}[(d + e x)^{(m+1)} (a + b \cdot \text{ArcSin}[c x])^{(n-1)}] / \text{Sqrt}[1 - c^2 x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$$
Rule 731

$$\text{Int}[(d) + (e) \cdot (x))^m \cdot (a + (c) \cdot (x)^2)^{(p)}, x\_Symbol] \rightarrow \text{Simp}[(e(d + e x)^{(m+1)} (a + c x^2)^{(p+1})) / ((m+1)(c d^2 + a e^2)), x] + \text{Dist}[(c d) / (c d^2 + a e^2), \text{Int}[(d + e x)^{(m+1)} (a + c x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, m, p\}, x \ \&\& \ \text{NeQ}[c d^2 + a e^2, 0] \ \&\& \ \text{EqQ}[m + 2 p + 3, 0]$$
Rule 725

$$\text{Int}[1 / (((d) + (e) \cdot (x)) \cdot \text{Sqrt}[(a) + (c) \cdot (x)^2]), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1 / (c d^2 + a e^2 - x^2), x], x, (a e - c d x) / \text{Sqrt}[a + c x^2]] /; \text{FreeQ}\{a, c, d, e\}, x]$$

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 4741

Int[((a\_.) + ArcSin[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Subst[Int[((a + b\*x)^n\*cos[x])/(c\*d + e\*sin[x]), x], x, ArcSin[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 4519

Int[(Cos[(c\_.) + (d\_.)\*(x\_)])\*((e\_.) + (f\_.)\*(x\_))^(m\_.)]/((a\_) + (b\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(e + f\*x)^(m + 1))/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a - Rt[a^2 - b^2, 2] - I\*b\*E^(I\*(c + d\*x))), x] + Int[((e + f\*x)^m\*E^(I\*(c + d\*x)))/(a + Rt[a^2 - b^2, 2] - I\*b\*E^(I\*(c + d\*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] & & PosQ[a^2 - b^2]

Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)]/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^3} dx &= -\text{Subst} \left( \int \frac{x^4 \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left( \int \left( \frac{e^2 \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^3} - \frac{2e \left( a + b \sin^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^2} + \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{\text{Subst} \left( \int \left( \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \text{Subst} \left( \int \left( -\frac{d(a + b \sin^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \sin^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{3 \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{16d} + \frac{3 \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{16d} + \frac{3 \text{Subst} \left( \int \frac{a + b \sin^{-1} \left( \frac{x}{c} \right)}{-de - d^2 x^2} dx, x, \frac{1}{x} \right)}{8d} \\
&= \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)^2} - \frac{5(a + b \csc^{-1}(cx))}{16d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)^2} + \frac{5(a + b \csc^{-1}(cx))}{16d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e}\sqrt{1 - \frac{1}{c^2 x^2}}}{16(-d)^{3/2} (c^2 d + e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \csc^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)}
\end{aligned}$$



**Mathematica [A]** time = 6.06086, size = 2060, normalized size = 1.82

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x^2)^3,x]

[Out] 
$$\frac{a x}{4 d (d + e x^2)^2} + \frac{3 a x}{8 d^2 (d + e x^2)} + \frac{3 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{8 d^{5/2} \sqrt{e}} + b \left( \frac{-3 \left( -\operatorname{ArcCsc}[c x] / \left( (-1) \sqrt{d} \sqrt{e} + e x \right) \right) + \left( \operatorname{ArcSin}\left[1 / (c x)\right] / \sqrt{e} - \operatorname{Log}\left[\frac{2 \sqrt{d} \sqrt{e} (\sqrt{e} + c (-1) c \sqrt{d} - \sqrt{-(c^2 d) - e}) \sqrt{1 - 1 / (c^2 x^2)}}{\sqrt{-(c^2 d) - e} (\sqrt{d} + I \sqrt{e} x)}\right]}{\sqrt{-(c^2 d) - e}}\right)}{\sqrt{d}} \right) / \left( 16 d^2 - \left( 3 \left( -\operatorname{ArcCsc}[c x] / \left( I \sqrt{d} \sqrt{e} + e x \right) \right) - \left( \operatorname{ArcSin}\left[1 / (c x)\right] / \sqrt{e} - \operatorname{Log}\left[\frac{2 \sqrt{d} \sqrt{e} (-\sqrt{e} + c (-1) c \sqrt{d} + \sqrt{-(c^2 d) - e}) \sqrt{1 - 1 / (c^2 x^2)}}{\sqrt{-(c^2 d) - e} (\sqrt{d} - I \sqrt{e} x)}\right]}{\sqrt{-(c^2 d) - e}}\right)}{\sqrt{d}} \right) \right) / \left( 16 d^2 + \left( \frac{I}{16} \right) \left( \frac{I c \sqrt{e} \sqrt{1 - 1 / (c^2 x^2)} x}{\sqrt{d} (c^2 d + e) (-1) \sqrt{d} + \sqrt{e} x} \right) - \operatorname{ArcCsc}[c x] / \left( \sqrt{e} (-1) \sqrt{d} + \sqrt{e} x \right)^2 - \operatorname{ArcSin}\left[1 / (c x)\right] / \left( d \sqrt{e} \right) + \left( \frac{I (2 c^2 d + e) \operatorname{Log}\left[\frac{4 d \sqrt{e} \sqrt{c^2 d + e} (I \sqrt{e} + c (c \sqrt{d} - \sqrt{c^2 d + e}) \sqrt{1 - 1 / (c^2 x^2)}}{(2 c^2 d + e) (-1) \sqrt{d} + \sqrt{e} x}\right]}{d (c^2 d + e)^{3/2}} \right) / d^{3/2} - \left( \frac{I}{16} \right) \left( \frac{(-1) c \sqrt{e} \sqrt{1 - 1 / (c^2 x^2)} x}{\sqrt{d} (c^2 d + e) (I \sqrt{d} + \sqrt{e} x)} \right) - \operatorname{ArcCsc}[c x] / \left( \sqrt{e} (I \sqrt{d} + \sqrt{e} x)^2 \right) - \operatorname{ArcSin}\left[1 / (c x)\right] / \left( d \sqrt{e} \right) + \left( \frac{I (2 c^2 d + e) \operatorname{Log}\left[\frac{-4 d \sqrt{e} \sqrt{c^2 d + e} (-1) \sqrt{e} + c (c \sqrt{d} + \sqrt{c^2 d + e}) \sqrt{1 - 1 / (c^2 x^2)}}{(2 c^2 d + e) (I \sqrt{d} + \sqrt{e} x)}\right]}{d (c^2 d + e)^{3/2}} \right) / d^{3/2} - \left( 3 \left( \operatorname{Pi}^2 - 4 \operatorname{Pi} \operatorname{ArcCsc}[c x] + 8 \operatorname{ArcCsc}[c x]^2 - 32 \operatorname{ArcSin}\left[\frac{\sqrt{1 - (I \sqrt{e})}}{c \sqrt{d}}\right] \right) / \sqrt{2} \right) \operatorname{ArcTan}\left[\frac{((-1) c \sqrt{d} + \sqrt{e}) \operatorname{Cot}\left[\frac{\operatorname{Pi} + 2 \operatorname{ArcCsc}[c x]}{4}\right]}{\sqrt{c^2 d + e}} \right] + \left( 4 I \right) \operatorname{Pi} \operatorname{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] - \left( 8 I \right) \operatorname{ArcCsc}[c x] \operatorname{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + \left( 16 I \right) \operatorname{ArcSin}\left[\frac{\sqrt{1 - (I \sqrt{e})}}{c \sqrt{d}}\right] / \sqrt{2} \right) \operatorname{Log}\left[1 + \frac{\sqrt{e} - \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + \left( 4 I \right) \operatorname{Pi} \operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] - \left( 8 I \right) \operatorname{ArcCsc}[c x] \operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] - \left( 16 I \right) \operatorname{ArcSin}\left[\frac{\sqrt{1 - (I \sqrt{e})}}{c \sqrt{d}}\right] / \sqrt{2} \right) \operatorname{Log}\left[1 + \frac{\sqrt{e} + \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + \left( 8 I \right) \operatorname{ArcCsc}[c x] \operatorname{Log}\left[1 - \operatorname{E}^{((2 I) \operatorname{ArcCsc}[c x])}\right] - \left( 4 I \right) \operatorname{Pi} \operatorname{Log}\left[\frac{\sqrt{e} + (I \sqrt{d}) / x}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + 8 \operatorname{PolyLog}\left[2, \frac{-\sqrt{e} + \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + 8 \operatorname{PolyLog}\left[2, -\frac{(\sqrt{e} + \sqrt{c^2 d + e})}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + 4 \operatorname{PolyLog}\left[2, \operatorname{E}^{((2 I) \operatorname{ArcCsc}[c x])}\right] \right) / \left( 128 d^{5/2} \sqrt{e} \right) + \left( 3 \left( \operatorname{Pi}^2 - 4 \operatorname{Pi} \operatorname{ArcCsc}[c x] + 8 \operatorname{ArcCsc}[c x]^2 - 32 \operatorname{ArcSin}\left[\frac{\sqrt{1 + (I \sqrt{e})}}{c \sqrt{d}}\right] \right) / \sqrt{2} \right) \operatorname{ArcTan}\left[\frac{(I c \sqrt{d} + \sqrt{e}) \operatorname{Cot}\left[\frac{\operatorname{Pi} + 2 \operatorname{ArcCsc}[c x]}{4}\right]}{\sqrt{c^2 d + e}} \right] + \left( 4 I \right) \operatorname{Pi} \operatorname{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] - \left( 8 I \right) \operatorname{ArcCsc}[c x] \operatorname{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + \left( 16 I \right) \operatorname{ArcSin}\left[\frac{\sqrt{1 + (I \sqrt{e})}}{c \sqrt{d}}\right] / \sqrt{2} \right) \operatorname{Log}\left[1 + \frac{-\sqrt{e} + \sqrt{c^2 d + e}}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + \left( 4 I \right) \operatorname{Pi} \operatorname{Log}\left[1 - \frac{(\sqrt{e} + \sqrt{c^2 d + e})}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] - \left( 8 I \right) \operatorname{ArcCsc}[c x] \operatorname{Log}\left[1 - \frac{(\sqrt{e} + \sqrt{c^2 d + e})}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] - \left( 16 I \right) \operatorname{ArcSin}\left[\frac{\sqrt{1 + (I \sqrt{e})}}{c \sqrt{d}}\right] / \sqrt{2} \right) \operatorname{Log}\left[1 - \frac{(\sqrt{e} + \sqrt{c^2 d + e})}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + \left( 8 I \right) \operatorname{ArcCsc}[c x] \operatorname{Log}\left[1 - \operatorname{E}^{((2 I) \operatorname{ArcCsc}[c x])}\right] - \left( 4 I \right) \operatorname{Pi} \operatorname{Log}\left[\frac{\sqrt{e} - (I \sqrt{d}) / x}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + 8 \operatorname{PolyLog}\left[2, \frac{(\sqrt{e} - \sqrt{c^2 d + e})}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + 8 \operatorname{PolyLog}\left[2, \frac{(\sqrt{e} + \sqrt{c^2 d + e})}{c \sqrt{d}}\right] \operatorname{E}^{(I \operatorname{ArcCsc}[c x])} \right] + 4 \operatorname{PolyLog}\left[2, \operatorname{E}^{((2 I) \operatorname{ArcCsc}[c x])}\right] \right) / \left( 128 d^{5/2} \sqrt{e} \right)$$

Maple [C] time = 2.95, size = 3166, normalized size = 2.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\arccsc(c*x))/(e*x^2+d)^3,x)$

[Out] 
$$-1/8*c^5*b*x^2/(c^2*e*x^2+c^2*d)^2/d/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^{(1/2)*e+3/8*a/d^2/(d*e)^{(1/2)}*\arctan(e*x/(d*e)^{(1/2)})+3/8*c^2*a/d^2*x/(c^2*e*x^2+c^2*d)+1/4*c^4*a*x/d/(c^2*e*x^2+c^2*d)^2+5/8*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)/d^3+5/8*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)/d^3-3/16*c^3*b/d/(c^2*d+e)*\sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\arccsc(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*c^3*b/d/(c^2*d+e)*\sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\arccsc(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*c*b/(c^2*d+e)/d^2*e*\sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\arccsc(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*c*b/(c^2*d+e)/d^2*e*\sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*\arccsc(c*x)*\ln((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I/c/x-(1-1/c^2/x^2)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+5/8*c^6*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*\arccsc(c*x)-5/8*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^{(1/2)+5/8*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^{(1/2)-5/4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2*e/d^3-9/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e^2*\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^4-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e^3*\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^5+7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)/d^4+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e^2*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)/d^5-9/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e^2*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^4-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e^3*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)^2/d^5+5/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*\arctan(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)/d^4*(e*(c^2*d+e))^{(1/2)-5/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)/d^4+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e^2*\operatorname{arctan}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e}*d)^{(1/2)})/(c^2*d+e)/d^5-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)}))/((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)})/(c^2*d+e)/d^5*(e*(c^2*d+e))^{(1/2)+7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}*e*(c^2*d+e))^{(1/2)+2*e}*d)^{(1/2)}}$$

$$\begin{aligned} & \left. \right)^{(1/2)+2e)d}^{(1/2)*e*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)))/((c^2*d+2* \\ & (e*(c^2*d+e))^{(1/2)+2e)*d)^{(1/2)))/(c^2*d+e)^2/d^4*(e*(c^2*d+e))^{(1/2)-1/8* \\ & c^5*b*x^4/(c^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^{(1/2)*e^2 \\ & +3/8*c^4*b*x^3/(c^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*\operatorname{arccsc}(c*x)*e^2+5/8*c^4*b* \\ & x/(c^2*e*x^2+c^2*d)^2/d/(c^2*d+e)*\operatorname{arccsc}(c*x)*e+1/c^4*b*(-(c^2*d-2*(e*(c^2* \\ & d+e))^{(1/2)+2e)*d)^{(1/2)*e*\operatorname{arctan}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)))/((-c^2*d \\ & +2*(e*(c^2*d+e))^{(1/2)-2e)*d)^{(1/2)))/(c^2*d+e)/d^5*(e*(c^2*d+e))^{(1/2)-7/4 \\ & /c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2e)*d)^{(1/2)*e*\operatorname{arctan}(d*c*(I/c/x+(1- \\ & 1/c^2/x^2)^{(1/2)))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2e)*d)^{(1/2)))/(c^2*d+e)^2 \\ & /d^4*(e*(c^2*d+e))^{(1/2)-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)+2e)*d)^{(1/ \\ & 2)*e^2*\operatorname{arctan}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)))/((-c^2*d+2*(e*(c^2*d+e))^{(1/2) \\ & )-2e)*d)^{(1/2)))/(c^2*d+e)^2/d^5*(e*(c^2*d+e))^{(1/2)+1/c^4*b*((c^2*d+2*(e*( \\ & c^2*d+e))^{(1/2)+2e)*d)^{(1/2)*e^2*\operatorname{arctanh}(d*c*(I/c/x+(1-1/c^2/x^2)^{(1/2)))/(( \\ & (c^2*d+2*(e*(c^2*d+e))^{(1/2)+2e)*d)^{(1/2)))/(c^2*d+e)^2/d^5*(e*(c^2*d+e))^{( \\ & 1/2)+3/8*c^6*b*x^3/(c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*\operatorname{arccsc}(c*x) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^3, x)
```

### 3.118 $\int x^5 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=403

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \csc^{-1}(cx))}{7e^3} - \frac{bx\sqrt{c^2x^2 - 1}}{2}$$

```
[Out] -(b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(1680*c^5*e^2*Sqrt[c^2*x^2]) - (b*(29*c^2*d - 25*e)*x*Sqrt[-1 + c^2*x^2]*
(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d
+ e*x^2)^(5/2))/(42*c*e^2*Sqrt[c^2*x^2]) + (d^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3)
+ ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3) - (8*b*c*d^(7/2)*x*ArcTan
[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])]/(105*e^3*Sqrt[c^2*x^2])) + (
b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*x*ArcTanh[(Sqrt[e]*S
qrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]/(1680*c^6*e^(5/2)*Sqrt[c^2*x^2]))
```

**Rubi [A]** time = 1.31223, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5239, 12, 1615, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \csc^{-1}(cx))}{7e^3} - \frac{bx\sqrt{c^2x^2 - 1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -(b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(1680*c^5*e^2*Sqrt[c^2*x^2]) - (b*(29*c^2*d - 25*e)*x*Sqrt[-1 + c^2*x^2]*
(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d
+ e*x^2)^(5/2))/(42*c*e^2*Sqrt[c^2*x^2]) + (d^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3)
+ ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3) - (8*b*c*d^(7/2)*x*ArcTan
[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])]/(105*e^3*Sqrt[c^2*x^2])) + (
b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*x*ArcTanh[(Sqrt[e]*S
qrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]/(1680*c^6*e^(5/2)*Sqrt[c^2*x^2]))
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
+ p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 157

```
Int[((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_)
))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p
, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt
```

Q[a, 0] || LtQ[b, 0])

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^5 \sqrt{d+ex^2} (a+b \csc^{-1}(cx)) dx &= \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5e^3} \\
 &= \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5e^3} \\
 &= -\frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} \\
 &= -\frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} \\
 &= -\frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} \\
 &= -\frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3} \\
 &= -\frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{d^2 (d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e^3}
 \end{aligned}$$

**Mathematica [C]** time = 0.647866, size = 365, normalized size = 0.91

$$\frac{\sqrt{d+ex^2} \left( 16ac^5 \left( -4d^2ex^2 + 8d^3 + 3de^2x^4 + 15e^3x^6 \right) + bex \sqrt{1 - \frac{1}{c^2x^2}} \left( c^4 \left( -41d^2 + 22dex^2 + 40e^2x^4 \right) + 2c^2e \left( 19d + 25ex^2 \right) \right) \right)}{1680c^5e^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*(-128\*c^4\*d^4\*Sqrt[1 + d/(e\*x^2)]\*(-1 + c^2\*x^2)\*AppellF1[1, 1/2, 1/2, 2, 1/(c^2\*x^2), -(d/(e\*x^2))]) + e\*(105\*c^6\*d^3 - 35\*c^4\*d^2\*e + 63\*c^2\*d\*e^2 + 75\*e^3)\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*AppellF1[1, 1/2, 1/2, 2, c^2\*x^2, -((e\*x^2)/d)])/(3360\*c^5\*e^3\*x\*(-1 + c^2\*x^2)\*Sqrt[d + e\*x^2]) + (Sqrt[d + e\*x^2]\*(16\*a\*c^5\*(8\*d^3 - 4\*d^2\*e\*x^2 + 3\*d\*e^2\*x^4 + 15\*e^3\*x^6) + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(75\*e^2 + 2\*c^2\*e\*(19\*d + 25\*e\*x^2) + c^4\*(-41\*d^2 + 22\*d\*e\*x^2 + 40\*e^2\*x^4)) + 16\*b\*c^5\*(8\*d^3 - 4\*d^2\*e\*x^2 + 3\*d\*e^2\*x^4 + 15\*e^3\*x^6)\*ArcCsc[c\*x]))/(1680\*c^5\*e^3)

**Maple [F]** time = 6.16, size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{arccsc}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x)

[Out] int(x^5\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 41.4373, size = 3779, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/6720\*(128\*b\*c^7\*sqrt(-d)\*d^3\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (105\*b\*c^6\*d^3 - 35\*b\*c^4\*d^2\*e + 63\*b\*c^2\*d\*



$$\begin{aligned}
& e^2 + 75*b*e^3) * \text{sqrt}(e) * \log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d* \\
& e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e) * \text{sqrt}(c^2*x^2 - 1) * \text{sqrt}(e*x \\
& ^2 + d) * \text{sqrt}(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c \\
& ^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4 \\
& *b*c^7*d^2*e*x^2 + 8*b*c^7*d^3) * \text{arccsc}(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5* \\
& d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2 \\
& ) * \text{sqrt}(c^2*x^2 - 1) * \text{sqrt}(e*x^2 + d)) / (c^7*e^3), -1/6720*(256*b*c^7*d^(7/2) \\
& * \text{arctan}(-1/2 * \text{sqrt}(c^2*x^2 - 1) * ((c^2*d - e)*x^2 - 2*d) * \text{sqrt}(e*x^2 + d) * \text{sqrt} \\
& (d) / (c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (105*b*c^6*d^3 - 35*b*c^4*d^2* \\
& e + 63*b*c^2*d*e^2 + 75*b*e^3) * \text{sqrt}(e) * \log(8*c^4*e^2*x^4 + c^4*d^2 - 6* \\
& c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e) * \text{sqrt}(c^ \\
& 2*x^2 - 1) * \text{sqrt}(e*x^2 + d) * \text{sqrt}(e) + e^2) - 4*(240*a*c^7*e^3*x^6 + 48*a*c^7 \\
& *d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3* \\
& b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3) * \text{arccsc}(c*x) + (40*b*c^5* \\
& e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 \\
& + 25*b*c^3*e^3)*x^2) * \text{sqrt}(c^2*x^2 - 1) * \text{sqrt}(e*x^2 + d)) / (c^7*e^3), 1/3360* \\
& (64*b*c^7 * \text{sqrt}(-d) * d^3 * \log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - \\
& d*e)*x^2 + 4 * \text{sqrt}(c^2*x^2 - 1) * ((c^2*d - e)*x^2 - 2*d) * \text{sqrt}(e*x^2 + d) * \text{sqrt} \\
& (-d) + 8*d^2) / x^4) - (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b* \\
& e^3) * \text{sqrt}(-e) * \text{arctan}(1/2 * (2*c^2*e*x^2 + c^2*d - e) * \text{sqrt}(c^2*x^2 - 1) * \text{sqrt} \\
& (e*x^2 + d) * \text{sqrt}(-e) / (c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(24 \\
& 0*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + \\
& 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3) \\
& ) * \text{arccsc}(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b* \\
& c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2) * \text{sqrt}(c^2*x^2 - 1) * \text{sqrt}(e*x^ \\
& 2 + d)) / (c^7*e^3), -1/3360*(128*b*c^7*d^(7/2) * \text{arctan}(-1/2 * \text{sqrt}(c^2*x^2 - 1) \\
& * ((c^2*d - e)*x^2 - 2*d) * \text{sqrt}(e*x^2 + d) * \text{sqrt}(d) / (c^2*d*e*x^4 + (c^2*d^2 - \\
& d*e)*x^2 - d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b* \\
& e^3) * \text{sqrt}(-e) * \text{arctan}(1/2 * (2*c^2*e*x^2 + c^2*d - e) * \text{sqrt}(c^2*x^2 - 1) * \text{sqrt}(e \\
& *x^2 + d) * \text{sqrt}(-e) / (c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(240* \\
& a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 1 \\
& 6*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3) * \\
& \text{arccsc}(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c* \\
& e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2) * \text{sqrt}(c^2*x^2 - 1) * \text{sqrt}(e*x^2 \\
& + d)) / (c^7*e^3)]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^5, x)
```

### 3.119 $\int x^3 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=294

$$\frac{d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{2bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}} - \frac{bx(15c^4d^2-10c^2de)}{120c^3e\sqrt{d+ex^2}}$$

```
[Out] (b*(c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^2) + (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(15*e^2*Sqrt[c^2*x^2]) - (b*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(3/2)*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 0.40881, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5239, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{2bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}} - \frac{bx(15c^4d^2-10c^2de)}{120c^3e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (b*(c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^2) + (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(15*e^2*Sqrt[c^2*x^2]) - (b*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(3/2)*Sqrt[c^2*x^2])
```

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
```

GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 573

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_)\*(x\_)^(n\_))^(r\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q\*(e + f\*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 154

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(h\*(a + b\*x)^m\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 2)), x] + Dist[1/(d\*f\*(m + n + p + 2)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*g\*(m + n + p + 2) - h\*(b\*c\*e\*m + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + (b\*d\*f\*g\*(m + n + p + 2) + h\*(a\*d\*f\*m - b\*(d\*e\*(m + n + 1) + c\*f\*(m + p + 1)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_))) / ((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)) / ((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^3 \sqrt{d+ex^2} (a+b \operatorname{csc}^{-1}(cx)) dx &= -\frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{5e^2} + \frac{(bcx) \int \frac{(d+ex^2)^{-1/2}}{15} dx}{15} \\
 &= -\frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{5e^2} + \frac{(bcx) \int \frac{(d+ex^2)^{-1/2}}{15} dx}{15} \\
 &= -\frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{5e^2} + \frac{(bcx) \operatorname{Subst} \int \frac{1}{15} dx}{15} \\
 &= \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{5e^2} \\
 &= \frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2} \\
 &= \frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2} \\
 &= \frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2} \\
 &= \frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{3e^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.72113, size = 326, normalized size = 1.11

$$\frac{\sqrt{d+ex^2} \left( 8ac^3(-2d^2+dex^2+3e^2x^4) + 8bc^3 \operatorname{csc}^{-1}(cx) (-2d^2+dex^2+3e^2x^4) + bex\sqrt{1-\frac{1}{c^2x^2}} (c^2(7d+6ex^2)+9e) \right)}{120c^3e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

[Out] (Sqrt[d + e\*x^2]\*(8\*a\*c^3\*(-2\*d^2 + d\*e\*x^2 + 3\*e^2\*x^4) + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(9\*e + c^2\*(7\*d + 6\*e\*x^2)) + 8\*b\*c^3\*(-2\*d^2 + d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsc[c\*x]))/(120\*c^3\*e^2) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*S

```

qrt[e]*Sqrt[c^2*d + e]*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*Sqrt[(c^2*(d + e*x
^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^
2*d + e])] + 16*c^7*d^(5/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x
^2])/Sqrt[d + e*x^2]])/(120*c^6*e^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])

```

**Maple [F]** time = 5.756, size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arccsc}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 16.4025, size = 3036, normalized size = 10.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2
*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2)
+ 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4
+ b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*
e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/480*(32*b*c
^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*c^4*d^2 -
10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8
*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*
sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16
*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x)
+ (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d))/(c^5*e^2), 1/240*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e
^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*
d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e -

```

```

9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(16*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d))/(c^5*e^2)]

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))*(e*x**2+d)**(1/2), x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2), x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^3, x)
```

### 3.120 $\int x\sqrt{d+ex^2} \left(a + b \csc^{-1}(cx)\right) dx$

**Optimal.** Leaf size=195

$$\frac{(d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e} - \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{bx(3c^2d+e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

[Out] (b\*x\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])/(6\*c\*Sqrt[c^2\*x^2]) + ((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*e) - (b\*c\*d^(3/2)\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*e\*Sqrt[c^2\*x^2]) + (b\*(3\*c^2\*d + e)\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(6\*c^2\*Sqrt[e]\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.211578, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5237, 446, 102, 157, 63, 217, 206, 93, 204}

$$\frac{(d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{3e} - \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{bx(3c^2d+e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*x\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])/(6\*c\*Sqrt[c^2\*x^2]) + ((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*e) - (b\*c\*d^(3/2)\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*e\*Sqrt[c^2\*x^2]) + (b\*(3\*c^2\*d + e)\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(6\*c^2\*Sqrt[e]\*Sqrt[c^2\*x^2])

#### Rule 5237

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsc[c\*x]))/(2\*e\*(p + 1)), x] + Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 102

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.))\*((e\_.) + (f\_.)\*(x\_)^(p\_.)), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2\*m, 2\*n, 2\*p]



Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{d+ex^2}(a+b\csc^{-1}(cx))dx &= \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{(bcx)\int\frac{(d+ex^2)^{3/2}}{x\sqrt{-1+c^2x^2}}dx}{3e\sqrt{c^2x^2}} \\
&= \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex^2)^{3/2}}{x\sqrt{-1+c^2x}}dx,x,x^2\right)}{6e\sqrt{c^2x^2}} \\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{(bx)\text{Subst}\left(\int\frac{c^2d^2+\frac{1}{2}e(3c^2d+e)}{x\sqrt{-1+c^2x}}dx,x,x^2\right)}{6ce\sqrt{c^2x^2}} \\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{(bcd^2x)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}}dx,x,x^2\right)}{6e\sqrt{c^2x^2}} \\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{(bcd^2x)\text{Subst}\left(\int\frac{1}{-d-x^2}dx,x,x^2\right)}{3e\sqrt{c^2x^2}} \\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} \\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.46289, size = 265, normalized size = 1.36

$$\frac{\sqrt{d+ex^2}\left(2ac(d+ex^2)+bex\sqrt{1-\frac{1}{c^2x^2}}+2bc\csc^{-1}(cx)(d+ex^2)\right)}{6ce} + \frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^5d^{3/2}\sqrt{d+ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)+\dots\right)}{6c^4e\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]),x]

[Out] (Sqrt[d + e\*x^2]\*(b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x + 2\*a\*c\*(d + e\*x^2) + 2\*b\*c\*(d + e\*x^2)\*ArcCsc[c\*x]))/(6\*c\*e) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*(3\*c^2\*d + e)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])] + 2\*c^5\*d^(3/2)\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(6\*c^4\*e\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 3.025, size = 0, normalized size = 0.

$$\int x(a + b\text{arccsc}(cx))\sqrt{ex^2 + d}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x)

[Out] int(x\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(ex^2 + d)^{\frac{3}{2}} a}{3e} + \frac{\left( e \int \frac{(c^2 ex^3 + c^2 dx) e^{\left(\frac{1}{2} \log(ex^2 + d) + \frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1)\right)}}{c^2 ex^2 + (c^2 ex^2 - e)(cx+1)(cx-1) - e} dx + (ex^2 \arctan(1, \sqrt{cx+1}\sqrt{cx-1}) + d \arctan(1, \sqrt{cx+1}\sqrt{cx-1})) \right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/3\*(e\*x^2 + d)^(3/2)\*a/e + 1/3\*(3\*e\*integrate(1/3\*(c^2\*e\*x^3 + c^2\*d\*x)\*e^(1/2\*log(e\*x^2 + d) + 1/2\*log(c\*x + 1) + 1/2\*log(c\*x - 1))/(c^2\*e\*x^2 + (c^2\*e\*x^2 - e)\*e^(log(c\*x + 1) + log(c\*x - 1)) - e), x) + (e\*x^2\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + d\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)))\*sqrt(e\*x^2 + d)\*b/e

**Fricas [A]** time = 6.77768, size = 2427, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/24\*(2\*b\*c^3\*sqrt(-d)\*d\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (3\*b\*c^2\*d + b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e), -1/24\*(4\*b\*c^3\*d^(3/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) - (3\*b\*c^2\*d + b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) - 4\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e), 1/12\*(b\*c^3\*sqrt(-d)\*d\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) - (3\*b\*c^2\*d + b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e), -1/12\*(2\*b\*c^3\*d^(3/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (3\*b\*c^2\*d + b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) - 2\*(2\*a\*c^3\*e\*x^2 + 2\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 + b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d)/(c^3\*e)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral(x\*(a + b\*acsc(c\*x))\*sqrt(d + e\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*x, x)

$$3.121 \quad \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x}, x\right)$$

[Out] Unintegrable[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x, x]

**Rubi [A]** time = 0.101547, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x, x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x} dx = \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

**Mathematica [A]** time = 5.34614, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x, x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x, x]

**Maple [A]** time = 6.236, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x, x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x, x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x,x)

[Out] Integral((a + b\*acsc(c\*x))\*sqrt(d + e\*x\*\*2)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x, x)

$$3.122 \quad \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^3}, x\right)$$

[Out] Unintegrable[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^3, x]

**Rubi [A]** time = 0.106718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^3, x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^3, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^3} dx = \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

**Mathematica [A]** time = 5.68755, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^3, x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^3, x]

**Maple [A]** time = 1.717, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^3} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^3, x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^3, x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^3, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*3,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^3, x)



$$3.123 \quad \int x^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(x^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)), x\right)$$

[Out] Unintegrable[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.096438, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int][x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int x^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx = \int x^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

**Mathematica [A]** time = 9.03404, size = 0, normalized size = 0.

$$\int x^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 5.807, size = 0, normalized size = 0.

$$\int x^2 (a + \text{arccsc}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2), x)

[Out] int(x^2\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 \operatorname{arccsc}(cx) + ax^2\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^2, x)
```

$$3.124 \quad \int \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\sqrt{d + ex^2} (a + b \csc^{-1}(cx)), x\right)$$

[Out] Unintegrable[Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.0337974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int][Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx = \int \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

**Mathematica [A]** time = 6.10874, size = 0, normalized size = 0.

$$\int \sqrt{d + ex^2} (a + b \csc^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 5.31, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a), x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsc(c\*x))\*sqrt(d + e\*x\*\*2), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a), x)

$$3.125 \quad \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^2}, x\right)$$

[Out] Unintegrable[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

**Rubi [A]** time = 0.0869433, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

**Mathematica [A]** time = 1.68221, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^2, x]

**Maple [A]** time = 4.628, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^2, x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^2, x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^2, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*2,x)

[Out] Integral((a + b\*acsc(c\*x))\*sqrt(d + e\*x\*\*2)/x\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^2, x)

$$3.126 \quad \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=328

$$\frac{bx\sqrt{1-c^2x^2}(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}(\sin^{-1}(cx),-\frac{e}{c^2d})}{9d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{3dx^3} - \frac{2bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}}$$

```
[Out] (-2*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*b*c^2*(c^2*d + 2*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(2*c^2*d + 3*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.431765, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {264, 5239, 12, 474, 583, 524, 427, 426, 424, 421, 419}

$$\frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{3dx^3} - \frac{2bc\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{bx\sqrt{1-c^2x^2}(c^2d+e)}{9d\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*b*c^2*(c^2*d + 2*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(2*c^2*d + 3*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

#### Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d+e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2-1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && ! (ILtQ[(m-1)/2, 0] && GtQ[m+2*p+3, 0])) || (IGtQ[(m+1)/2, 0] && ! (ILtQ[p, 0] && GtQ[m+2*p+3, 0])) || (ILtQ[(m+2*p+1)/2, 0] && ! ILtQ[(m-1)/2, 0]))
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 474

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(
q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
+ a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

#### Rule 583

```
Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

#### Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

#### Rule 427

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

#### Rule 426

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
]
```

#### Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 421

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

#### Rule 419



```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4} dx &= -\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(bcx) \int -\frac{(d+ex^2)^{3/2}}{3dx^4\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} - \frac{(bcx) \int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{-1+c^2x^2}} dx}{3d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(bcx) \int \frac{-2d(c^2d+2e)-e(c^2d+2e)}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{9d\sqrt{c^2x^2}} \\
&= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} \\
&= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} \\
&= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} \\
&= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} \\
&= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3}
\end{aligned}$$

**Mathematica [C]** time = 0.626642, size = 247, normalized size = 0.75

$$\frac{\sqrt{d+ex^2}\left(3a(d+ex^2)+bcx\sqrt{1-\frac{1}{c^2x^2}}(2c^2dx^2+d+4ex^2)+3b\csc^{-1}(cx)(d+ex^2)\right)}{9dx^3} + \frac{ibcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}+1}\left(2c^2d+2e\right)}{9d\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^4,x]

[Out] -(Sqrt[d + e\*x^2]\*(3\*a\*(d + e\*x^2) + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + 2\*c^2\*d\*x^2 + 4\*e\*x^2) + 3\*b\*(d + e\*x^2)\*ArcCsc[c\*x]))/(9\*d\*x^3) + ((I/9)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(2\*c^2\*d\*(c^2\*d + 2\*e)\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] - (2\*c^4\*d^2 + 5\*c^2\*d\*e + 3\*e^2)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d)))]/(Sqrt[-c^2]\*d\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 9.214, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^4} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^4, x)

$$3.127 \quad \int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=453

$$\frac{bx\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{2e(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{15d^2x^3}$$

```
[Out] -(b*c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(225*d^2*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(225*d*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3
/2))/(25*d*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(5*
d*x^5) + (2*e*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(15*d^2*x^3) + (b*c^2*
(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*Elli
pticE[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]
*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.620064, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {271, 264, 5239, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{15d^2x^3} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} - \frac{bc\sqrt{c^2x^2-1}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6, x]
```

```
[Out] -(b*c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(225*d^2*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(225*d*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3
/2))/(25*d*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(5*
d*x^5) + (2*e*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(15*d^2*x^3) + (b*c^2*
(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*Elli
pticE[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]
*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rule 271**

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*
(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

**Rule 264**

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 580

```
Int[((g_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)])], x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplersqrtQ[-(b/a), -(d/c)])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}(a+b \operatorname{csc}^{-1}(cx))}{x^6} dx &= -\frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{15d^2x^3} + \frac{(bcx) \int \frac{(d+ex^2)}{15d^2x^2} dx}{\sqrt{d+ex^2}} \\
&= -\frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{15d^2x^3} + \frac{(bcx) \int \frac{(d+ex^2)}{x^2} dx}{15d^2} \\
&= -\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{15d^2x^3} \\
&= -\frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b \operatorname{csc}^{-1}(cx))}{5dx^5}
\end{aligned}$$

**Mathematica [C]** time = 0.698121, size = 325, normalized size = 0.72

$$\frac{\sqrt{d+ex^2} \left( 15a(3d^2+dex^2-2e^2x^4) + bcx\sqrt{1-\frac{1}{c^2x^2}}(3d^2(8c^4x^4+4c^2x^2+3) + dex^2(19c^2x^2+8) - 31e^2x^4) + 15b \operatorname{csc}^{-1}(cx) \right)}{225d^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/x^6,x]

[Out]  $-(\text{Sqrt}[d + e*x^2]*(15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) + 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*\text{ArcCsc}[c*x])/(225*d^2*x^5) + ((I/225)*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[1 + (e*x^2)/d]*(c^2*d*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2))*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d))] + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d)))])/(\text{Sqrt}[-c^2]*d^2*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])$

**Maple [F]** time = 7.089, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^6} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x)

[Out] int((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^6, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*6,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/x^6, x)

### 3.128 $\int x^3 (d + ex^2)^{3/2} (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=374

$$-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2} - \frac{bx\sqrt{c^2x^2-1}(3c^4d^2-38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \dots$$

```
[Out] -(b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(560*c^5*e*Sqrt[c^2*x^2]) + (b*(13*c^2*d + 25*e)*x*Sqrt[-1 + c^2*x^2]*(d +
e*x^2)^(3/2))/(840*c^3*e*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x
^2)^(5/2))/(42*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]
))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^2) + (2*b*c*d^(7/
2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(35*e^2*Sqrt[c^2
*x^2]) - (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*x*ArcTanh[(
Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[c^2
*x^2])
```

**Rubi [A]** time = 0.537857, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5239, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2} - \frac{bx\sqrt{c^2x^2-1}(3c^4d^2-38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

```
[Out] -(b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(560*c^5*e*Sqrt[c^2*x^2]) + (b*(13*c^2*d + 25*e)*x*Sqrt[-1 + c^2*x^2]*(d +
e*x^2)^(3/2))/(840*c^3*e*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x
^2)^(5/2))/(42*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]
))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^2) + (2*b*c*d^(7/
2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(35*e^2*Sqrt[c^2
*x^2]) - (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*x*ArcTanh[(
Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[c^2
*x^2])
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```



```
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 573

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
)*(e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

### Rule 154

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_
))^(p_)*((g_) + (h_)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 157

```
Int[((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_)*((g_) + (h_)*(x_
)))/((a_) + (b_)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

### Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 93

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)/((e_) + (f_)*(x
```

```

_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

```

### Rule 204

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

### Rubi steps

$$\begin{aligned}
\int x^3 (d + ex^2)^{3/2} (a + b \csc^{-1}(cx)) dx &= -\frac{d(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \csc^{-1}(cx))}{7e^2} + \frac{(bcx) \int \frac{(d+ex^2)^{3/2}}{35e}}{\sqrt{\dots}} \\
&= -\frac{d(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \csc^{-1}(cx))}{7e^2} + \frac{(bcx) \int \frac{(d+ex^2)^{3/2}}{35e}}{\sqrt{\dots}} \\
&= -\frac{d(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \csc^{-1}(cx))}{7e^2} + \frac{(bcx) \text{Subst}}{\sqrt{\dots}} \\
&= \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \csc^{-1}(cx))}{7e^2} \\
&= \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1 + c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d + ex^2)^{7/2} (a + b \csc^{-1}(cx))}{7e^2} \\
&= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{840c^3e\sqrt{c^2x^2}} \\
&= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{840c^3e\sqrt{c^2x^2}} \\
&= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{840c^3e\sqrt{c^2x^2}} \\
&= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{840c^3e\sqrt{c^2x^2}} \\
&= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d + 25e)x\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{840c^3e\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.600757, size = 339, normalized size = 0.91

$$\frac{\sqrt{d + ex^2} \left( -48ac^5 (2d - 5ex^2) (d + ex^2)^2 + bex\sqrt{1 - \frac{1}{c^2x^2}} (c^4 (57d^2 + 106dex^2 + 40e^2x^4) + 2c^2e (82d + 25ex^2) + 75e^2) - 4d^2 \right)}{1680c^5e^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*(32\*c^4\*d^4\*Sqrt[1 + d/(e\*x^2)]\*(-1 + c^2\*x^2)\*AppellF1[1, 1/2, 1/2, 2, 1/(c^2\*x^2), -(d/(e\*x^2))]) + e\*(-35\*c^6\*d^3 + 35\*c^4\*d^2\*e + 63\*c^2\*d\*e^2 + 25\*e^3)\*Sqrt[1 - 1/(c^2\*x^2)]\*x^4\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*AppellF1[1, 1/2, 1/2, 2, c^2\*x^2, -((e\*x^2)/d)])/(1120\*c^5\*e^2\*x\*(-1 + c^2\*x^2)\*Sqrt[d + e\*x^2]) + (Sqrt[d + e\*x^2]\*(-48\*a\*c^5\*(2\*d - 5\*e\*x^2)\*(d + e\*x^2)^2 + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(75\*e^2 + 2\*c^2\*e\*(82\*d + 25\*e\*x^2) + c^4\*(57\*d^2 + 106\*d\*e\*x^2 + 40\*e^2\*x^4)) - 48\*b\*c^5\*(2\*d - 5\*e\*x^2)\*(d + e\*x^2)^2\*ArcCsc[c\*x]))/(1680\*c^5\*e^2)

**Maple [F]** time = 3.198, size = 0, normalized size = 0.

$$\int x^3 (ex^2 + d)^{\frac{3}{2}} (a + \text{barccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x)

[Out] int(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 38.9588, size = 3768, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] [1/6720\*(96\*b\*c^7\*sqrt(-d)\*d^3\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) - 3\*(35\*b\*c^6\*d^3 - 35\*b\*c^4\*d^2\*e - 63\*b\*c^2\*d\*e^2 - 25\*b\*e^3)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(240\*a\*c^7\*e^3\*x^6 + 384\*a\*c^7\*d\*e^2\*x^4 + 48\*a\*c^7\*d^2\*e\*x^2 - 96\*a\*c^7\*d^3 + 48\*(5\*b\*c^7\*e^3\*x^6 + 8\*b\*c^7\*d\*e^2\*x^4 + b\*c^7\*d^2\*e\*x^2 - 2\*b\*c^7\*d^3)\*arccsc(c\*x) + (40\*b\*c^5\*e^3\*x^4 + 57\*b\*c^5\*d^2\*e + 164\*b\*c^3\*d\*e^2 + 75\*b\*c\*e^3 + 2\*(53\*b\*c^5\*d\*e^2 + 25\*b\*c^3\*e^3)\*x^2)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d))/(c^7\*e^2), 1/6720\*(192\*b\*c^7\*d^(7/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) - 3\*(35\*b\*c^6\*d^3 - 35\*b\*c^4\*d^2

```

2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^
2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*
x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*
d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c
^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x
^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25
*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), 1/3360*(48*
b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d)
+ 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e
^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*
x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a
*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48
*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcc
sc(c*x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3
+ 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d
))/(c^7*e^2), 1/3360*(96*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*
d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^
2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*s
qrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 +
d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a*c^7*
e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b
*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*
x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*
(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^7*e^2)]

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}}(b \operatorname{arccsc}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^3, x)
```

### 3.129 $\int x (d + ex^2)^{3/2} (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=262

$$\frac{(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e} + \frac{bx (15c^4d^2 + 10c^2de + 3e^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}} - \frac{bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2}}{5e}$$

```
[Out] (b*(7*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[c^2*x^2]) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(5*e*Sqrt[c^2*x^2]) + (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(40*c^4*Sqrt[e]*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 0.288391, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {5237, 446, 102, 154, 157, 63, 217, 206, 93, 204}

$$\frac{(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e} + \frac{bx (15c^4d^2 + 10c^2de + 3e^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}} - \frac{bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (b*(7*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[c^2*x^2]) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(5*e*Sqrt[c^2*x^2]) + (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(40*c^4*Sqrt[e]*Sqrt[c^2*x^2])
```

#### Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sqrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 102

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1)) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 154

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(h\*(a + b\*x)^m\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 2)), x] + Dist[1/(d\*f\*(m + n + p + 2)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*g\*(m + n + p + 2) - h\*(b\*c\*e\*m + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + (b\*d\*f\*g\*(m + n + p + 2) + h\*(a\*d\*f\*m - b\*(d\*e\*(m + n + 1) + c\*f\*(m + p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_))) / ((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)) / ((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int x(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))dx &= \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bcx)\int\frac{(d+ex^2)^{5/2}}{x\sqrt{-1+c^2x^2}}dx}{5e\sqrt{c^2x^2}} \\
&= \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex^2)^{5/2}}{x\sqrt{-1+c^2x}}dx,x,x^2\right)}{10e\sqrt{c^2x^2}} \\
&= \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex^2)^{5/2}}{x\sqrt{-1+c^2x}}dx,x,x^2\right)}{10e\sqrt{c^2x^2}} \\
&= \frac{b(7c^2d+3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} \\
&= \frac{b(7c^2d+3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} \\
&= \frac{b(7c^2d+3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} \\
&= \frac{b(7c^2d+3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} \\
&= \frac{b(7c^2d+3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} \\
&= \frac{b(7c^2d+3e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e}
\end{aligned}$$

**Mathematica [A]** time = 0.714831, size = 304, normalized size = 1.16

$$\frac{\sqrt{d+ex^2}\left(8ac^3(d+ex^2)^2+bex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2(9d+2ex^2)+3e\right)+8bc^3\csc^{-1}(cx)(d+ex^2)^2\right)}{40c^3e} + \frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(\sqrt{c^2}\sqrt{e}\sqrt{d+ex^2}\right)}{10c\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (Sqrt[d + e\*x^2]\*(8\*a\*c^3\*(d + e\*x^2)^2 + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(3\*e + c^2\*(9\*d + 2\*e\*x^2)) + 8\*b\*c^3\*(d + e\*x^2)^2\*ArcCsc[c\*x]))/(40\*c^3\*e) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*(15\*c^4\*d^2 + 10\*c^2\*d\*e + 3\*e^2)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])]) + 8\*c^7\*d^(5/2)\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]]))/(40\*c^6\*e\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 1.773, size = 0, normalized size = 0.

$$\int x(ex^2+d)^{\frac{3}{2}}(a+b\text{arccsc}(cx))dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)`

[Out] `int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(ex^2 + d)^{\frac{5}{2}} a}{5e} + \frac{\left( e \int \frac{(c^2 e^2 x^5 + 2c^2 dex^3 + c^2 d^2 x) e^{\left(\frac{1}{2} \log(cx^2 + d) + \frac{1}{2} \log(cx+1) + \frac{1}{2} \log(cx-1)\right)}}{c^2 ex^2 + (c^2 ex^2 - e)(cx+1)(cx-1) - e} dx + \left( e^2 x^4 \arctan\left(1, \sqrt{cx+1} \sqrt{cx-1}\right) + 2 dex^2 \arctan\left(1, \sqrt{cx+1} \sqrt{cx-1}\right) + d^2 \arctan\left(1, \sqrt{cx+1} \sqrt{cx-1}\right) \right) \sqrt{e^2 x^4 + 2 dex^2 + d^2}}{5e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out] `1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*(5*e*integrate(1/5*(c^2*e^2*x^5 + 2*c^2*d*e*x^3 + c^2*d^2*x)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x) + (e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*d*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(e*x^2 + d)*b/e`

**Fricas [A]** time = 16.3016, size = 3006, normalized size = 11.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out] `[1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d))/(c^5*e), -1/160*(16*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d))/(c^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d))/(c^5*e), -1/80*(8*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)`



```
)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^
2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(8*a*c^5
*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*
x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*
sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e)]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}}(b \operatorname{arccsc}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x, x)
```

$$3.130 \quad \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x}, x \right)$$

[Out] Unintegrable[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x, x]

**Rubi [A]** time = 0.116081, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x,x]

[Out] Defer[Int] [((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x} dx = \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

**Mathematica [A]** time = 5.96237, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x,x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x, x]

**Maple [A]** time = 1.941, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x,x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \arccsc(cx))\sqrt{ex^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \arccsc(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x, x)`

$$3.131 \quad \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^3}, x \right)$$

[Out] Unintegrable[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^3, x]

**Rubi [A]** time = 0.122081, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^3, x]

[Out] Defer[Int][((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^3, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^3} dx = \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

**Mathematica [A]** time = 4.79437, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^3, x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^3, x]

**Maple [A]** time = 1.263, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^3} (ex^2 + d)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^3, x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx))\sqrt{ex^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**3,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arccsc}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^3, x)`

$$3.132 \quad \int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(x^2 (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)), x\right)$$

[Out] Unintegrable[x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.123709, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int][x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx = \int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Mathematica [A]** time = 9.56484, size = 0, normalized size = 0.

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 3.258, size = 0, normalized size = 0.

$$\int x^2 (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)), x)

[Out] int(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(aex^4 + adx^2 + (bex^4 + bdx^2)\text{arccsc}(cx)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e\*x^4 + a\*d\*x^2 + (b\*e\*x^4 + b\*d\*x^2)\*arccsc(c\*x))\*sqrt(e\*x^2 + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsc(c\*x)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}}(b\text{arccsc}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsc(c\*x) + a)\*x^2, x)

$$\mathbf{3.133} \quad \int (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\left(d + ex^2\right)^{3/2} (a + b \operatorname{csc}^{-1}(cx)), x\right)$$

[Out] Unintegrable[(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.0452941, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int] [(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx = \int (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Mathematica [A]** time = 6.67558, size = 0, normalized size = 0.

$$\int (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 2.795, size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)), x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)), x)



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \arccsc(cx)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsc(c\*x)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}}(b \arccsc(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsc(c\*x) + a), x)

$$3.134 \quad \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^2}, x \right)$$

[Out] Unintegrable[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^2, x]

**Rubi [A]** time = 0.106317, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^2,x]

[Out] Defer[Int][((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^2} dx = \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

**Mathematica [A]** time = 12.6123, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^2,x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^2, x]

**Maple [A]** time = 3.097, size = 0, normalized size = 0.

$$\int \frac{a + \operatorname{arccsc}(cx)}{x^2} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^2,x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx))\sqrt{ex^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arccsc}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^2, x)`

$$3.135 \quad \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^4}, x \right)$$

[Out] Unintegrable[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^4, x]

**Rubi [A]** time = 0.10664, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^4, x]

[Out] Defer[Int] [((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^4, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^4} dx = \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^4} dx$$

**Mathematica [A]** time = 6.42733, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^4, x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^4, x]

**Maple [A]** time = 6.571, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^4} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^4, x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \arccsc(cx))\sqrt{ex^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^4, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**4,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \arccsc(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^4, x)`

$$3.136 \quad \int \frac{(d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=416

$$\frac{bx\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{(d+ex^2)^{5/2}(a+b \csc^{-1}(cx))}{5dx^5} - \frac{b}{c}$$

```
[Out] -(b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(75*d*Sqrt[c^2*x^2]) - (4*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(75*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(
25*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*d*x^5) +
(b*c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*
x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2
)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d
))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.572568, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {264, 5239, 12, 474, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{(d+ex^2)^{5/2}(a+b \csc^{-1}(cx))}{5dx^5} - \frac{bc\sqrt{c^2x^2-1}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{bx\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{b}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -(b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(75*d*Sqrt[c^2*x^2]) - (4*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(75*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(
25*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*d*x^5) +
(b*c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*
x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2
)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d
))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

#### Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
```

\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 474

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*e\*(m + 1)), x] - Dist[1/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(c\*b - a\*d)\*(m + 1) + c\*n\*(b\*c\*(p + 1) + a\*d\*(q - 1)) + d\*((c\*b - a\*d)\*(m + 1) + c\*b\*n\*(p + q))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

### Rule 580

Int[((g\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(e\*(g\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*g\*(m + 1)), x] - Dist[1/(a\*g^n\*(m + 1)), Int[(g\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^(q - 1)\*Simp[c\*(b\*e - a\*f)\*(m + 1) + e\*n\*(b\*c\*(p + 1) + a\*d\*q) + d\*((b\*e - a\*f)\*(m + 1) + b\*e\*n\*(p + q + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f\*x^n, c + d\*x^n])

### Rule 583

Int[((g\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(e\*(g\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*c\*g\*(m + 1)), x] + Dist[1/(a\*c\*g^n\*(m + 1)), Int[(g\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^q\*Simp[a\*f\*c\*(m + 1) - e\*(b\*c + a\*d)\*(m + n + 1) - e\*n\*(b\*c\*p + a\*d\*q) - b\*e\*d\*(m + n\*(p + q + 2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

### Rule 524

Int[((e\_) + (f\_)\*(x\_)^(n\_))/(Sqrt[(a\_) + (b\_)\*(x\_)^(n\_)]\*Sqrt[(c\_) + (d\_)\*(x\_)^(n\_)]), x\_Symbol] := Dist[f/b, Int[Sqrt[a + b\*x^n]/Sqrt[c + d\*x^n], x], x] + Dist[(b\*e - a\*f)/b, Int[1/(Sqrt[a + b\*x^n]\*Sqrt[c + d\*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

### Rule 427

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[Sqrt[a + b\*x^2]/Sqrt[1 + (d\*x^2)/c], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]

### Rule 426

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Dist[Sqrt[a + b\*x^2]/Sqrt[1 + (b\*x^2)/a], Int[Sqrt[1 + (b\*x^2)/a]/Sqrt[c + d\*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[
(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2} (a+b \csc^{-1}(cx))}{x^6} dx &= -\frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5} + \frac{(bcx) \int -\frac{(d+ex^2)^{5/2}}{5dx^6 \sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5} - \frac{(bcx) \int \frac{(d+ex^2)^{5/2}}{x^6 \sqrt{-1+c^2x^2}} dx}{5d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-4d(c^2x^2+1))}{x^4 \sqrt{-1+c^2x^2}} dx}{25d\sqrt{c^2x^2}} \\
&= -\frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5} \\
&= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \csc^{-1}(cx))}{5dx^5}
\end{aligned}$$

**Mathematica [C]** time = 0.657712, size = 303, normalized size = 0.73

$$\frac{\sqrt{d+ex^2} \left( 15a(d+ex^2)^2 + bcx \sqrt{1 - \frac{1}{c^2x^2}} \left( d^2(8c^4x^4 + 4c^2x^2 + 3) + dex^2(23c^2x^2 + 11) + 23e^2x^4 \right) + 15b \csc^{-1}(cx) (d+ex^2) \right)}{75dx^5}$$



Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -(Sqrt[d + e*x^2]*(15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(23*e^2
*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) + 15*b*
(d + e*x^2)^2*ArcCsc[c*x]))/(75*d*x^5) + ((I/75)*b*c*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[I*
ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 34*c^2*d
*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d)))])/(Sqrt[-c^
2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple [F]** time = 5.984, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^6} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx))\sqrt{ex^2 + d}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^6,
x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsc(c\*x))/x\*\*6,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arccsc}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^6,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsc(c\*x) + a)/x^6, x)

$$3.137 \quad \int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=554

$$\frac{2bx\sqrt{1-c^2x^2}(c^2d+e)(204c^4d^2e+120c^6d^3+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)+2e(d+ex^2)^{5/2}}{3675d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

```
[Out] -(b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d^2*Sqrt[c^2*x^2]) - (b*c*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[c^2*x^2]) - (b*c*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^4*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(7*d*x^7) + (2*e*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e^2 - 105*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.891112, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {271, 264, 5239, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e(d+ex^2)^{5/2}(a+b \operatorname{csc}^{-1}(cx))}{35d^2x^5} - \frac{(d+ex^2)^{5/2}(a+b \operatorname{csc}^{-1}(cx))}{7dx^7} - \frac{bc\sqrt{c^2x^2-1}(528c^4d^2e+240c^6d^3+193c^2de^2-247e^3)}{3675d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8, x]
```

```
[Out] -(b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d^2*Sqrt[c^2*x^2]) - (b*c*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[c^2*x^2]) - (b*c*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^4*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(7*d*x^7) + (2*e*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e^2 - 105*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

### Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5239

```
Int[((a_) + ArcCsc[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 580

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1
))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 427

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
```

#### Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2} (a+b \operatorname{csc}^{-1}(cx))}{x^8} dx &= -\frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{35d^2x^5} + \frac{(bcx) \int \frac{(d+ex^2)}{35d^2x^8}}{\sqrt{d+ex^2}} \\
&= -\frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{35d^2x^5} + \frac{(bcx) \int \frac{(d+ex^2)}{35d^2x^8}}{35d^2} \\
&= -\frac{bc\sqrt{-1+c^2x^2} (d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{35d^2x^5} \\
&= -\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2} (d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2} (d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+b \operatorname{csc}^{-1}(cx))}{35d^2x^5} \\
&= -\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} - \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}}{1225dx^4\sqrt{c^2x^2}} \\
&= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2)}{1225dx^4\sqrt{c^2x^2}} \\
&= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2)}{1225dx^4\sqrt{c^2x^2}} \\
&= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2)}{1225dx^4\sqrt{c^2x^2}} \\
&= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2)}{1225dx^4\sqrt{c^2x^2}} \\
&= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2)}{1225dx^4\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.851825, size = 383, normalized size = 0.69

$$\frac{\sqrt{d+ex^2} \left( 105a(5d-2ex^2)(d+ex^2)^2 + bcx\sqrt{1-\frac{1}{c^2x^2}}(3d^2ex^2(176c^4x^4+83c^2x^2+61) + 15d^3(16c^6x^6+8c^4x^4+6c^2x^2)) \right)}{3675d^2x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/x^8, x]

[Out] -(Sqrt[d + e\*x^2]\*(105\*a\*(5\*d - 2\*e\*x^2)\*(d + e\*x^2)^2 + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(-247\*e^3\*x^6 + d\*e^2\*x^4\*(71 + 193\*c^2\*x^2) + 3\*d^2\*e\*x^2\*(61 + 83\*c^2\*x^2 + 176\*c^4\*x^4) + 15\*d^3\*(5 + 6\*c^2\*x^2 + 8\*c^4\*x^4 + 16\*c^6\*x^6)) + 105\*b\*(5\*d - 2\*e\*x^2)\*(d + e\*x^2)^2\*ArcCsc[c\*x]))/(3675\*d^2\*x^7) + ((I/3675)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(240\*c^6\*d^3 + 528\*c^4\*d^2\*e + 193\*c^2\*d\*e^2 - 247\*e^3)\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] - 2\*(120\*c^8\*d^4 + 324\*c^6\*d^3\*e + 221\*c^4\*d^2\*e^2 - 88\*c^2\*d\*e^3 - 105\*e^4)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))]))/(Sqrt[-c^2]\*d^2\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 5.132, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^8} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^8,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^8,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^8,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx))\sqrt{ex^2 + d}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x))/x^8,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d)/x^8, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsc(c\*x))/x\*\*8,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arccsc}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^8, x)
```



$$3.138 \quad \int \frac{x^5 (a + b \csc^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=321

$$\frac{d^2 \sqrt{d+ex^2} (a + b \csc^{-1}(cx))}{e^3} - \frac{2d (d+ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^3} - \frac{8bcd^{5/2} x \tan^{-1}\left(\frac{d+ex^2}{\sqrt{d+ex^2}}\right)}{15e^3 \sqrt{c^2 x^2}}$$

```
[Out] -(b*(19*c^2*d - 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e^2*Sqr
t[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[c^2
*x^2]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 - (2*d*(d + e*x^2)^(
3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))
/(5*e^3) - (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(15*e^3*Sqrt[c^2*x^2]) + (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*x*ArcT
anh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqr
t[c^2*x^2])
```

**Rubi [A]** time = 1.04819, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5239, 12, 1615, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d^2 \sqrt{d+ex^2} (a + b \csc^{-1}(cx))}{e^3} - \frac{2d (d+ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^3} - \frac{8bcd^{5/2} x \tan^{-1}\left(\frac{d+ex^2}{\sqrt{d+ex^2}}\right)}{15e^3 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -(b*(19*c^2*d - 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e^2*Sqr
t[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[c^2
*x^2]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 - (2*d*(d + e*x^2)^(
3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))
/(5*e^3) - (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(15*e^3*Sqrt[c^2*x^2]) + (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*x*ArcT
anh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqr
t[c^2*x^2])
```

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5239

```
Int[((a_) + ArcCsc[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x
_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 1615

```
Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f
_)*(x_))^(p_), x_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
+ p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 154

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_
))^(p_)*((g_) + (h_)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 157

```
Int[((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_)*((g_) + (h_)*(x_
)))/((a_) + (b_)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*(e + f*x)^p/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

### Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt
```

Q[a, 0] || LtQ[b, 0])

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (a + b \csc^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \csc^{-1}(cx))}{5e^3} \\
 &= \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} \\
 &= -\frac{b (19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} + \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} \\
 &= -\frac{b (19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} + \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} \\
 &= -\frac{b (19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} + \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} \\
 &= -\frac{b (19c^2 d - 9e) x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{c^2 x^2}} + \frac{bx \sqrt{-1 + c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.745021, size = 328, normalized size = 1.02

$$\frac{\sqrt{d + ex^2} \left( 8ac^3 (8d^2 - 4dex^2 + 3e^2x^4) + 8bc^3 \csc^{-1}(cx) (8d^2 - 4dex^2 + 3e^2x^4) + bex \sqrt{1 - \frac{1}{c^2 x^2}} (c^2 (6ex^2 - 13d) + 9e) \right)}{120c^3 e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(8\*a\*c^3\*(8\*d^2 - 4\*d\*e\*x^2 + 3\*e^2\*x^4) + b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(9\*e + c^2\*(-13\*d + 6\*e\*x^2)) + 8\*b\*c^3\*(8\*d^2 - 4\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsc[c\*x]))/(120\*c^3\*e^3) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*(45\*c^4\*d^2 - 10\*c^2\*d\*e + 9\*e^2)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])]) + 64\*c^7\*d^(5/2)\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]]))/(120\*c^6\*e^3\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 6.129, size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{arccsc}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

[Out] int(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 20.0825, size = 3063, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] [1/480\*(64\*b\*c^5\*sqrt(-d)\*d^2\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (45\*b\*c^4\*d^2 - 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + 8\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*arccsc(c\*x) + (6\*b\*c^3\*e^2\*x^2 - 13\*b\*c^3\*d\*e + 9\*b\*c^3\*e^2)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^5\*e^3), -1/480\*(128\*b\*c^5\*d^(5/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt

```
(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), -1/240*(64*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3)]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(1/2), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^5/sqrt(e\*x^2 + d), x)

$$3.139 \quad \int \frac{x^3 (a + b \csc^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=225

$$-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{bx(3c^2d-e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{d+ex^2}}{c\sqrt{d}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

[Out] (b\*x\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])/(6\*c\*e\*Sqrt[c^2\*x^2]) - (d\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e^2 + ((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*e^2) + (2\*b\*c\*d^(3/2)\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*e^2\*Sqrt[c^2\*x^2]) - (b\*(3\*c^2\*d - e)\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(6\*c^2\*e^(3/2)\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.327374, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 5239, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{bx(3c^2d-e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{d+ex^2}}{c\sqrt{d}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (b\*x\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])/(6\*c\*e\*Sqrt[c^2\*x^2]) - (d\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e^2 + ((d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*e^2) + (2\*b\*c\*d^(3/2)\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*e^2\*Sqrt[c^2\*x^2]) - (b\*(3\*c^2\*d - e)\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(6\*c^2\*e^(3/2)\*Sqrt[c^2\*x^2])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

$\text{Int}[(a\_)(u\_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b\_)(v\_)] /; \text{FreeQ}[b, x]$

Rule 573

$\text{Int}[(x\_)^{(m\_)}*((a\_)+(b\_)(x\_)^{(n\_}))^{(p\_)}*((c\_)+(d\_)(x\_)^{(n\_}))^{(q\_)}*((e\_)+(f\_)(x\_)^{(n\_}))^{(r\_)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n)-1}*(a+bx)^p*(c+dx)^q*(e+fx)^r, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p, q, r\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 154

$\text{Int}[(a\_)+(b\_)(x\_)^{(m\_)}*((c\_)+(d\_)(x\_)^{(n\_)}*((e\_)+(f\_)(x\_)^{(p\_)}*((g\_)+(h\_)(x\_))^{(n\_)})), x\_Symbol] \rightarrow \text{Simp}[(h*(a+bx)^m*(c+dx)^{(n+1)}*(e+fx)^{(p+1)})/(d*f*(m+n+p+2)), x] + \text{Dist}[1/(d*f*(m+n+p+2)), \text{Int}[(a+bx)^{(m-1)}*(c+dx)^n*(e+fx)^p*\text{Simp}[a*d*f*g*(m+n+p+2)-h*(b*c*e*m+a*(d*e*(n+1)+c*f*(p+1)))+(b*d*f*g*(m+n+p+2)+h*(a*d*f*m-b*(d*e*(m+n+1)+c*f*(m+p+1)))*x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{NeQ}[m+n+p+2, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*n, 2*p]$

Rule 157

$\text{Int}[(c\_)+(d\_)(x\_)^{(n\_)}*((e\_)+(f\_)(x\_)^{(p\_)}*((g\_)+(h\_)(x\_))^{(n\_)}))/((a\_)+(b\_)(x\_)), x\_Symbol] \rightarrow \text{Dist}[h/b, \text{Int}[(c+dx)^n*(e+fx)^p, x], x] + \text{Dist}[(b*g-a*h)/b, \text{Int}[(c+dx)^n*(e+fx)^p/(a+bx), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x]$

Rule 63

$\text{Int}[(a\_)+(b\_)(x\_)^{(m\_)}*((c\_)+(d\_)(x\_)^{(n\_)}), x\_Symbol] \rightarrow \text{With}\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c-(a*d)/b+(d*x^p)/b)^n, x], x, (a+bx)^{(1/p)}], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c-a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a\_)+(b\_)(x\_)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1-b*x^2), x], x, x/\text{Sqrt}[a+bx^2]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 206

$\text{Int}[(a\_)+(b\_)(x\_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 93

$\text{Int}[(a\_)+(b\_)(x\_)^{(m\_)}*((c\_)+(d\_)(x\_)^{(n\_)}))/((e\_)+(f\_)(x\_)), x\_Symbol] \rightarrow \text{With}\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1)-1)}]/(b*e-a*f-(d*e-c*f)*x^q), x], x, (a+bx)^{(1/q)}/(c+dx)^{(1/q)}], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[m+n+1, 0] \ \&\& \ \text{RationalQ}[n] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{SimplerQ}[a+bx, c+dx]$

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{x^3 (a + b \operatorname{csc}^{-1}(cx))}{\sqrt{d + ex^2}} dx &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} + \frac{(bcx) \int \frac{(-2d+ex^2)\sqrt{d+ex^2}}{3e^2x\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\ &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} + \frac{(bcx) \int \frac{(-2d+ex^2)\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}} dx}{3e^2\sqrt{c^2x^2}} \\ &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{-1+c^2x}} dx\right)}{6e^2\sqrt{c^2x^2}} \\ &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} + \dots \\ &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} - \dots \\ &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} - \dots \\ &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} - \dots \\ &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^2} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.515055, size = 271, normalized size = 1.2

$$\frac{\sqrt{d + ex^2} \left( -4acd + 2acex^2 + bex\sqrt{1 - \frac{1}{c^2x^2}} + 2bc \operatorname{csc}^{-1}(cx) (ex^2 - 2d) \right)}{6ce^2} - \frac{bx\sqrt{1 - \frac{1}{c^2x^2}} \left( 4c^5d^{3/2}\sqrt{d + ex^2} \tan^{-1} \left( \frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}} \right) \right)}{6c^4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

```
[Out] (Sqrt[d + e*x^2]*(-4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^2)]*x + 2*a*c*e*x^2 + 2*b*c*(-2*d + e*x^2)*ArcCsc[c*x]))/(6*c*e^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^2]*(3*c^2*d - e)*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])]) + 4*c^5*d^(3/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(6*c^4*e^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```



**Maple [F]** time = 6.874, size = 0, normalized size = 0.

$$\int x^3 (a + \operatorname{arccsc}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 8.08869, size = 2449, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/24\*(4\*b\*c^3\*sqrt(-d)\*d\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) - (3\*b\*c^2\*d - b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(2\*a\*c^3\*e\*x^2 - 4\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d))/(c^3\*e^2), 1/24\*(8\*b\*c^3\*d^(3/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) - (3\*b\*c^2\*d - b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 4\*(2\*a\*c^3\*e\*x^2 - 4\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d))/(c^3\*e^2), 1/12\*(2\*b\*c^3\*sqrt(-d)\*d\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 - 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + (3\*b\*c^2\*d - b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*(2\*a\*c^3\*e\*x^2 - 4\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d))/(c^3\*e^2), 1/12\*(4\*b\*c^3\*d^(3/2)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) + (3\*b\*c^2\*d - b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^2 + c^2\*d - e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(-e)/(c^3\*e^2\*x^4 - c\*d\*e + (c^3\*d\*e - c\*e^2)\*x^2)) + 2\*(2\*a\*c^3\*e\*x^2 - 4\*a\*c^3\*d + sqrt(c^2\*x^2 - 1)\*b\*c\*e + 2\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d))/(c^3

\*e<sup>2</sup>)]

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^3/sqrt(e\*x^2 + d), x)

$$3.140 \quad \int \frac{x(a+b \csc^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=132

$$\frac{\sqrt{d+ex^2}(a+b \csc^{-1}(cx))}{e} - \frac{bc\sqrt{d} \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}} + \frac{bx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

[Out] (Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e - (b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(e\*Sqrt[c^2\*x^2]) + (b\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(Sqrt[e]\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.150841, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {5237, 446, 105, 63, 217, 206, 93, 204}

$$\frac{\sqrt{d+ex^2}(a+b \csc^{-1}(cx))}{e} - \frac{bc\sqrt{d} \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}} + \frac{bx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e - (b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(e\*Sqrt[c^2\*x^2]) + (b\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(Sqrt[e]\*Sqrt[c^2\*x^2])

#### Rule 5237

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsc[c\*x]))/(2\*e\*(p + 1)), x] + Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 105

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Dist[b/f, Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x] - Dist[(b\*e - a\*f)/f, Int[((a + b\*x)^(m - 1)\*(c + d\*x)^n)/(e + f\*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ

[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 93

Int[(((a\_) + (b\_)\*(x\_)^m)\*((c\_) + (d\_)\*(x\_)^n))/((e\_) + (f\_)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{x(a + b \csc^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{e} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}} dx}{e\sqrt{c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{e} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}} dx, x, x^2\right)}{2e\sqrt{c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{e} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx, x, x^2\right)}{2\sqrt{c^2x^2}} + \frac{(bcdx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x^2}} dx, x, x^2\right)}{2e\sqrt{c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{e} + \frac{(bx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}} dx, x, \sqrt{-1+c^2x^2}\right)}{c\sqrt{c^2x^2}} + \frac{(bcdx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x^2}} dx, x, x^2\right)}{2e\sqrt{c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{e} - \frac{bc\sqrt{dx} \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{(bx) \operatorname{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}} dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{c\sqrt{c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{e} - \frac{bc\sqrt{dx} \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{bx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.223011, size = 210, normalized size = 1.59

$$\frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{e} + \frac{bx\sqrt{1 - \frac{1}{c^2x^2}} \left( c^3\sqrt{d}\sqrt{d + ex^2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right) + \sqrt{c^2}\sqrt{e}\sqrt{c^2d + e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}} \sinh^{-1}\left(\frac{c\sqrt{e}\sqrt{c^2x^2}}{\sqrt{c^2}\sqrt{c^2d+e}}\right) \right)}{c^2e\sqrt{c^2x^2 - 1}\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])] + c^3*Sqrt[d]*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(c^2*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple [F]** time = 4.49, size = 0, normalized size = 0.

$$\int x(a + b \operatorname{arccsc}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

**Fricas [A]** time = 4.29964, size = 1928, normalized size = 14.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), -1/4*(2*b*c*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), 1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*
```

```

b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^
2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 4*sqrt(e*x
^2 + d)*(b*c*arccsc(c*x) + a*c)/(c*e), -1/2*(b*c*sqrt(d)*arctan(-1/2*sqrt(
c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 +
(c^2*d^2 - d*e)*x^2 - d^2)) + b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d -
e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*
d*e - c*e^2)*x^2)) - 2*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e)]

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(a + b \operatorname{arccsc}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*arccsc(c*x))/sqrt(d + e*x**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

$$3.141 \quad \int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

**Rubi [A]** time = 0.096927, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

[Out] Defer[Int][(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex^2}} dx = \int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

**Mathematica [A]** time = 1.66954, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

**Maple [A]** time = 5.735, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x} \frac{1}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^(1/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{ex^3 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/(e\*x^3 + d\*x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{x\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsc(c\*x))/(x\*sqrt(d + e\*x\*\*2)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex^2 + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(sqrt(e\*x^2 + d)\*x), x)



$$3.142 \quad \int \frac{a+b \csc^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x^3 \sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

**Rubi [A]** time = 0.110658, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

[Out] Defer[Int][(a + b\*ArcCsc[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx = \int \frac{a+b \csc^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

**Mathematica [A]** time = 13.0237, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x^3 \sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

**Maple [A]** time = 1.263, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x^3} \frac{1}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^3/(e\*x^2+d)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))/x^3/(e\*x^2+d)^(1/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^3/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{ex^5 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^3/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/(e\*x^5 + d\*x^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*3/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex^2 + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^3/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(sqrt(e\*x^2 + d)\*x^3), x)

$$3.143 \quad \int \frac{x^2(a+b \operatorname{csc}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{x^2(a+b \operatorname{csc}^{-1}(cx))}{\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[(x^2\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

**Rubi [A]** time = 0.092479, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^2(a+b \operatorname{csc}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Defer[Int] [(x^2\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

Rubi steps

$$\int \frac{x^2(a+b \operatorname{csc}^{-1}(cx))}{\sqrt{d+ex^2}} dx = \int \frac{x^2(a+b \operatorname{csc}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Mathematica [A]** time = 12.9801, size = 0, normalized size = 0.

$$\int \frac{x^2(a+b \operatorname{csc}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

**Maple [A]** time = 4.946, size = 0, normalized size = 0.

$$\int x^2(a+b \operatorname{arccsc}(cx)) \frac{1}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

[Out] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arccsc}(cx) + ax^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsc(c\*x) + a\*x^2)/sqrt(e\*x^2 + d), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^2/sqrt(e\*x^2 + d), x)

$$3.144 \quad \int \frac{a+b \csc^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/Sqrt[d + e\*x^2], x]

**Rubi [A]** time = 0.0307962, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/Sqrt[d + e\*x^2], x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/Sqrt[d + e\*x^2], x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{\sqrt{d+ex^2}} dx = \int \frac{a+b \csc^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

**Mathematica [A]** time = 0.986144, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/Sqrt[d + e\*x^2], x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/Sqrt[d + e\*x^2], x]

**Maple [A]** time = 4.268, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)/sqrt(e\*x^2 + d), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsc(c\*x))/sqrt(d + e\*x\*\*2), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/sqrt(e\*x^2 + d), x)

$$3.145 \quad \int \frac{a+b \csc^{-1}(cx)}{x^2 \sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=247

$$\frac{bx\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{\sqrt{d+ex^2}(a+b \csc^{-1}(cx))}{dx} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} + \dots$$

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[c^2*x^2])) - (Sqrt[d + e
*x^2]*(a + b*ArcCsc[c*x]))/(d*x) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2
]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2
)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.2665, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {264, 5239, 12, 475, 21, 423, 427, 426, 424, 421, 419}

$$\frac{\sqrt{d+ex^2}(a+b \csc^{-1}(cx))}{dx} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{bx\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{F}\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}}{d\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[c^2*x^2])) - (Sqrt[d + e
*x^2]*(a + b*ArcCsc[c*x]))/(d*x) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2
]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2
)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])
```

#### Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps



$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^2 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{dx^2 \sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x^2 \sqrt{-1+c^2x^2}} dx}{d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} + \frac{(bcx) \int \frac{-e+c^2ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} + \frac{(bcex) \int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}} dx}{d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} + \frac{(bc^3x) \int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}} dx}{d\sqrt{c^2x^2}} - \frac{(bc(c^2d + e))}{d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} + \frac{(bc^3x\sqrt{1 - c^2x^2}) \int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}} dx}{d\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}} - \frac{(bc(c^2d + e))}{d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} + \frac{(bc^3x\sqrt{1 - c^2x^2}\sqrt{d + ex^2}) \int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}} dx}{d\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}\sqrt{1 + \frac{ex^2}{d}}} - \frac{(bc(c^2d + e))}{d\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{dx} + \frac{bc^2x\sqrt{1 - c^2x^2}\sqrt{d + ex^2}E(\sin^{-1}(cx))}{d\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}\sqrt{1 + \frac{ex^2}{d}}} - \frac{(bc(c^2d + e))}{d\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.187339, size = 140, normalized size = 0.57

$$\frac{bcex\sqrt{1 - \frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}} + 1E\left(\sin^{-1}\left(\sqrt{-\frac{e}{d}}x\right) \middle| -\frac{c^2d}{e}\right)}{d\sqrt{1 - c^2x^2}\sqrt{-\frac{e}{d}}\sqrt{d + ex^2}} - \frac{\sqrt{d + ex^2}\left(a + bcx\sqrt{1 - \frac{1}{c^2x^2}} + b \csc^{-1}(cx)\right)}{dx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*Sqrt[d + e\*x^2]), x]

[Out] -((Sqrt[d + e\*x^2]\*(a + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x + b\*ArcCsc[c\*x]))/(d\*x) + (b\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*EllipticE[ArcSin[Sqrt[-(e/d)]\*x], -(c^2\*d)/e]))/(d\*Sqrt[-(e/d)]\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 4.654, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^(1/2), x)

[Out] `int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^4 + d*x^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^2 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(1/2),x)`

[Out] `Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x**2)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex^2 + dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)`

$$3.146 \quad \int \frac{a+b \csc^{-1}(cx)}{x^4 \sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=362

$$\frac{2bx\sqrt{1-c^2x^2}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{2e\sqrt{d+ex^2}(a+b \csc^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2}}{9d}$$

```
[Out] -(b*c*(2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*Sqrt[c^2*x^2]) - (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d - 5*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.568787, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {271, 264, 5239, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e\sqrt{d+ex^2}(a+b \csc^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2}(a+b \csc^{-1}(cx))}{3dx^3} - \frac{bc\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{2bx\sqrt{1-c^2x^2}(c^2d-3e)\sqrt{d+ex^2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -(b*c*(2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*Sqrt[c^2*x^2]) - (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d - 5*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

#### Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

#### Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 580

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

### Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

### Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

### Rule 427

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

### Rule 426

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

### Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/(Sqrt[a + b\*x^2]\*Sqrt[1 + (d\*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \csc^{-1}(cx)}{x^4 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3d^2x} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{3d^2x^4\sqrt{-1+c^2x^2}} dx}{\sqrt{c^2x^2}} \\
 &= -\frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3d^2x} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{x^4\sqrt{-1+c^2x^2}} dx}{3d^2\sqrt{c^2x^2}} \\
 &= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3d^2x} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{x^4\sqrt{-1+c^2x^2}} dx}{3d^2\sqrt{c^2x^2}} \\
 &= -\frac{bc(2c^2d - 5e)\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} \\
 &= -\frac{bc(2c^2d - 5e)\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} \\
 &= -\frac{bc(2c^2d - 5e)\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} \\
 &= -\frac{bc(2c^2d - 5e)\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} \\
 &= -\frac{bc(2c^2d - 5e)\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3} \\
 &= -\frac{bc(2c^2d - 5e)\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{3dx^3}
 \end{aligned}$$

**Mathematica [C]** time = 0.658398, size = 249, normalized size = 0.69

$$\frac{\sqrt{d + ex^2} \left( 3a(d - 2ex^2) + bcx\sqrt{1 - \frac{1}{c^2x^2}}(2c^2dx^2 + d - 5ex^2) + 3b \csc^{-1}(cx)(d - 2ex^2) \right)}{9d^2x^3} + \frac{ibcx\sqrt{1 - \frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d} + 1}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^4\*Sqrt[d + e\*x^2]), x]

[Out] -(Sqrt[d + e\*x^2]\*(b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + 2\*c^2\*d\*x^2 - 5\*e\*x^2) + 3\*a\*(d - 2\*e\*x^2) + 3\*b\*(d - 2\*e\*x^2)\*ArcCsc[c\*x]))/(9\*d^2\*x^3) + ((I/9)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(2\*c^2\*d - 5\*e)\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x))

```
pticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e +
3*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d)))]/(Sqrt[-c^2]*d^2*S
qrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple [F]** time = 8.776, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^4} \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{ex^6 + dx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^6 + d*x^4), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex^2 + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

$$3.147 \quad \int \frac{x^5 (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=252

$$-\frac{d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \frac{8bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}} + \dots$$

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + (8*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e^3*Sqrt[c^2*x^2]) - (b*(9*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*e^(5/2)*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 1.07354, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 5239, 12, 1615, 157, 63, 217, 206, 93, 204}

$$-\frac{d^2 (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \csc^{-1}(cx))}{3e^3} + \frac{8bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + (8*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e^3*Sqrt[c^2*x^2]) - (b*(9*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*e^(5/2)*Sqrt[c^2*x^2])
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
```



\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 1615

Int[(Px\_)\*((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k\*(a + b\*x)^(m + q - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*b^(q - 1)\*(m + n + p + q + 1)), x] + Dist[1/(d\*f\*b^q\*(m + n + p + q + 1)), Int[(a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^p\*ExpandToSum[d\*f\*b^q\*(m + n + p + q + 1)\*Px - d\*f\*k\*(m + n + p + q + 1)\*(a + b\*x)^q + k\*(a + b\*x)^(q - 2)\*(a^2\*d\*f\*(m + n + p + q + 1) - b\*(b\*c\*e\*(m + q - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*(m + q) + n + p) - b\*(d\*e\*(m + q + n) + c\*f\*(m + q + p)))\*x], x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_))) / ((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= -\frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3} + \frac{(bc)}{e^3} \\
 &= -\frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3} + \frac{(bc)}{e^3} \\
 &= -\frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3} + \frac{(bc)}{e^3} \\
 &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3} \\
 &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3} \\
 &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3} \\
 &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3} \\
 &= \frac{bx\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2 (a + b \operatorname{csc}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx))}{3e^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.607621, size = 302, normalized size = 1.2

$$\frac{-2ac(8d^2 + 4dex^2 - e^2x^4) + bex\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2) - 2bc \operatorname{csc}^{-1}(cx)(8d^2 + 4dex^2 - e^2x^4)}{6ce^3\sqrt{d + ex^2}} - \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}(16c^5d^{3/2}\sqrt{d + ex^2})}{6ce^3\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] (b\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + e\*x^2) - 2\*a\*c\*(8\*d^2 + 4\*d\*e\*x^2 - e^2\*x^4) - 2\*b\*c\*(8\*d^2 + 4\*d\*e\*x^2 - e^2\*x^4)\*ArcCsc[c\*x])/(6\*c\*e^3\*Sqrt[d + e\*x^2]) - (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*(9\*c^2\*d - e)\*Sqrt[e]\*Sqrt[c^2\*d + e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])]) + 16\*c^5\*d^(3/2)\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(6\*c^4\*e^3\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 3.643, size = 0, normalized size = 0.

$$\int x^5 (a + \operatorname{barccsc}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 7.19572, size = 3167, normalized size = 12.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2
+ c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 16*(b*c^
3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c
^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b
*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^
3*d*e^3), 1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^2
*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c
^2*d^2 - d*e)*x^2 - d^2)) - (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^
2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*
x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(
e) + e^2) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*
e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*e^2*x^2 + b*c*d
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9
*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
- c*d*e + (c^3*d*e - c*e^2)*x^2)) + 8*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*
log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*
(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*
c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x
^2 - 1)*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*(16*(b*c^3*d*e*x^
2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d
)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b
*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*
x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16
*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x)
```

```
+ (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d)/(c^3*e^4*x^2
+ c^3*d*e^3)]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

$$3.148 \quad \int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=156

$$\frac{\sqrt{d+ex^2} (a + b \csc^{-1}(cx))}{e^2} + \frac{d (a + b \csc^{-1}(cx))}{e^2 \sqrt{d+ex^2}} - \frac{2bc \sqrt{dx} \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^2 \sqrt{c^2x^2}} + \frac{bx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2} \sqrt{c^2x^2}}$$

[Out] (d\*(a + b\*ArcCsc[c\*x]))/(e^2\*Sqrt[d + e\*x^2]) + (Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e^2 - (2\*b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(e^2\*Sqrt[c^2\*x^2]) + (b\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(e^(3/2)\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.321309, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 5239, 12, 573, 157, 63, 217, 206, 93, 204}

$$\frac{\sqrt{d+ex^2} (a + b \csc^{-1}(cx))}{e^2} + \frac{d (a + b \csc^{-1}(cx))}{e^2 \sqrt{d+ex^2}} - \frac{2bc \sqrt{dx} \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^2 \sqrt{c^2x^2}} + \frac{bx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2} \sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] (d\*(a + b\*ArcCsc[c\*x]))/(e^2\*Sqrt[d + e\*x^2]) + (Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e^2 - (2\*b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(e^2\*Sqrt[c^2\*x^2]) + (b\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(e^(3/2)\*Sqrt[c^2\*x^2])

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 5239

Int[((a\_) + ArcCsc[(c\_)\*(x\_)])\*(b\_))\*((f\_)\*(x\_)^(m\_))\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 573

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] :=> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

### Rule 157

```
Int[(((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_)*((g_) + (h_)*(x_
)))/((a_) + (b_)*(x_)), x_Symbol] :=> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*(e + f*x)^p/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

### Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :=> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :=> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 93

```
Int[(((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_))/((e_) + (f_)*(x
_)), x_Symbol] :=> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= \frac{d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^2} + \frac{(bcx) \int \frac{2d + ex^2}{e^2 x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}} dx}{\sqrt{c^2 x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^2} + \frac{(bcx) \int \frac{2d + ex^2}{x \sqrt{-1 + c^2 x^2} \sqrt{d + ex^2}} dx}{e^2 \sqrt{c^2 x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^2} + \frac{(bcx) \text{Subst} \left( \int \frac{2d + ex}{x \sqrt{-1 + c^2 x} \sqrt{d + ex}} dx, x, x \right)}{2e^2 \sqrt{c^2 x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^2} + \frac{(bcdx) \text{Subst} \left( \int \frac{1}{x \sqrt{-1 + c^2 x} \sqrt{d + ex}} dx, x, x \right)}{e^2 \sqrt{c^2 x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^2} + \frac{(2bcdx) \text{Subst} \left( \int \frac{1}{-d - x^2} dx, x, \frac{\sqrt{d + ex^2}}{\sqrt{-1 + c^2 x}} \right)}{e^2 \sqrt{c^2 x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^2} - \frac{2bc\sqrt{dx} \tan^{-1} \left( \frac{\sqrt{d + ex^2}}{\sqrt{d}\sqrt{-1 + c^2 x^2}} \right)}{e^2 \sqrt{c^2 x^2}} + \frac{(bx) \text{Subst} \left( \int \frac{1}{-d - x^2} dx, x, \frac{\sqrt{d + ex^2}}{\sqrt{-1 + c^2 x}} \right)}{e^2 \sqrt{c^2 x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^2} - \frac{2bc\sqrt{dx} \tan^{-1} \left( \frac{\sqrt{d + ex^2}}{\sqrt{d}\sqrt{-1 + c^2 x^2}} \right)}{e^2 \sqrt{c^2 x^2}} + \frac{bx \text{Subst} \left( \int \frac{1}{-d - x^2} dx, x, \frac{\sqrt{d + ex^2}}{\sqrt{-1 + c^2 x}} \right)}{e^2 \sqrt{c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.348843, size = 220, normalized size = 1.41

$$\frac{(2d + ex^2)(a + b \csc^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{bx \sqrt{1 - \frac{1}{c^2 x^2}} \left( 2c^3 \sqrt{d} \sqrt{d + ex^2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{c^2 x^2 - 1}}{\sqrt{d + ex^2}} \right) + \sqrt{c^2} \sqrt{e} \sqrt{c^2 d + e} \sqrt{\frac{c^2 (d + ex^2)}{c^2 d + e}} \sinh^{-1} \left( \frac{c \sqrt{d + ex^2}}{\sqrt{d + ex^2}} \right) \right)}{c^2 e^2 \sqrt{c^2 x^2 - 1} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] ((2\*d + e\*x^2)\*(a + b\*ArcCsc[c\*x]))/(e^2\*Sqrt[d + e\*x^2]) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])] + 2\*c^3\*Sqrt[d]\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(c^2\*e^2\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 3.068, size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x)

[Out] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>3</sup>\*(a+b\*arccsc(c\*x))/(e\*x<sup>2</sup>+d)<sup>(3/2)</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 4.97111, size = 2364, normalized size = 15.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>3</sup>\*(a+b\*arccsc(c\*x))/(e\*x<sup>2</sup>+d)<sup>(3/2)</sup>,x, algorithm="fricas")

[Out] [1/4\*((b\*e\*x<sup>2</sup> + b\*d)\*sqrt(e)\*log(8\*c<sup>4</sup>\*e<sup>2</sup>\*x<sup>4</sup> + c<sup>4</sup>\*d<sup>2</sup> - 6\*c<sup>2</sup>\*d\*e + 8\*(c<sup>4</sup>\*d\*e - c<sup>2</sup>\*e<sup>2</sup>)\*x<sup>2</sup> + 4\*(2\*c<sup>3</sup>\*e\*x<sup>2</sup> + c<sup>3</sup>\*d - c\*e)\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(e) + e<sup>2</sup>) + 2\*(b\*c\*e\*x<sup>2</sup> + b\*c\*d)\*sqrt(-d)\*log(((c<sup>4</sup>\*d<sup>2</sup> - 6\*c<sup>2</sup>\*d\*e + e<sup>2</sup>)\*x<sup>4</sup> - 8\*(c<sup>2</sup>\*d<sup>2</sup> - d\*e)\*x<sup>2</sup> + 4\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*((c<sup>2</sup>\*d - e)\*x<sup>2</sup> - 2\*d)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(-d) + 8\*d<sup>2</sup>)/x<sup>4</sup>) + 4\*(a\*c\*e\*x<sup>2</sup> + 2\*a\*c\*d + (b\*c\*e\*x<sup>2</sup> + 2\*b\*c\*d)\*arccsc(c\*x))\*sqrt(e\*x<sup>2</sup> + d))/(c\*e<sup>3</sup>\*x<sup>2</sup> + c\*d\*e<sup>2</sup>), -1/4\*(4\*(b\*c\*e\*x<sup>2</sup> + b\*c\*d)\*sqrt(d)\*arctan(-1/2\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*((c<sup>2</sup>\*d - e)\*x<sup>2</sup> - 2\*d)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(d)/(c<sup>2</sup>\*d\*e\*x<sup>4</sup> + (c<sup>2</sup>\*d<sup>2</sup> - d\*e)\*x<sup>2</sup> - d<sup>2</sup>)) - (b\*e\*x<sup>2</sup> + b\*d)\*sqrt(e)\*log(8\*c<sup>4</sup>\*e<sup>2</sup>\*x<sup>4</sup> + c<sup>4</sup>\*d<sup>2</sup> - 6\*c<sup>2</sup>\*d\*e + 8\*(c<sup>4</sup>\*d\*e - c<sup>2</sup>\*e<sup>2</sup>)\*x<sup>2</sup> + 4\*(2\*c<sup>3</sup>\*e\*x<sup>2</sup> + c<sup>3</sup>\*d - c\*e)\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(e) + e<sup>2</sup>) - 4\*(a\*c\*e\*x<sup>2</sup> + 2\*a\*c\*d + (b\*c\*e\*x<sup>2</sup> + 2\*b\*c\*d)\*arccsc(c\*x))\*sqrt(e\*x<sup>2</sup> + d))/(c\*e<sup>3</sup>\*x<sup>2</sup> + c\*d\*e<sup>2</sup>), -1/2\*((b\*e\*x<sup>2</sup> + b\*d)\*sqrt(-e)\*arctan(1/2\*(2\*c<sup>2</sup>\*e\*x<sup>2</sup> + c<sup>2</sup>\*d - e)\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(-e)/(c<sup>3</sup>\*e<sup>2</sup>\*x<sup>4</sup> - c\*d\*e + (c<sup>3</sup>\*d\*e - c\*e<sup>2</sup>)\*x<sup>2</sup>)) - (b\*c\*e\*x<sup>2</sup> + b\*c\*d)\*sqrt(-d)\*log(((c<sup>4</sup>\*d<sup>2</sup> - 6\*c<sup>2</sup>\*d\*e + e<sup>2</sup>)\*x<sup>4</sup> - 8\*(c<sup>2</sup>\*d<sup>2</sup> - d\*e)\*x<sup>2</sup> + 4\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*((c<sup>2</sup>\*d - e)\*x<sup>2</sup> - 2\*d)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(-d) + 8\*d<sup>2</sup>)/x<sup>4</sup>) - 2\*(a\*c\*e\*x<sup>2</sup> + 2\*a\*c\*d + (b\*c\*e\*x<sup>2</sup> + 2\*b\*c\*d)\*arccsc(c\*x))\*sqrt(e\*x<sup>2</sup> + d))/(c\*e<sup>3</sup>\*x<sup>2</sup> + c\*d\*e<sup>2</sup>), -1/2\*(2\*(b\*c\*e\*x<sup>2</sup> + b\*c\*d)\*sqrt(d)\*arctan(-1/2\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*((c<sup>2</sup>\*d - e)\*x<sup>2</sup> - 2\*d)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(d)/(c<sup>2</sup>\*d\*e\*x<sup>4</sup> + (c<sup>2</sup>\*d<sup>2</sup> - d\*e)\*x<sup>2</sup> - d<sup>2</sup>)) + (b\*e\*x<sup>2</sup> + b\*d)\*sqrt(-e)\*arctan(1/2\*(2\*c<sup>2</sup>\*e\*x<sup>2</sup> + c<sup>2</sup>\*d - e)\*sqrt(c<sup>2</sup>\*x<sup>2</sup> - 1)\*sqrt(e\*x<sup>2</sup> + d)\*sqrt(-e)/(c<sup>3</sup>\*e<sup>2</sup>\*x<sup>4</sup> - c\*d\*e + (c<sup>3</sup>\*d\*e - c\*e<sup>2</sup>)\*x<sup>2</sup>)) - 2\*(a\*c\*e\*x<sup>2</sup> + 2\*a\*c\*d + (b\*c\*e\*x<sup>2</sup> + 2\*b\*c\*d)\*arccsc(c\*x))\*sqrt(e\*x<sup>2</sup> + d))/(c\*e<sup>3</sup>\*x<sup>2</sup> + c\*d\*e<sup>2</sup>)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

$$3.149 \quad \int \frac{x(a+b \csc^{-1}(cx))}{(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=79

$$\frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{de}\sqrt{c^2x^2}} - \frac{a+b \csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

[Out] -((a + b\*ArcCsc[c\*x])/(e\*Sqrt[d + e\*x^2])) + (b\*c\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(Sqrt[d]\*e\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.117238, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {5237, 446, 93, 204}

$$\frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{de}\sqrt{c^2x^2}} - \frac{a+b \csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2),x]

[Out] -((a + b\*ArcCsc[c\*x])/(e\*Sqrt[d + e\*x^2])) + (b\*c\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(Sqrt[d]\*e\*Sqrt[c^2\*x^2])

#### Rule 5237

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)]\*(b\_.))\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsc[c\*x]))/(2\*e\*(p + 1)), x] + Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= -\frac{a + b \operatorname{csc}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{e\sqrt{c^2x^2}} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex^2}} dx, x, x^2\right)}{2e\sqrt{c^2x^2}} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{-d-x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} \\
&= -\frac{a + b \operatorname{csc}^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{de}\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.145208, size = 96, normalized size = 1.22

$$-\frac{a + b \operatorname{csc}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{bcx\sqrt{1 - \frac{1}{c^2x^2}} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)}{\sqrt{de}\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] -((a + b\*ArcCsc[c\*x])/(e\*Sqrt[d + e\*x^2])) - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(Sqrt[d]\*e\*Sqrt[-1 + c^2\*x^2])

**Maple [F]** time = 1.764, size = 0, normalized size = 0.

$$\int x(a + b \operatorname{arccsc}(cx))(ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x)

[Out] int(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="maxima")

[Out] Timed out

---

**Fricas [A]** time = 3.67538, size = 625, normalized size = 7.91

$$\left[ \frac{(bex^2 + bd)\sqrt{-d} \log\left(\frac{(c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 + 4\sqrt{c^2x^2 - 1}((c^2d - e)x^2 - 2d)\sqrt{ex^2 + d}\sqrt{-d} + 8d^2}{x^4}\right) + 4\sqrt{ex^2 + d}(bd \operatorname{arccsc}(cx) + ad)}{4(de^2x^2 + d^2e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [-1/4\*((b\*e\*x^2 + b\*d)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + 4\*sqrt(e\*x^2 + d)\*(b\*d\*arccsc(c\*x) + a\*d))/(d\*e^2\*x^2 + d^2\*e), 1/2\*((b\*e\*x^2 + b\*d)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) - 2\*sqrt(e\*x^2 + d)\*(b\*d\*arccsc(c\*x) + a\*d))/(d\*e^2\*x^2 + d^2\*e)]

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x/(e\*x^2 + d)^(3/2), x)

$$3.150 \quad \int \frac{a+b \csc^{-1}(cx)}{x(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

**Rubi [A]** time = 0.124985, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

[Out] Defer[Int][(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{3/2}} dx = \int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 18.2164, size = 0, normalized size = 0.

$$\int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

**Maple [A]** time = 3.144, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x} (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^(3/2), x)

[Out] `int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{e^2x^5 + 2dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(3/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)`

$$3.151 \quad \int \frac{a+b \csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

**Rubi [A]** time = 0.136165, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

[Out] Defer[Int][(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx = \int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 23.7964, size = 0, normalized size = 0.

$$\int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

**Maple [A]** time = 1.072, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^3} (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^3/(e\*x^2+d)^(3/2), x)

[Out] `int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{e^2x^7 + 2dex^5 + d^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(3/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)`



$$3.152 \quad \int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

**Rubi [A]** time = 0.113119, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int] [(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 9.30999, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

**Maple [A]** time = 2.306, size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x)

[Out] `int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 \operatorname{arccsc}(cx) + ax^4)\sqrt{ex^2 + d}}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^4}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)`

$$3.153 \quad \int \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

**Rubi [A]** time = 0.101641, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int] [(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 4.77435, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

**Maple [A]** time = 2.06, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x)

[Out] `int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 \operatorname{arccsc}(cx) + ax^2)\sqrt{ex^2 + d}}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)`

$$3.154 \quad \int \frac{a+b \csc^{-1}(cx)}{(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=108

$$\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{x(a+b \csc^{-1}(cx))}{d\sqrt{d+ex^2}}$$

[Out] (x\*(a + b\*ArcCsc[c\*x]))/(d\*Sqrt[d + e\*x^2]) + (b\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[c\*x], -(e/(c^2\*d))])/(d\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**Rubi [A]** time = 0.0911184, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {191, 5229, 12, 421, 419}

$$\frac{x(a+b \csc^{-1}(cx))}{d\sqrt{d+ex^2}} + \frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}F\left(\sin^{-1}(cx)\left|-\frac{e}{c^2d}\right.\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/(d + e\*x^2)^(3/2), x]

[Out] (x\*(a + b\*ArcCsc[c\*x]))/(d\*Sqrt[d + e\*x^2]) + (b\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[c\*x], -(e/(c^2\*d))])/(d\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

### Rule 5229

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 421

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/(Sqrt[a + b\*x^2]\*Sqrt[1 + (d\*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

### Rule 419

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)])/(Sqrt[a]\*Sqrt[c]\*Rt

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplifierSqrtQ[-(b/a), -(d/c)])
```

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^{3/2}} dx &= \frac{x(a + b \csc^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{1}{d\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{\sqrt{c^2x^2}} \\
 &= \frac{x(a + b \csc^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{d\sqrt{c^2x^2}} \\
 &= \frac{x(a + b \csc^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{\left(bcx\sqrt{1 + \frac{ex^2}{d}}\right) \int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} dx}{d\sqrt{c^2x^2}\sqrt{d + ex^2}} \\
 &= \frac{x(a + b \csc^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{\left(bcx\sqrt{1 - c^2x^2}\sqrt{1 + \frac{ex^2}{d}}\right) \int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}} dx}{d\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}} \\
 &= \frac{x(a + b \csc^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{bx\sqrt{1 - c^2x^2}\sqrt{1 + \frac{ex^2}{d}} F\left(\sin^{-1}(cx) \mid -\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.192367, size = 112, normalized size = 1.04

$$\frac{bcx\sqrt{1 - \frac{1}{c^2x^2}}\sqrt{1 - c^2x^2}\sqrt{\frac{ex^2}{d}} + 1 \operatorname{EllipticF}\left(\sin^{-1}(cx), -\frac{e}{c^2d}\right)}{d(c^3x^2 - c)\sqrt{d + ex^2}} + \frac{x(a + b \csc^{-1}(cx))}{d\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2), x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

**Maple [F]** time = 1.883, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2), x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2), x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x
)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

$$3.155 \quad \int \frac{a+b \csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=275

$$\frac{bx\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{2ex(a+b \csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b \csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}$$

[Out] -((b\*c\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])/(d^2\*Sqrt[c^2\*x^2])) - (a + b\*ArcCsc[c\*x])/(d\*x\*Sqrt[d + e\*x^2]) - (2\*e\*x\*(a + b\*ArcCsc[c\*x]))/(d^2\*Sqrt[d + e\*x^2]) + (b\*c^2\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2]\*EllipticE[ArcSin[c\*x], -(e/(c^2\*d))])/(d^2\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]) - (b\*(c^2\*d + 2\*e)\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[c\*x], -(e/(c^2\*d))])/(d^2\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**Rubi [A]** time = 0.317526, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {271, 191, 5239, 12, 583, 524, 427, 426, 424, 421, 419}

$$-\frac{2ex(a+b \csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b \csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{bx\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x^2)^(3/2)), x]

[Out] -((b\*c\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])/(d^2\*Sqrt[c^2\*x^2])) - (a + b\*ArcCsc[c\*x])/(d\*x\*Sqrt[d + e\*x^2]) - (2\*e\*x\*(a + b\*ArcCsc[c\*x]))/(d^2\*Sqrt[d + e\*x^2]) + (b\*c^2\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2]\*EllipticE[ArcSin[c\*x], -(e/(c^2\*d))])/(d^2\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]) - (b\*(c^2\*d + 2\*e)\*x\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[c\*x], -(e/(c^2\*d))])/(d^2\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

#### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^(m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (I



$\text{GtQ}[(m + 1)/2, 0] \ \&\& \ !(\text{ILtQ}[p, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0]) \ || \ (\text{ILtQ}[(m + 2*p + 1)/2, 0] \ \&\& \ !\text{ILtQ}[(m - 1)/2, 0])$

### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \ :> \ \text{Dist}[a, \text{Int}[u, x], x] \ /; \ \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] \ /; \ \text{FreeQ}[b, x]$

### Rule 583

$\text{Int}[(g_*)(x_)^{(m_*)}((a_*) + (b_*)(x_)^{(n_*)})^{(p_*)}((c_*) + (d_*)(x_)^{(n_*)})^{(q_*)}((e_*) + (f_*)(x_)^{(n_*)}), x\_Symbol] \ :> \ \text{Simp}[(e*(g*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}/(a*c*g*(m+1)), x] + \text{Dist}[1/(a*c*g^n*(m+1)), \text{Int}[(g*x)^{(m+n)}*(a + b*x^n)^p*(c + d*x^n)^q*\text{Simp}[a*f*c*(m+1) - e*(b*c + a*d)*(m+n+1) - e*n*(b*c*p + a*d*q) - b*e*d*(m+n*(p+q+2) + 1)*x^n, x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, g, p, q\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1]$

### Rule 524

$\text{Int}[(e_*) + (f_*)(x_)^{(n_*)}/(\text{Sqrt}[(a_*) + (b_*)(x_)^{(n_*)}]*\text{Sqrt}[(c_*) + (d_*)(x_)^{(n_*)}]), x\_Symbol] \ :> \ \text{Dist}[f/b, \text{Int}[\text{Sqrt}[a + b*x^n]/\text{Sqrt}[c + d*x^n], x], x] + \text{Dist}[(b*e - a*f)/b, \text{Int}[1/(\text{Sqrt}[a + b*x^n]*\text{Sqrt}[c + d*x^n]), x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \ !(\text{EqQ}[n, 2] \ \&\& \ ((\text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]) \ || \ (\text{NegQ}[b/a] \ \&\& \ (\text{PosQ}[d/c] \ || \ (\text{GtQ}[a, 0] \ \&\& \ (!\text{GtQ}[c, 0] \ || \ \text{SimplerSqrtQ}[-(b/a), -(d/c)]))))))$

### Rule 427

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)(x_)^2]/\text{Sqrt}[(c_*) + (d_*)(x_)^2], x\_Symbol] \ :> \ \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (d*x^2)/c], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ !\text{GtQ}[c, 0]$

### Rule 426

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)(x_)^2]/\text{Sqrt}[(c_*) + (d_*)(x_)^2], x\_Symbol] \ :> \ \text{Dist}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (b*x^2)/a], \text{Int}[\text{Sqrt}[1 + (b*x^2)/a]/\text{Sqrt}[c + d*x^2], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ !\text{GtQ}[a, 0]$

### Rule 424

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)(x_)^2]/\text{Sqrt}[(c_*) + (d_*)(x_)^2], x\_Symbol] \ :> \ \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0]$

### Rule 421

$\text{Int}[1/(\text{Sqrt}[(a_*) + (b_*)(x_)^2]*\text{Sqrt}[(c_*) + (d_*)(x_)^2]), x\_Symbol] \ :> \ \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[1 + (d*x^2)/c]), x], x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ !\text{GtQ}[c, 0]$

### Rule 419

$\text{Int}[1/(\text{Sqrt}[(a_*) + (b_*)(x_)^2]*\text{Sqrt}[(c_*) + (d_*)(x_)^2]), x\_Symbol] \ :> \ \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ !(\text{NegQ}[b/a] \ \&\& \ \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

## Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{x^2 (d + ex^2)^{3/2}} dx &= -\frac{a + b \csc^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \csc^{-1}(cx))}{d^2\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{-d-2ex^2}{d^2x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{\sqrt{c^2x^2}} \\
&= -\frac{a + b \csc^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \csc^{-1}(cx))}{d^2\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{-d-2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{d^2\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \csc^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \csc^{-1}(cx))}{d^2\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{-2de+c^2dex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{d^3\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \csc^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \csc^{-1}(cx))}{d^2\sqrt{d + ex^2}} + \frac{(bc^3x) \int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}} dx}{d^2\sqrt{c^2x^2}} - (bc^3x) \int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}} dx \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \csc^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \csc^{-1}(cx))}{d^2\sqrt{d + ex^2}} + \frac{(bc^3x\sqrt{1 - c^2x^2}) \int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}} dx}{d^2\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \csc^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \csc^{-1}(cx))}{d^2\sqrt{d + ex^2}} + \frac{(bc^3x\sqrt{1 - c^2x^2}\sqrt{d + ex^2})}{d^2\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}} \\
&= -\frac{bc\sqrt{-1 + c^2x^2}\sqrt{d + ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a + b \csc^{-1}(cx)}{dx\sqrt{d + ex^2}} - \frac{2ex (a + b \csc^{-1}(cx))}{d^2\sqrt{d + ex^2}} + \frac{bc^2x\sqrt{1 - c^2x^2}\sqrt{d + ex^2}E\left(\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1 + c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.477304, size = 213, normalized size = 0.77

$$\frac{-a(d + 2ex^2) - bcx\sqrt{1 - \frac{1}{c^2x^2}(d + ex^2)} - b \csc^{-1}(cx)(d + 2ex^2)}{d^2x\sqrt{d + ex^2}} + \frac{ibcx\sqrt{1 - \frac{1}{c^2x^2}\frac{ex^2}{d}} + 1 \left( c^2dE\left(i \sinh^{-1}\left(\sqrt{-c^2}x\right)\right) - \frac{e}{c^2d} \right)}{\sqrt{-c^2d^2}\sqrt{1 - c^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^2\*(d + e\*x^2)^(3/2)), x]

[Out]  $(-(b*c*\text{Sqrt}[1 - 1/(c^2*x^2)])*x*(d + e*x^2)) - a*(d + 2*e*x^2) - b*(d + 2*e*x^2)*\text{ArcCsc}[c*x])/(d^2*x*\text{Sqrt}[d + e*x^2]) + (I*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{Sqrt}[1 + (e*x^2)/d]*(c^2*d*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d))] - (c^2*d + 2*e)*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-c^2]*x], -(e/(c^2*d)))]/(\text{Sqrt}[-c^2]*d^2*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])$

**Maple [F]** time = 3.06, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2} (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^(3/2), x)

[Out] int((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^(3/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{arccsc}(cx) + a)}{e^2x^6 + 2dex^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)/(e^2\*x^6 + 2\*d\*e\*x^4 + d^2\*x^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*2/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x^2), x)

$$3.156 \quad \int \frac{x^5 (a + b \csc^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=243

$$-\frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} + \frac{bcdx \sqrt{c^2 x^2 - 1}}{3e^2 \sqrt{c^2 x^2} (c^2 d + e) \sqrt{d + ex^2}} - \frac{8bc \sqrt{dx} \tan^{-1}(\dots)}{3e^3}$$

[Out] (b\*c\*d\*x\*Sqrt[-1 + c^2\*x^2])/(3\*e^2\*(c^2\*d + e)\*Sqrt[c^2\*x^2]\*Sqrt[d + e\*x^2]) - (d^2\*(a + b\*ArcCsc[c\*x]))/(3\*e^3\*(d + e\*x^2)^(3/2)) + (2\*d\*(a + b\*ArcCsc[c\*x]))/(e^3\*Sqrt[d + e\*x^2]) + (Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e^3 - (8\*b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*e^3\*Sqrt[c^2\*x^2]) + (b\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(e^(5/2)\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 1.0566, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 5239, 12, 1614, 157, 63, 217, 206, 93, 204}

$$-\frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} + \frac{bcdx \sqrt{c^2 x^2 - 1}}{3e^2 \sqrt{c^2 x^2} (c^2 d + e) \sqrt{d + ex^2}} - \frac{8bc \sqrt{dx} \tan^{-1}(\dots)}{3e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*d\*x\*Sqrt[-1 + c^2\*x^2])/(3\*e^2\*(c^2\*d + e)\*Sqrt[c^2\*x^2]\*Sqrt[d + e\*x^2]) - (d^2\*(a + b\*ArcCsc[c\*x]))/(3\*e^3\*(d + e\*x^2)^(3/2)) + (2\*d\*(a + b\*ArcCsc[c\*x]))/(e^3\*Sqrt[d + e\*x^2]) + (Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]))/e^3 - (8\*b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*e^3\*Sqrt[c^2\*x^2]) + (b\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(e^(5/2)\*Sqrt[c^2\*x^2])

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 5239

Int[((a\_) + ArcCsc[(c\_)\*(x\_)])\*(b\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (I

$\text{GtQ}[(m + 1)/2, 0] \&\& \text{!(ILtQ}[p, 0] \&\& \text{GtQ}[m + 2*p + 3, 0]) \text{ || (ILtQ}[(m + 2*p + 1)/2, 0] \&\& \text{!ILtQ}[(m - 1)/2, 0])$

### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \text{ :> Dist}[a, \text{Int}[u, x], x] \text{ /; FreeQ}[a, x] \&\& \text{!Match} \text{Q}[u, (b_*)(v_)] \text{ /; FreeQ}[b, x]$

### Rule 1614

$\text{Int}[(Px_*)((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}*((e_.) + (f_.)*(x_))^{(p_)}], x\_Symbol] \text{ :> With}[\{Qx = \text{PolynomialQuotient}[Px, a + b*x, x], R = \text{PolynomialRemainder}[Px, a + b*x, x]\}, \text{Simp}[(b*R*(a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)})/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n*(e + f*x)^p * \text{ExpandToSum}[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{PolyQ}[Px, x] \&\& \text{LtQ}[m, -1] \&\& \text{IntegersQ}[2*m, 2*n, 2*p]$

### Rule 157

$\text{Int}[(c_.) + (d_.)*(x_))^{(n_)}*((e_.) + (f_.)*(x_))^{(p_)}*((g_.) + (h_.)*(x_))^{(q_)}], x\_Symbol] \text{ :> Dist}[h/b, \text{Int}[(c + d*x)^n*(e + f*x)^p], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n*(e + f*x)^p/(a + b*x), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x]$

### Rule 63

$\text{Int}[(a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}], x\_Symbol] \text{ :> With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] \text{ /; FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x\_Symbol] \text{ :> Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

### Rule 206

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}], x\_Symbol] \text{ :> Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \text{ || LtQ}[b, 0])$

### Rule 93

$\text{Int}[(a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}], x\_Symbol] \text{ :> With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m + 1) - 1)}*(c + d*x)^n], x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n] \&\& \text{LtQ}[-1, m, 0] \&\& \text{SimplerQ}[a + b*x, c + d*x]$

### Rule 204

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}], x\_Symbol] \text{ :> -Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[$

a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (a + b \csc^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} + \frac{(bcx) \int \frac{8d^2 + 12dex^2 + 3e^2x^4}{3e^3x\sqrt{d + ex^2}} dx}{\sqrt{d + ex^2}} \\
 &= -\frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} + \frac{(bcx) \int \frac{8d^2 + 12dex^2 + 3e^2x^4}{x\sqrt{d + ex^2}} dx}{3e^3 \sqrt{d + ex^2}} \\
 &= -\frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} + \frac{(bcx) \text{Subst}\left(\int \frac{8d^2 + 12dex^2 + 3e^2x^4}{x\sqrt{d + ex^2}} dx, x, \sqrt{d + ex^2}\right)}{3e^3 \sqrt{d + ex^2}} \\
 &= \frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2}\sqrt{d + ex^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} \\
 &= \frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2}\sqrt{d + ex^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} \\
 &= \frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2}\sqrt{d + ex^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3} \\
 &= \frac{bcdx\sqrt{-1 + c^2x^2}}{3e^2 (c^2d + e) \sqrt{c^2x^2}\sqrt{d + ex^2}} - \frac{d^2 (a + b \csc^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \csc^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \csc^{-1}(cx))}{e^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.552179, size = 311, normalized size = 1.28

$$\frac{a(c^2d + e)(8d^2 + 12dex^2 + 3e^2x^4) + b(c^2d + e)\csc^{-1}(cx)(8d^2 + 12dex^2 + 3e^2x^4) + bcdex\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2)}{3e^3(c^2d + e)(d + ex^2)^{3/2}} + \frac{bx\sqrt{1 - \frac{1}{c^2x^2}}}{\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*d\*e\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(d + e\*x^2) + a\*(c^2\*d + e)\*(8\*d^2 + 12\*d\*e\*x^2 + 3\*e^2\*x^4) + b\*(c^2\*d + e)\*(8\*d^2 + 12\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsc[c\*x])/(3\*e^3\*(c^2\*d + e)\*(d + e\*x^2)^(3/2)) + (b\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*(3\*Sqrt[c^2]\*Sqrt[e]\*Sqrt[c^2\*d + e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d + e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[-1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d + e])] + 8\*c^3\*Sqrt[d]\*Sqrt[d + e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])/Sqrt[d + e\*x^2]])/(3\*c^2\*e^3\*Sqrt[-1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

---

**Maple [F]** time = 3.697, size = 0, normalized size = 0.

$$\int x^5 (a + \operatorname{arccsc}(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 7.85175, size = 4487, normalized size = 18.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] [1/12\*(3\*(b\*c^2\*d^3 + (b\*c^2\*d\*e^2 + b\*e^3)\*x^4 + b\*d^2\*e + 2\*(b\*c^2\*d^2\*e + b\*d\*e^2)\*x^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) + 8\*(b\*c^3\*d^3 + b\*c\*d^2\*e + (b\*c^3\*d\*e^2 + b\*c\*e^3)\*x^4 + 2\*(b\*c^3\*d^2\*e + b\*c\*d\*e^2)\*x^2)\*sqrt(-d)\*log(((c^4\*d^2 - 6\*c^2\*d\*e + e^2)\*x^4 - 8\*(c^2\*d^2 - d\*e)\*x^2 + 4\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(-d) + 8\*d^2)/x^4) + 4\*(8\*a\*c^3\*d^3 + 8\*a\*c\*d^2\*e + 3\*(a\*c^3\*d\*e^2 + a\*c\*e^3)\*x^4 + 12\*(a\*c^3\*d^2\*e + a\*c\*d\*e^2)\*x^2 + (8\*b\*c^3\*d^3 + 8\*b\*c\*d^2\*e + 3\*(b\*c^3\*d\*e^2 + b\*c\*e^3)\*x^4 + 12\*(b\*c^3\*d^2\*e + b\*c\*d\*e^2)\*x^2)\*arccsc(c\*x) + (b\*c\*d\*e^2\*x^2 + b\*c\*d^2\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^3\*d^3\*e^3 + c\*d^2\*e^4 + (c^3\*d\*e^5 + c\*e^6)\*x^4 + 2\*(c^3\*d^2\*e^4 + c\*d\*e^5)\*x^2), -1/12\*(16\*(b\*c^3\*d^3 + b\*c\*d^2\*e + (b\*c^3\*d\*e^2 + b\*c\*e^3)\*x^4 + 2\*(b\*c^3\*d^2\*e + b\*c\*d\*e^2)\*x^2)\*sqrt(d)\*arctan(-1/2\*sqrt(c^2\*x^2 - 1)\*((c^2\*d - e)\*x^2 - 2\*d)\*sqrt(e\*x^2 + d)\*sqrt(d)/(c^2\*d\*e\*x^4 + (c^2\*d^2 - d\*e)\*x^2 - d^2)) - 3\*(b\*c^2\*d^3 + (b\*c^2\*d\*e^2 + b\*e^3)\*x^4 + b\*d^2\*e + 2\*(b\*c^2\*d^2\*e + b\*d\*e^2)\*x^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 - 6\*c^2\*d\*e + 8\*(c^4\*d\*e - c^2\*e^2)\*x^2 + 4\*(2\*c^3\*e\*x^2 + c^3\*d - c\*e)\*sqrt(c^2\*x^2 - 1)\*sqrt(e\*x^2 + d)\*sqrt(e) + e^2) - 4\*(8\*a\*c^3\*d^3 + 8\*a\*c\*d^2\*e + 3\*(a\*c^3\*d\*e^2 + a\*c\*e^3)\*x^4 + 12\*(a\*c^3\*d^2\*e + a\*c\*d\*e^2)\*x^2 + (8\*b\*c^3\*d^3 + 8\*b\*c\*d^2\*e + 3\*(b\*c^3\*d\*e^2 + b\*c\*e^3)\*x^4 + 12\*(b\*c^3\*d^2\*e + b\*c\*d\*e^2)\*x^2)\*arccsc(c\*x) + (b\*c\*d\*e^2\*x^2 + b\*c\*d^2\*e)\*sqrt(c^2\*x^2 - 1))\*sqrt(e\*x^2 + d))/(c^3\*d^3\*e^3 + c\*d^2\*e^4 + (c^3\*d\*e^5 + c\*e^6)\*x^4 + 2\*(c^3\*d^2\*e^4 + c\*d\*e^5)\*x^2), -1/6\*(3\*(b\*c^2\*d^3 + (b\*c^2\*d\*e^2 + b\*e^3)\*x^4 + b\*d^2\*e + 2\*(b\*c^2\*d^2\*e + b\*d\*e^2)\*x^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*

```

x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) - 4*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 +
b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6
*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d -
e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*(8*a*c^3*d^3 + 8*
a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^
2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e + b*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*
x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x
^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(8*(b*c^3*d^3 + b*c*d^2*e + (b*c^
3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arctan(-1
/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d
*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)
*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
- c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3
*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8
*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x
^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4
+ c*d*e^5)*x^2)]

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```



$$3.157 \quad \int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=163

$$-\frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} + \frac{2bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3\sqrt{de^2}\sqrt{c^2x^2}} - \frac{bcx\sqrt{c^2x^2-1}}{3e\sqrt{c^2x^2}(c^2d + e)\sqrt{d + ex^2}}$$

```
[Out] -(b*c*x*Sqrt[-1 + c^2*x^2])/(3*e*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2])
+ (d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsc[c*x])/
(e^2*Sqrt[d + e*x^2]) + (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 +
c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[c^2*x^2])
```

**Rubi [A]** time = 0.246188, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {266, 43, 5239, 12, 573, 152, 93, 204}

$$-\frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} + \frac{2bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3\sqrt{de^2}\sqrt{c^2x^2}} - \frac{bcx\sqrt{c^2x^2-1}}{3e\sqrt{c^2x^2}(c^2d + e)\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

```
[Out] -(b*c*x*Sqrt[-1 + c^2*x^2])/(3*e*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2])
+ (d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsc[c*x])/
(e^2*Sqrt[d + e*x^2]) + (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 +
c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[c^2*x^2])
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
)*(e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :=> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

### Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] :=> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

### Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x
_)), x_Symbol] :=> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \csc^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(bcx) \int \frac{-2d-3ex^2}{3e^2 x \sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(bcx) \int \frac{-2d-3ex^2}{x \sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{3e^2 \sqrt{c^2x^2}} \\
&= \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(bcx) \text{Subst} \left( \int \frac{-2d-3ex}{x \sqrt{-1+c^2x}(d+ex)^{3/2}} dx, x, x^2 \right)}{6e^2 \sqrt{c^2x^2}} \\
&= -\frac{bcx \sqrt{-1 + c^2x^2}}{3e (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \text{Subst} \left( \int \frac{-2d-3ex}{x \sqrt{-1+c^2x}(d+ex)^{3/2}} dx, x, x^2 \right)}{3de^2 \sqrt{c^2x^2}} \\
&= -\frac{bcx \sqrt{-1 + c^2x^2}}{3e (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \text{Subst} \left( \int \frac{-2d-3ex}{x \sqrt{-1+c^2x}(d+ex)^{3/2}} dx, x, x^2 \right)}{3de^2 \sqrt{c^2x^2}} \\
&= -\frac{bcx \sqrt{-1 + c^2x^2}}{3e (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(2bcx) \text{Subst} \left( \int \frac{-2d-3ex}{x \sqrt{-1+c^2x}(d+ex)^{3/2}} dx, x, x^2 \right)}{3de^2 \sqrt{c^2x^2}} \\
&= -\frac{bcx \sqrt{-1 + c^2x^2}}{3e (c^2d + e) \sqrt{c^2x^2} \sqrt{d + ex^2}} + \frac{d(a + b \csc^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \csc^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{2bcx \tan^{-1} \left( \frac{\sqrt{d} \sqrt{c^2x}}{\sqrt{d+ex}} \right)}{3\sqrt{de^2} \sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.264813, size = 173, normalized size = 1.06

$$\frac{-a(c^2d + e)(2d + 3ex^2) - bcex \sqrt{1 - \frac{1}{c^2x^2}} (d + ex^2) - b(c^2d + e) \csc^{-1}(cx) (2d + 3ex^2)}{3e^2 (c^2d + e) (d + ex^2)^{3/2}} - \frac{2bcx \sqrt{1 - \frac{1}{c^2x^2}} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{c^2x}}{\sqrt{d+ex}} \right)}{3\sqrt{de^2} \sqrt{c^2x^2 - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out]  $(-(b*c*e*\text{Sqrt}[1 - 1/(c^2*x^2)])*x*(d + e*x^2)) - a*(c^2*d + e)*(2*d + 3*e*x^2) - b*(c^2*d + e)*(2*d + 3*e*x^2)*\text{ArcCsc}[c*x]/(3*e^2*(c^2*d + e)*(d + e*x^2)^{(3/2)}) - (2*b*c*\text{Sqrt}[1 - 1/(c^2*x^2)]*x*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[-1 + c^2*x^2])/(\text{Sqrt}[d + e*x^2])])/(3*\text{Sqrt}[d]*e^2*\text{Sqrt}[-1 + c^2*x^2])$

**Maple [F]** time = 3.102, size = 0, normalized size = 0.

$$\int x^3 (a + b \arccsc(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2), x)

[Out] int(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 5.3895, size = 1378, normalized size = 8.45

$$\left[ \frac{(bc^2d^3 + (bc^2de^2 + be^3)x^4 + bd^2e + 2(bc^2d^2e + bde^2)x^2)\sqrt{-d} \log\left(\frac{(c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 + 4\sqrt{c^2x^2 - 1}((c^2d - e)x^2 - 2d)\sqrt{ex^2 - 1}}{x^4}\right)}{6(c^2d^4e^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*\sqrt{-d}*\log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*\sqrt{c^2*x^2 - 1}*((c^2*d - e)*x^2 - 2*d)*\sqrt{e*x^2 + d}*\sqrt{-d} + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*\arccsc(c*x) \\ & + (b*d*e^2*x^2 + b*d^2*e)*\sqrt{c^2*x^2 - 1}*\sqrt{e*x^2 + d}]/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2), \\ & 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*\sqrt{d}*\arctan(-1/2*\sqrt{c^2*x^2 - 1}*((c^2*d - e)*x^2 - 2*d)*\sqrt{e*x^2 + d}*\sqrt{d}/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*\arccsc(c*x) + (b*d*e^2*x^2 + b*d^2*e)*\sqrt{c^2*x^2 - 1}*\sqrt{e*x^2 + d}]/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)] \end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

$$3.158 \quad \int \frac{x(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=138

$$-\frac{a+b \csc^{-1}(cx)}{3e(d+ex^2)^{3/2}} + \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}} + \frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

[Out] (b\*c\*x\*Sqrt[-1 + c^2\*x^2])/(3\*d\*(c^2\*d + e)\*Sqrt[c^2\*x^2]\*Sqrt[d + e\*x^2]) - (a + b\*ArcCsc[c\*x])/(3\*e\*(d + e\*x^2)^(3/2)) + (b\*c\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*d^(3/2)\*e\*Sqrt[c^2\*x^2])

**Rubi [A]** time = 0.135041, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {5237, 446, 96, 93, 204}

$$-\frac{a+b \csc^{-1}(cx)}{3e(d+ex^2)^{3/2}} + \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}} + \frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*x\*Sqrt[-1 + c^2\*x^2])/(3\*d\*(c^2\*d + e)\*Sqrt[c^2\*x^2]\*Sqrt[d + e\*x^2]) - (a + b\*ArcCsc[c\*x])/(3\*e\*(d + e\*x^2)^(3/2)) + (b\*c\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 + c^2\*x^2])])/(3\*d^(3/2)\*e\*Sqrt[c^2\*x^2])

#### Rule 5237

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsc[c\*x]))/(2\*e\*(p + 1)), x] + Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[c^2\*x^2]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[c^2\*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 96

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.))\*((e\_.) + (f\_.)\*(x\_)^(p\_.)), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[(a\*d\*f\*(m + 1) + b\*c\*f\*(n + 1) + b\*d\*e\*(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

#### Rule 93

```
Int[(((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))/((e_) + (f_)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{x(a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{a + b \operatorname{csc}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{(bcx) \int \frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{3e\sqrt{c^2x^2}} \\ &= -\frac{a + b \operatorname{csc}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}(d+ex)^{3/2}} dx, x, x^2\right)}{6e\sqrt{c^2x^2}} \\ &= \frac{bcx\sqrt{-1+c^2x^2}}{3d(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} - \frac{a + b \operatorname{csc}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}} dx, x, x^2\right)}{6de\sqrt{c^2x^2}} \\ &= \frac{bcx\sqrt{-1+c^2x^2}}{3d(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} - \frac{a + b \operatorname{csc}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{-d-x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3de\sqrt{c^2x^2}} \\ &= \frac{bcx\sqrt{-1+c^2x^2}}{3d(c^2d + e)\sqrt{c^2x^2}\sqrt{d + ex^2}} - \frac{a + b \operatorname{csc}^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.206035, size = 157, normalized size = 1.14

$$\frac{-ad(c^2d + e) + bcex\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2) - bd(c^2d + e)\operatorname{csc}^{-1}(cx)}{3de(c^2d + e)(d + ex^2)^{3/2}} - \frac{bcx\sqrt{1 - \frac{1}{c^2x^2}}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

```
[Out] (-a*d*(c^2*d + e) + b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - b*d*(c^2*d + e)*ArcCsc[c*x])/(3*d*e*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(3*d^(3/2)*e*Sqrt[-1 + c^2*x^2])
```

**Maple [F]** time = 1.777, size = 0, normalized size = 0.

$$\int x(a + b \operatorname{arccsc}(cx))(ex^2 + d)^{-5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)`

[Out] `int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 5.19978, size = 1188, normalized size = 8.61

$$\left[ \frac{(bc^2d^3 + (bc^2de^2 + be^3)x^4 + bd^2e + 2(bc^2d^2e + bde^2)x^2)\sqrt{-d} \log\left(\frac{(c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 + 4\sqrt{c^2x^2 - 1}((c^2d - e)x^2 - 2d)\sqrt{ex^2 - 1}}{x^4}\right)}{12(c^2d^5e + d^4e^2 + (c^2d^3e^3 + d^2e^4)x^4 + 2d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `[-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arccsc(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d)/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arccsc(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d)/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)`

[Out] Timed out



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

$$3.159 \quad \int \frac{a+b \csc^{-1}(cx)}{x(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

**Rubi [A]** time = 0.123205, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{5/2}} dx = \int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{5/2}} dx$$

**Mathematica [A]** time = 34.1612, size = 0, normalized size = 0.

$$\int \frac{a + b \csc^{-1}(cx)}{x (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

**Maple [A]** time = 1.883, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x} (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(e\*x^2+d)^(5/2), x)

[Out] `int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{arccsc}(cx) + a)}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(5/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)`

$$3.160 \quad \int \frac{a+b \csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

**Rubi [A]** time = 0.140413, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx = \int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

**Mathematica [A]** time = 43.1475, size = 0, normalized size = 0.

$$\int \frac{a + b \csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

**Maple [A]** time = 1.093, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^3} (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^3/(e\*x^2+d)^(5/2), x)

[Out] `int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{arccsc}(cx) + a)}{e^3x^9 + 3de^2x^7 + 3d^2ex^5 + d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(5/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)`

$$3.161 \quad \int \frac{x^6 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{x^6 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(x^6\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

**Rubi [A]** time = 0.120836, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^6 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^6\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Defer[Int] [(x^6\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^6 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^6 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Mathematica [A]** time = 13.4056, size = 0, normalized size = 0.

$$\int \frac{x^6 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^6\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Integrate[(x^6\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

**Maple [A]** time = 3.532, size = 0, normalized size = 0.

$$\int x^6 (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2), x)

[Out] `int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^6 \operatorname{arccsc}(cx) + ax^6)\sqrt{ex^2 + d}}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^6*arccsc(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^6}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)`

$$3.162 \quad \int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

**Rubi [A]** time = 0.109561, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Defer[Int] [(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Mathematica [A]** time = 11.6749, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Integrate[(x^4\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

**Maple [A]** time = 2.412, size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2), x)



[Out] `int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 \operatorname{arccsc}(cx) + ax^4)\sqrt{ex^2 + d}}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)`

$$3.163 \quad \int \frac{x^2(a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=276

$$\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{x^3(a+b \csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}}$$

```
[Out] (b*c*x^2*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]
) + (x^3*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c^2*x*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*(c^2*d
+ e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (b*x*Sqrt[1 -
c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*
Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.283373, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {264, 5239, 12, 471, 423, 427, 426, 424, 421, 419}

$$\frac{x^3(a+b \csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} + \frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

```
[Out] (b*c*x^2*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]
) + (x^3*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c^2*x*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*(c^2*d
+ e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (b*x*Sqrt[1 -
c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*
Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

#### Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

#### Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p},
x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \csc^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{x^3 (a + b \csc^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{(bcx) \int \frac{x^2}{3d\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{x^3 (a + b \csc^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{(bcx) \int \frac{x^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{3d\sqrt{c^2x^2}} \\
&= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{(bcx) \int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}} dx}{3d(c^2d+e)\sqrt{c^2x^2}} \\
&= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{(bcx) \int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} dx}{3de\sqrt{c^2x^2}} - \frac{(bc^3x) \int \dots}{3de(c^2d+e)} \\
&= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{(bc^3x\sqrt{1-c^2x^2}) \int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}} dx}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \dots \\
&= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}) \int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}} dx}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E(\sin^{-1}(cx))}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}
\end{aligned}$$

**Mathematica [A]** time = 0.27801, size = 185, normalized size = 0.67

$$\frac{x^2 \left( ax(c^2d + e) + bc\sqrt{1 - \frac{1}{c^2x^2}}(d + ex^2) + bx(c^2d + e)\csc^{-1}(cx) \right)}{3d(c^2d + e)(d + ex^2)^{3/2}} - \frac{bcx\sqrt{1 - \frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}} + 1E\left(\sin^{-1}\left(\sqrt{\frac{-e}{d}}x\right) \middle| -\frac{c^2d}{e}\right)}{3d\sqrt{1 - c^2x^2}\sqrt{-\frac{e}{d}}(c^2d + e)\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (x^2\*(a\*(c^2\*d + e)\*x + b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*(d + e\*x^2) + b\*(c^2\*d + e)\*x\*ArcCsc[c\*x]))/(3\*d\*(c^2\*d + e)\*(d + e\*x^2)^(3/2)) - (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*EllipticE[ArcSin[Sqrt[-(e/d)]\*x], -((c^2\*d)/e)))/(3\*d\*Sqrt[-(e/d)]\*(c^2\*d + e)\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 2.079, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2), x)

[Out] int(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}a\left(\frac{x}{(ex^2+d)^{\frac{3}{2}}e} - \frac{x}{\sqrt{ex^2+d}de}\right) + b\int\frac{x^2\arctan(1,\sqrt{cx+1}\sqrt{cx-1})}{(e^2x^4+2dex^2+d^2)\sqrt{ex^2+d}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*a\*(x/((e\*x^2 + d)^(3/2)\*e) - x/(sqrt(e\*x^2 + d)\*d\*e)) + b\*integrate(x^2\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/((e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e\*x^2 + d)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2\arccsc(cx) + ax^2)\sqrt{ex^2+d}}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsc(c\*x) + a\*x^2)\*sqrt(e\*x^2 + d)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\arccsc(cx) + a)x^2}{(ex^2+d)^{\frac{5}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*x^2/(e\*x^2 + d)^(5/2), x)

$$3.164 \quad \int \frac{a+b \csc^{-1}(cx)}{(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=296

$$\frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{2x(a+b \csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b \csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

```
[Out] -(b*c*e*x^2*Sqrt[-1 + c^2*x^2])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]) + (x*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsc[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*b*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

**Rubi [A]** time = 0.237614, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.55$ , Rules used = {192, 191, 5229, 12, 527, 524, 427, 426, 424, 421, 419}

$$\frac{2x(a+b \csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b \csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}} + \frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2), x]
```

```
[Out] -(b*c*e*x^2*Sqrt[-1 + c^2*x^2])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]) + (x*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsc[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*b*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

#### Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]
```

#### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

#### Rule 5229

```
Int[((a_.) + ArcCsc[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
```

0])

### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_) /; \text{FreeQ}[b, x]]$

### Rule 527

$\text{Int}[(a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_)^{(n_*)})^{(q_*)}*((e_*) + (f_*)*(x_)^{(n_*)}), x\_Symbol] \rightarrow -\text{Simp}[(b*e - a*f)*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}/(a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, q\}, x] \ \&\& \ \text{LtQ}[p, -1]$

### Rule 524

$\text{Int}[(e_*) + (f_*)*(x_)^{(n_*)}]/(\text{Sqrt}[(a_*) + (b_*)*(x_)^{(n_*)}]*\text{Sqrt}[(c_*) + (d_*)*(x_)^{(n_*)}), x\_Symbol] \rightarrow \text{Dist}[f/b, \text{Int}[\text{Sqrt}[a + b*x^n]/\text{Sqrt}[c + d*x^n], x], x] + \text{Dist}[(b*e - a*f)/b, \text{Int}[1/(\text{Sqrt}[a + b*x^n]*\text{Sqrt}[c + d*x^n]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \ !(\text{EqQ}[n, 2] \ \&\& \ ((\text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]) \ || \ (\text{NegQ}[b/a] \ \&\& \ (\text{PosQ}[d/c] \ || \ (\text{GtQ}[a, 0] \ \&\& \ (!\text{GtQ}[c, 0] \ || \ \text{SimplerSqrtQ}[-(b/a), -(d/c)]))))))$

### Rule 427

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_)^2], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (d*x^2)/c], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ !\text{GtQ}[c, 0]$

### Rule 426

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_)^2], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (b*x^2)/a], \text{Int}[\text{Sqrt}[1 + (b*x^2)/a]/\text{Sqrt}[c + d*x^2], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ !\text{GtQ}[a, 0]$

### Rule 424

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0]$

### Rule 421

$\text{Int}[1/(\text{Sqrt}[(a_*) + (b_*)*(x_)^2]*\text{Sqrt}[(c_*) + (d_*)*(x_)^2]), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[1 + (d*x^2)/c]), x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ !\text{GtQ}[c, 0]$

### Rule 419

$\text{Int}[1/(\text{Sqrt}[(a_*) + (b_*)*(x_)^2]*\text{Sqrt}[(c_*) + (d_*)*(x_)^2]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ !(\text{NegQ}[b/a] \ \&\& \ \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^{5/2}} dx &= \frac{x(a + b \csc^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \csc^{-1}(cx))}{3d^2\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{3d+2ex^2}{3d^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{\sqrt{c^2x^2}} \\
&= \frac{x(a + b \csc^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \csc^{-1}(cx))}{3d^2\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{3d+2ex^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} dx}{3d^2\sqrt{c^2x^2}} \\
&= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a + b \csc^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \csc^{-1}(cx))}{3d^2\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{d(3c^2d+2e)+}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}}{3d^3(c^2d+e)\sqrt{c^2x^2}} \\
&= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a + b \csc^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \csc^{-1}(cx))}{3d^2\sqrt{d + ex^2}} + \frac{(2bcx) \int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}}{3d^2\sqrt{c^2x^2}} \\
&= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a + b \csc^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \csc^{-1}(cx))}{3d^2\sqrt{d + ex^2}} + \frac{(bc^3x\sqrt{1-c^2x^2})}{3d^2(c^2d+e)\sqrt{c^2x^2}} \\
&= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a + b \csc^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \csc^{-1}(cx))}{3d^2\sqrt{d + ex^2}} + \frac{(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2})}{3d^2(c^2d+e)\sqrt{c^2x^2}} \\
&= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a + b \csc^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \csc^{-1}(cx))}{3d^2\sqrt{d + ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.554377, size = 249, normalized size = 0.84

$$\frac{x\left(a(c^2d+e)(3d+2ex^2) - bcex\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2) + b(c^2d+e)\csc^{-1}(cx)(3d+2ex^2)\right)}{3d^2(c^2d+e)(d+ex^2)^{3/2}} + \frac{ibcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}+1}\left(2(c^2d+e)\sqrt{d+ex^2}\right)}{3d^2(c^2d+e)\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(d + e\*x^2)^(5/2), x]

[Out] (x\*(-(b\*c\*e\*Sqrt[1 - 1/(c^2\*x^2)])\*x\*(d + e\*x^2)) + a\*(c^2\*d + e)\*(3\*d + 2\*e\*x^2) + b\*(c^2\*d + e)\*(3\*d + 2\*e\*x^2)\*ArcCsc[c\*x])/(3\*d^2\*(c^2\*d + e)\*(d + e\*x^2)^(3/2)) + ((I/3)\*b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*EllipticE[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))] + 2\*(c^2\*d + e)\*EllipticF[I\*ArcSinh[Sqrt[-c^2]\*x], -(e/(c^2\*d))])/(Sqrt[-c^2]\*d^2\*(c^2\*d + e)\*Sqrt[1 - c^2\*x^2]\*Sqrt[d + e\*x^2])

**Maple [F]** time = 1.879, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccsc}(cx)) (ex^2 + d)^{-5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/(e\*x^2+d)^(5/2), x)



[Out]  $\text{int}((a+b*\text{arccsc}(c*x))/(e*x^2+d)^{(5/2)}, x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a \left( \frac{2x}{\sqrt{ex^2 + dd^2}} + \frac{x}{(ex^2 + d)^{\frac{3}{2}} d} \right) + b \int \frac{\arctan(1, \sqrt{cx + 1} \sqrt{cx - 1})}{(e^2 x^4 + 2 dex^2 + d^2) \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\text{arccsc}(c*x))/(e*x^2+d)^{(5/2)}, x, \text{algorithm}="maxima")$

[Out]  $1/3*a*(2*x/(\text{sqrt}(e*x^2 + d)*d^2) + x/((e*x^2 + d)^{(3/2)}*d)) + b*\text{integrate}(\text{arctan2}(1, \text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*\text{sqrt}(e*x^2 + d)), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{ex^2 + d}(b \text{arccsc}(cx) + a)}{e^3 x^6 + 3 de^2 x^4 + 3 d^2 ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\text{arccsc}(c*x))/(e*x^2+d)^{(5/2)}, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(\text{sqrt}(e*x^2 + d)*(b*\text{arccsc}(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\text{acsc}(c*x))/(e*x**2+d)**(5/2), x)$

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \text{arccsc}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\text{arccsc}(c*x))/(e*x^2+d)^{(5/2)}, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}((b*\text{arccsc}(c*x) + a)/(e*x^2 + d)^{(5/2)}, x)$

### 3.165 $\int (fx)^m (d + ex^2)^3 (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=585

$$\frac{3d^2e(fx)^{m+3}(a + b \csc^{-1}(cx))}{f^3(m+3)} + \frac{d^3(fx)^{m+1}(a + b \csc^{-1}(cx))}{f(m+1)} + \frac{3de^2(fx)^{m+5}(a + b \csc^{-1}(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a + b \csc^{-1}(cx))}{f^7(m+7)}$$

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqrt[-1 + c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (b*e^3*x*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcCsc[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsc[c*x]))/(f^7*(7 + m)) + (b*((c^6*d^3*(2 + m)*(4 + m)*(6 + m))/(1 + m) + (e*(1 + m)*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/((3 + m)*(5 + m)*(7 + m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(c^5*f*(1 + m)*(2 + m)*(4 + m)*(6 + m)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

**Rubi [A]** time = 2.40489, antiderivative size = 566, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {270, 5239, 1809, 1267, 459, 365, 364}

$$\frac{3d^2e(fx)^{m+3}(a + b \csc^{-1}(cx))}{f^3(m+3)} + \frac{d^3(fx)^{m+1}(a + b \csc^{-1}(cx))}{f(m+1)} + \frac{3de^2(fx)^{m+5}(a + b \csc^{-1}(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a + b \csc^{-1}(cx))}{f^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]), x]
```

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqrt[-1 + c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (b*e^3*x*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcCsc[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsc[c*x]))/(f^7*(7 + m)) + (b*c*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/((3 + m)*(5 + m)*(7 + m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

#### Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

### Rule 5239

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rule 1809

Int[(Pq\_)\*((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(c\*x)^(m + q - 1)\*(a + b\*x^2)^(p + 1))/(b\*c^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(b\*(m + q + 2\*p + 1)), Int[(c\*x)^m\*(a + b\*x^2)^p\*ExpandToSum[b\*(m + q + 2\*p + 1)\*Pq - b\*f\*(m + q + 2\*p + 1)\*x^q - a\*f\*(m + q - 1)\*x^(q - 2), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])

### Rule 1267

Int[((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Simp[(c^p\*(f\*x)^(m + 4\*p - 1)\*(d + e\*x^2)^(q + 1))/(e\*f^(4\*p - 1)\*(m + 4\*p + 2\*q + 1)), x] + Dist[1/(e\*(m + 4\*p + 2\*q + 1)), Int[(f\*x)^m\*(d + e\*x^2)^q\*ExpandToSum[e\*(m + 4\*p + 2\*q + 1)\*((a + b\*x^2 + c\*x^4)^p - c^p\*x^(4\*p)) - d\*c^p\*(m + 4\*p - 1)\*x^(4\*p - 2), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4\*p + 2\*q + 1, 0]

### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 365

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2)^3 (a + b \operatorname{csc}^{-1}(cx)) dx &= \frac{d^3 (fx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{f(1+m)} + \frac{3d^2 e (fx)^{3+m} (a + b \operatorname{csc}^{-1}(cx))}{f^3(3+m)} + \frac{3de^2 (fx)^{5+m} (a + b \operatorname{csc}^{-1}(cx))}{f^5(5+m)} \\
&= \frac{be^3 x (fx)^{5+m} \sqrt{-1 + c^2 x^2}}{cf^5(6+m)(7+m)\sqrt{c^2 x^2}} + \frac{d^3 (fx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{f(1+m)} + \frac{3d^2 e (fx)^{3+m} (a + b \operatorname{csc}^{-1}(cx))}{f^3(3+m)} \\
&= \frac{be^2 (e(5+m)^2 + 3c^2 d (42 + 13m + m^2)) x (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{c^3 f^3 (4+m)(5+m)(6+m)(7+m)\sqrt{c^2 x^2}} + \frac{be^3 x (fx)^{5+m} (a + b \operatorname{csc}^{-1}(cx))}{cf^5(6+m)(7+m)} \\
&= \frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (840 + 638m + 105m^2))}{c^5 f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)} \\
&= \frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (840 + 638m + 105m^2))}{c^5 f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)} \\
&= \frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (840 + 638m + 105m^2))}{c^5 f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}
\end{aligned}$$

**Mathematica [A]** time = 1.67206, size = 402, normalized size = 0.69

$$x(fx)^m \left( \frac{3ad^2 ex^2}{m+3} + \frac{ad^3}{m+1} + \frac{3ade^2 x^4}{m+5} + \frac{ae^3 x^6}{m+7} - \frac{3bcd^2 ex^3 \sqrt{1 - \frac{1}{c^2 x^2}} {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; c^2 x^2\right)}{(m+3)^2 \sqrt{1 - c^2 x^2}} - \frac{bcd^3 x \sqrt{1 - \frac{1}{c^2 x^2}} {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; c^2 x^2\right)}{(m+1)^2 \sqrt{1 - c^2 x^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^3\*(a + b\*ArcCsc[c\*x]), x]

[Out] x\*(f\*x)^m\*((a\*d^3)/(1+m) + (3\*a\*d^2\*e\*x^2)/(3+m) + (3\*a\*d\*e^2\*x^4)/(5+m) + (a\*e^3\*x^6)/(7+m) + (b\*d^3\*ArcCsc[c\*x])/(1+m) + (3\*b\*d^2\*e\*x^2\*ArcCsc[c\*x])/(3+m) + (3\*b\*d\*e^2\*x^4\*ArcCsc[c\*x])/(5+m) + (b\*e^3\*x^6\*ArcCsc[c\*x])/(7+m) - (b\*c\*d^3\*sqrt[1 - 1/(c^2\*x^2)]\*x\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/((1+m)^2\*sqrt[1 - c^2\*x^2]) - (3\*b\*c\*d^2\*e\*sqrt[1 - 1/(c^2\*x^2)]\*x^3\*Hypergeometric2F1[1/2, (3+m)/2, (5+m)/2, c^2\*x^2])/((3+m)^2\*sqrt[1 - c^2\*x^2]) - (3\*b\*c\*d\*e^2\*sqrt[1 - 1/(c^2\*x^2)]\*x^5\*Hypergeometric2F1[1/2, (5+m)/2, (7+m)/2, c^2\*x^2])/((5+m)^2\*sqrt[1 - c^2\*x^2]) - (b\*c\*e^3\*sqrt[1 - 1/(c^2\*x^2)]\*x^7\*Hypergeometric2F1[1/2, (7+m)/2, (9+m)/2, c^2\*x^2])/((7+m)^2\*sqrt[1 - c^2\*x^2]))

**Maple [F]** time = 8.362, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d)^3 (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)`

[Out] `int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

`integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsc(cx))(f*x)^m, x)`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out] `integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsc(c*x))*(f*x)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsc(c*x)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^3 (b \operatorname{arccsc}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^3*(b*arccsc(c*x) + a)*(f*x)^m, x)`

### 3.166 $\int (fx)^m (d + ex^2)^2 (a + b \operatorname{csc}^{-1}(cx)) dx$

**Optimal.** Leaf size=371

$$\frac{d^2(fx)^{m+1} (a + b \operatorname{csc}^{-1}(cx))}{f(m+1)} + \frac{2de(fx)^{m+3} (a + b \operatorname{csc}^{-1}(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5} (a + b \operatorname{csc}^{-1}(cx))}{f^5(m+5)} + \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1} (c^4d^2)}{f^5(m+5)}$$

[Out] (b\*e\*(e\*(3 + m)^2 + 2\*c^2\*d\*(20 + 9\*m + m^2))\*x\*(f\*x)^(1 + m)\*Sqrt[-1 + c^2\*x^2])/(c^3\*f\*(2 + m)\*(3 + m)\*(4 + m)\*(5 + m)\*Sqrt[c^2\*x^2]) + (b\*e^2\*x\*(f\*x)^(3 + m)\*Sqrt[-1 + c^2\*x^2])/(c\*f^3\*(4 + m)\*(5 + m)\*Sqrt[c^2\*x^2]) + (d^2\*(f\*x)^(1 + m)\*(a + b\*ArcCsc[c\*x]))/(f\*(1 + m)) + (2\*d\*e\*(f\*x)^(3 + m)\*(a + b\*ArcCsc[c\*x]))/(f^3\*(3 + m)) + (e^2\*(f\*x)^(5 + m)\*(a + b\*ArcCsc[c\*x]))/(f^5\*(5 + m)) + (b\*(c^4\*d^2\*(2 + m)\*(3 + m)\*(4 + m)\*(5 + m) + e\*(1 + m)^2\*(e\*(3 + m)^2 + 2\*c^2\*d\*(20 + 9\*m + m^2)))\*x\*(f\*x)^(1 + m)\*Sqrt[1 - c^2\*x^2]\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2\*x^2])/(c^3\*f\*(1 + m)^2\*(2 + m)\*(3 + m)\*(4 + m)\*(5 + m)\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2])

**Rubi [A]** time = 0.432032, antiderivative size = 352, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.304, Rules used = {270, 5239, 12, 1267, 459, 365, 364}

$$\frac{d^2(fx)^{m+1} (a + b \operatorname{csc}^{-1}(cx))}{f(m+1)} + \frac{2de(fx)^{m+3} (a + b \operatorname{csc}^{-1}(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5} (a + b \operatorname{csc}^{-1}(cx))}{f^5(m+5)} + \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1} \left( \frac{e(2c^2)}{c^4} \right)}{f^5(m+5)}$$

Antiderivative was successfully verified.

[In] Int[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*e\*(e\*(3 + m)^2 + 2\*c^2\*d\*(20 + 9\*m + m^2))\*x\*(f\*x)^(1 + m)\*Sqrt[-1 + c^2\*x^2])/(c^3\*f\*(2 + m)\*(3 + m)\*(4 + m)\*(5 + m)\*Sqrt[c^2\*x^2]) + (b\*e^2\*x\*(f\*x)^(3 + m)\*Sqrt[-1 + c^2\*x^2])/(c\*f^3\*(4 + m)\*(5 + m)\*Sqrt[c^2\*x^2]) + (d^2\*(f\*x)^(1 + m)\*(a + b\*ArcCsc[c\*x]))/(f\*(1 + m)) + (2\*d\*e\*(f\*x)^(3 + m)\*(a + b\*ArcCsc[c\*x]))/(f^3\*(3 + m)) + (e^2\*(f\*x)^(5 + m)\*(a + b\*ArcCsc[c\*x]))/(f^5\*(5 + m)) + (b\*c\*(d^2/(1 + m)^2 + (e\*(e\*(3 + m)^2 + 2\*c^2\*d\*(20 + 9\*m + m^2)))/(c^4\*(2 + m)\*(3 + m)\*(4 + m)\*(5 + m)))\*x\*(f\*x)^(1 + m)\*Sqrt[1 - c^2\*x^2]\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2\*x^2])/(f\*Sqrt[c^2\*x^2]\*Sqrt[-1 + c^2\*x^2])

#### Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1267

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^(
q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
+ 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x]
/; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]
&& !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2)^2 (a + b \operatorname{csc}^{-1}(cx)) dx &= \frac{d^2 (fx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{f(1+m)} + \frac{2de (fx)^{3+m} (a + b \operatorname{csc}^{-1}(cx))}{f^3(3+m)} + \frac{e^2 (fx)^{5+m} (a + b \operatorname{csc}^{-1}(cx))}{f^5(5+m)} \\
&= \frac{d^2 (fx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{f(1+m)} + \frac{2de (fx)^{3+m} (a + b \operatorname{csc}^{-1}(cx))}{f^3(3+m)} + \frac{e^2 (fx)^{5+m} (a + b \operatorname{csc}^{-1}(cx))}{f^5(5+m)} \\
&= \frac{be^2 x (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{cf^3(4+m)(5+m)\sqrt{c^2 x^2}} + \frac{d^2 (fx)^{1+m} (a + b \operatorname{csc}^{-1}(cx))}{f(1+m)} + \frac{2de (fx)^{3+m} (a + b \operatorname{csc}^{-1}(cx))}{f^3(3+m)} \\
&= \frac{be (e(3+m)^2 + 2c^2 d (20 + 9m + m^2)) x (fx)^{1+m} \sqrt{-1 + c^2 x^2}}{c^3 f(2+m)(4+m)(15 + 8m + m^2) \sqrt{c^2 x^2}} + \frac{be^2 x (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{cf^3(4+m)(5+m)\sqrt{c^2 x^2}} \\
&= \frac{be (e(3+m)^2 + 2c^2 d (20 + 9m + m^2)) x (fx)^{1+m} \sqrt{-1 + c^2 x^2}}{c^3 f(2+m)(4+m)(15 + 8m + m^2) \sqrt{c^2 x^2}} + \frac{be^2 x (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{cf^3(4+m)(5+m)\sqrt{c^2 x^2}} \\
&= \frac{be (e(3+m)^2 + 2c^2 d (20 + 9m + m^2)) x (fx)^{1+m} \sqrt{-1 + c^2 x^2}}{c^3 f(2+m)(4+m)(15 + 8m + m^2) \sqrt{c^2 x^2}} + \frac{be^2 x (fx)^{3+m} \sqrt{-1 + c^2 x^2}}{cf^3(4+m)(5+m)\sqrt{c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.845237, size = 293, normalized size = 0.79

$$x(fx)^m \left( \frac{ad^2}{m+1} + \frac{2adex^2}{m+3} + \frac{ae^2x^4}{m+5} - \frac{bcd^2x\sqrt{1 - \frac{1}{c^2x^2}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; c^2x^2\right)}{(m+1)^2\sqrt{1 - c^2x^2}} - \frac{2bcdex^3\sqrt{1 - \frac{1}{c^2x^2}} {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; c^2x^2\right)}{(m+3)^2\sqrt{1 - c^2x^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcCsc[c\*x]), x]

[Out] x\*(f\*x)^m\*((a\*d^2)/(1 + m) + (2\*a\*d\*e\*x^2)/(3 + m) + (a\*e^2\*x^4)/(5 + m) + (b\*d^2\*ArcCsc[c\*x])/(1 + m) + (2\*b\*d\*e\*x^2\*ArcCsc[c\*x])/(3 + m) + (b\*e^2\*x^4\*ArcCsc[c\*x])/(5 + m) - (b\*c\*d^2\*sqrt[1 - 1/(c^2\*x^2)]\*x\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2\*x^2])/((1 + m)^2\*sqrt[1 - c^2\*x^2]) - (2\*b\*c\*d\*e\*sqrt[1 - 1/(c^2\*x^2)]\*x^3\*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2\*x^2])/((3 + m)^2\*sqrt[1 - c^2\*x^2]) - (b\*c\*e^2\*sqrt[1 - 1/(c^2\*x^2)]\*x^5\*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2\*x^2])/((5 + m)^2\*sqrt[1 - c^2\*x^2]))

**Maple [F]** time = 4.826, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d)^2 (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)), x)

[Out] int((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)), x)



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2)\arccsc(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e^2\*x^4 + 2\*a\*d\*e\*x^2 + a\*d^2 + (b\*e^2\*x^4 + 2\*b\*d\*e\*x^2 + b\*d^2)\*arccsc(c\*x))\*(f\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*\*2\*(a+b\*acsc(c\*x)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \arccsc(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsc(c\*x) + a)\*(f\*x)^m, x)

### 3.167 $\int (fx)^m (d + ex^2) (a + b \csc^{-1}(cx)) dx$

**Optimal.** Leaf size=215

$$\frac{d(fx)^{m+1} (a + b \csc^{-1}(cx))}{f(m+1)} + \frac{e(fx)^{m+3} (a + b \csc^{-1}(cx))}{f^3(m+3)} + \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1} (c^2d(m+2)(m+3) + e(m+1)^2)}{cf(m+1)^2(m+2)(m+3)\sqrt{c^2x^2}\sqrt{c^2x^2-1}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}, c^2x^2\right)$$

[Out] (b\*e\*x\*(f\*x)^(1+m)\*Sqrt[-1+c^2\*x^2])/(c\*f\*(6+5\*m+m^2)\*Sqrt[c^2\*x^2]) + (d\*(f\*x)^(1+m)\*(a+b\*ArcCsc[c\*x]))/(f\*(1+m)) + (e\*(f\*x)^(3+m)\*(a+b\*ArcCsc[c\*x]))/(f^3\*(3+m)) + (b\*(e\*(1+m)^2+c^2\*d\*(2+m)\*(3+m))\*x\*(f\*x)^(1+m)\*Sqrt[1-c^2\*x^2]\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/(c\*f\*(1+m)^2\*(2+m)\*(3+m)\*Sqrt[c^2\*x^2]\*Sqrt[-1+c^2\*x^2])

**Rubi [A]** time = 0.193828, antiderivative size = 202, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {14, 5239, 12, 459, 365, 364}

$$\frac{d(fx)^{m+1} (a + b \csc^{-1}(cx))}{f(m+1)} + \frac{e(fx)^{m+3} (a + b \csc^{-1}(cx))}{f^3(m+3)} + \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1} \left( \frac{e}{c^{2(m+2)(m+3)}} + \frac{d}{(m+1)^2} \right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}, c^2x^2\right)$$

Antiderivative was successfully verified.

[In] Int[(f\*x)^m\*(d + e\*x^2)\*(a + b\*ArcCsc[c\*x]),x]

[Out] (b\*e\*x\*(f\*x)^(1+m)\*Sqrt[-1+c^2\*x^2])/(c\*f\*(6+5\*m+m^2)\*Sqrt[c^2\*x^2]) + (d\*(f\*x)^(1+m)\*(a+b\*ArcCsc[c\*x]))/(f\*(1+m)) + (e\*(f\*x)^(3+m)\*(a+b\*ArcCsc[c\*x]))/(f^3\*(3+m)) + (b\*c\*(d/(1+m)^2 + e/(c^2\*(2+m)\*(3+m))))\*x\*(f\*x)^(1+m)\*Sqrt[1-c^2\*x^2]\*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2\*x^2])/(f\*Sqrt[c^2\*x^2]\*Sqrt[-1+c^2\*x^2])

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 5239

Int[((a\_.) + ArcCsc[(c\_)\*(x\_)])\*(b\_.)\*((f\_)\*(x\_))^(m\_.)\*((d\_.) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsc[c\*x], u, x] + Dist[(b\*c\*x)/Sqrt[c^2\*x^2], Int[SimplifyIntegrand[u/(x\*Sqrt[c^2\*x^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

### Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])
```

### Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

### Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2) (a + b \csc^{-1}(cx)) dx &= \frac{d(fx)^{1+m} (a + b \csc^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \csc^{-1}(cx))}{f^3(3+m)} + \frac{(bcx) \int \frac{(fx)^m (d(3+m) + ex^2) (a + b \csc^{-1}(cx))}{\sqrt{c^2x^2 - 1}} dx}{\sqrt{c^2x^2 - 1}} \\ &= \frac{d(fx)^{1+m} (a + b \csc^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \csc^{-1}(cx))}{f^3(3+m)} + \frac{(bcx) \int \frac{(fx)^m (d(3+m) + ex^2) (a + b \csc^{-1}(cx))}{\sqrt{c^2x^2 - 1}} dx}{(3 + 4m + 1)\sqrt{c^2x^2 - 1}} \\ &= \frac{bex(fx)^{1+m} \sqrt{-1 + c^2x^2}}{cf(6 + 5m + m^2) \sqrt{c^2x^2}} + \frac{d(fx)^{1+m} (a + b \csc^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \csc^{-1}(cx))}{f^3(3+m)} \\ &= \frac{bex(fx)^{1+m} \sqrt{-1 + c^2x^2}}{cf(6 + 5m + m^2) \sqrt{c^2x^2}} + \frac{d(fx)^{1+m} (a + b \csc^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \csc^{-1}(cx))}{f^3(3+m)} \\ &= \frac{bex(fx)^{1+m} \sqrt{-1 + c^2x^2}}{cf(6 + 5m + m^2) \sqrt{c^2x^2}} + \frac{d(fx)^{1+m} (a + b \csc^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \csc^{-1}(cx))}{f^3(3+m)} \end{aligned}$$

**Mathematica [A]** time = 0.511153, size = 171, normalized size = 0.8

$$x(fx)^m \left( \frac{\frac{(m+3)(d(m+3)+e(m+1)x^2)(a+b \csc^{-1}(cx))}{m+1} - \frac{bcex^3 \sqrt{1 - \frac{1}{c^2x^2}} {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; c^2x^2\right)}{\sqrt{1-c^2x^2}}}{(m+3)^2} - \frac{bcdx \sqrt{1 - \frac{1}{c^2x^2}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; c^2x^2\right)}{(m+1)^2 \sqrt{1-c^2x^2}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]), x]
```

```
[Out] x*(f*x)^m*(-((b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(1 + m)^2*Sqrt[1 - c^2*x^2])) + (((3 + m)*(d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcCsc[c*x]))/(1 + m) - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/Sqrt[1 - c^2*x^2])
```

$2*x^2])/(3 + m)^2)$

**Maple [F]** time = 3.074, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d)(a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)`

[Out] `int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*(f*x)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(e*x**2+d)*(a+b*acsc(c*x)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arccsc}(cx) + a)(fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

$$3.168 \quad \int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{d + ex^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{d + ex^2}, x \right)$$

[Out] Unintegrable[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

**Rubi [A]** time = 0.0751858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

[Out] Defer[Int][((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{d + ex^2} dx = \int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{d + ex^2} dx$$

**Mathematica [A]** time = 1.92326, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2), x]

**Maple [A]** time = 2.106, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{arccsc}(cx))}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d), x)

[Out] int((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a) (fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d),x, algorithm="maxima")

[Out] integrate((b\*arccsc(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b \operatorname{arccsc}(cx) + a) (fx)^m}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a) (fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

$$3.169 \quad \int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable} \left( \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2}, x \right)$$

[Out] Unintegrable[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2, x]

**Rubi [A]** time = 0.0741915, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2, x]

[Out] Defer[Int][((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2, x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx$$

**Mathematica [A]** time = 2.64339, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2, x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^2, x]

**Maple [A]** time = 2.455, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + \operatorname{arccsc}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)`

[Out] `int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

[Out] `integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")`

[Out] `integral((b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)`

$$\mathbf{3.170} \quad \int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Optimal.** Leaf size=27

$$\text{Unintegrable}\left((d + ex^2)^{3/2} (fx)^m (a + b \operatorname{csc}^{-1}(cx)), x\right)$$

[Out] Unintegrable[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.11548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int] [(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx = \int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Mathematica [A]** time = 1.0844, size = 0, normalized size = 0.

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 3.12, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsc}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)), x)

[Out] int((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arccsc}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsc(c\*x) + a)\*(f\*x)^m, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \operatorname{arccsc}(cx)\right)\sqrt{ex^2 + d} (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsc(c\*x))\*sqrt(e\*x^2 + d)\*(f\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsc(c\*x)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arccsc}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsc(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsc(c\*x) + a)\*(f\*x)^m, x)

$$3.171 \quad \int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Optimal.** Leaf size=27

$$\text{Unintegrable}\left(\sqrt{d + ex^2}(fx)^m (a + b \operatorname{csc}^{-1}(cx)), x\right)$$

[Out] Unintegrable[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

**Rubi [A]** time = 0.0972978, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

[Out] Defer[Int] [(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

Rubi steps

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx)) dx = \int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

**Mathematica [A]** time = 0.108481, size = 0, normalized size = 0.

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csc}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

[Out] Integrate[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsc[c\*x]), x]

**Maple [A]** time = 5.697, size = 0, normalized size = 0.

$$\int (fx)^m (a + b \operatorname{arccsc}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2), x)

[Out] int((f\*x)^m\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)(fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*(f\*x)^m, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*(f\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*acsc(c\*x))\*(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a)(fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsc(c\*x) + a)\*(f\*x)^m, x)

$$3.172 \quad \int \frac{(fx)^m (a + b \csc^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=27

$$\text{Unintegrable} \left( \frac{(fx)^m (a + b \csc^{-1}(cx))}{\sqrt{d + ex^2}}, x \right)$$

[Out] Unintegrable[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

**Rubi [A]** time = 0.100059, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \csc^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Defer[Int] [((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

Rubi steps

$$\int \frac{(fx)^m (a + b \csc^{-1}(cx))}{\sqrt{d + ex^2}} dx = \int \frac{(fx)^m (a + b \csc^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

**Mathematica [A]** time = 1.40697, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \csc^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/Sqrt[d + e\*x^2], x]

**Maple [A]** time = 7.631, size = 0, normalized size = 0.

$$\int (fx)^m (a + b \operatorname{arccsc}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

[Out] int((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate((b\*arccsc(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((b\*arccsc(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*acsc(c\*x))/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

$$3.173 \quad \int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=27

$$\text{Unintegrable} \left( \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

**Rubi [A]** time = 0.117209, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int] [((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 1.63662, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{csc}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsc[c\*x]))/(d + e\*x^2)^(3/2), x]

**Maple [A]** time = 4.121, size = 0, normalized size = 0.

$$\int (fx)^m (a + \operatorname{arccsc}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsc(c\*x))/(e\*x^2+d)^(3/2), x)



[Out] `int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a) (fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arccsc}(cx) + a) (fx)^m}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a) (fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)`

$$3.174 \quad \int \frac{x^{11}(a+b \csc^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$$

**Optimal.** Leaf size=401

$$-\frac{(1-c^4x^4)^{5/2}(a+b \csc^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b \csc^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b \csc^{-1}(cx))}{2c^{12}} - \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^9}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

[Out] (-4\*b\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + c^2\*x^2])/(15\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) + (7\*b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(3/2))/(90\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) - (13\*b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(5/2))/(150\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) + (3\*b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(7/2))/(70\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) - (b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(9/2))/(90\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) - (Sqrt[1 - c^4\*x^4]\*(a + b\*ArcCsc[c\*x]))/(2\*c^12) + ((1 - c^4\*x^4)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*c^12) - ((1 - c^4\*x^4)^(5/2)\*(a + b\*ArcCsc[c\*x]))/(10\*c^12) + (4\*b\*Sqrt[1 - c^2\*x^2]\*ArcTanh[Sqrt[1 + c^2\*x^2]])/(15\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x)

**Rubi [A]** time = 2.52723, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {266, 43, 5247, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$-\frac{(1-c^4x^4)^{5/2}(a+b \csc^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b \csc^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b \csc^{-1}(cx))}{2c^{12}} - \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^9}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(x^11\*(a + b\*ArcCsc[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] (-4\*b\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + c^2\*x^2])/(15\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) + (7\*b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(3/2))/(90\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) - (13\*b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(5/2))/(150\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) + (3\*b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(7/2))/(70\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) - (b\*Sqrt[1 - c^2\*x^2]\*(1 + c^2\*x^2)^(9/2))/(90\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x) - (Sqrt[1 - c^4\*x^4]\*(a + b\*ArcCsc[c\*x]))/(2\*c^12) + ((1 - c^4\*x^4)^(3/2)\*(a + b\*ArcCsc[c\*x]))/(3\*c^12) - ((1 - c^4\*x^4)^(5/2)\*(a + b\*ArcCsc[c\*x]))/(10\*c^12) + (4\*b\*Sqrt[1 - c^2\*x^2]\*ArcTanh[Sqrt[1 + c^2\*x^2]])/(15\*c^13\*Sqrt[1 - 1/(c^2\*x^2)]\*x)

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] := With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*sqrt[1 - 1/(c^2*x^2)])], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{11}(a + b \operatorname{csc}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4}(a + b \operatorname{csc}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2}(a + b \operatorname{csc}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4}(a + b \operatorname{csc}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2}(a + b \operatorname{csc}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4}(a + b \operatorname{csc}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2}(a + b \operatorname{csc}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4}(a + b \operatorname{csc}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2}(a + b \operatorname{csc}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4}(a + b \operatorname{csc}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2}(a + b \operatorname{csc}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4}(a + b \operatorname{csc}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2}(a + b \operatorname{csc}^{-1}(cx))}{10c^{12}} \\
&= -\frac{4b\sqrt{1 - c^2 x^2}\sqrt{1 + c^2 x^2}}{15c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} - \frac{\sqrt{1 - c^4 x^4}(a + b \operatorname{csc}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2}(a + b \operatorname{csc}^{-1}(cx))}{3c^{12}} \\
&= -\frac{4b\sqrt{1 - c^2 x^2}\sqrt{1 + c^2 x^2}}{15c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} + \frac{7b\sqrt{1 - c^2 x^2}(1 + c^2 x^2)^{3/2}}{90c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} - \frac{13b\sqrt{1 - c^2 x^2}(1 + c^2 x^2)^{5/2}}{150c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} + \frac{3b\sqrt{1 - c^2 x^2}(1 + c^2 x^2)^{7/2}}{7c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} \\
&= -\frac{4b\sqrt{1 - c^2 x^2}\sqrt{1 + c^2 x^2}}{15c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} + \frac{7b\sqrt{1 - c^2 x^2}(1 + c^2 x^2)^{3/2}}{90c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} - \frac{13b\sqrt{1 - c^2 x^2}(1 + c^2 x^2)^{5/2}}{150c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}} + \frac{3b\sqrt{1 - c^2 x^2}(1 + c^2 x^2)^{7/2}}{7c^{13}\sqrt{1 - \frac{1}{c^2 x^2}x}}
\end{aligned}$$

**Mathematica [A]** time = 0.290755, size = 194, normalized size = 0.48

$$105a\sqrt{1 - c^4 x^4}(3c^8 x^8 + 4c^4 x^4 + 8) + \frac{bcx\sqrt{1 - \frac{1}{c^2 x^2}}\sqrt{1 - c^4 x^4}(35c^8 x^8 + 5c^6 x^6 + 78c^4 x^4 + 36c^2 x^2 + 768)}{c^2 x^2 - 1} + 840b \tan^{-1}\left(\frac{cx\sqrt{1 - \frac{1}{c^2 x^2}}}{\sqrt{1 - c^4 x^4}}\right) + 105b\sqrt{1 - c^4 x^4}$$


---

3150c<sup>12</sup>

Antiderivative was successfully verified.

[In] Integrate[(x<sup>11</sup>\*(a + b\*ArcCsc[c\*x]))/Sqrt[1 - c<sup>4</sup>\*x<sup>4</sup>], x]

[Out] -(105\*a\*Sqrt[1 - c<sup>4</sup>\*x<sup>4</sup>]\*(8 + 4\*c<sup>4</sup>\*x<sup>4</sup> + 3\*c<sup>8</sup>\*x<sup>8</sup>) + (b\*c\*Sqrt[1 - 1/(c<sup>2</sup>\*x<sup>2</sup>)]\*x\*Sqrt[1 - c<sup>4</sup>\*x<sup>4</sup>]\*(768 + 36\*c<sup>2</sup>\*x<sup>2</sup> + 78\*c<sup>4</sup>\*x<sup>4</sup> + 5\*c<sup>6</sup>\*x<sup>6</sup> + 35\*c<sup>8</sup>\*x<sup>8</sup>))/(-1 + c<sup>2</sup>\*x<sup>2</sup>) + 105\*b\*Sqrt[1 - c<sup>4</sup>\*x<sup>4</sup>]\*(8 + 4\*c<sup>4</sup>\*x<sup>4</sup> + 3\*c<sup>8</sup>\*x<sup>8</sup>)\*ArcCsc[c\*x] + 840\*b\*ArcTan[(c\*Sqrt[1 - 1/(c<sup>2</sup>\*x<sup>2</sup>)]\*x)/Sqrt[1 - c<sup>4</sup>\*x<sup>4</sup>]]

4]])/(3150\*c^12)

**Maple [F]** time = 22.796, size = 0, normalized size = 0.

$$\int x^{11} (a + b \operatorname{arccsc}(cx)) \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11\*(a+b\*arccsc(c\*x))/(-c^4\*x^4+1)^(1/2),x)

[Out] int(x^11\*(a+b\*arccsc(c\*x))/(-c^4\*x^4+1)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{30} a \left( \frac{3(-c^4 x^4 + 1)^{\frac{5}{2}}}{c^{12}} - \frac{10(-c^4 x^4 + 1)^{\frac{3}{2}}}{c^{12}} + \frac{15 \sqrt{-c^4 x^4 + 1}}{c^{12}} \right) + \frac{c^{12} \int \frac{(3c^{10}x^{11} + 3c^8x^9 + 4c^6x^7 + 4c^4x^5 + 8c^2x^3 + 8x)e^{\left(-\frac{1}{2} \log(c^2x^2 + 1)\right)}}{(cx+1)(cx-1)\sqrt{-cx+1}c^{10} + \sqrt{-cx+1}c^{10}} dx}{c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(a+b\*arccsc(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/30\*a\*(3\*(-c^4\*x^4 + 1)^(5/2)/c^12 - 10\*(-c^4\*x^4 + 1)^(3/2)/c^12 + 15\*sqrt(-c^4\*x^4 + 1)/c^12) + 1/30\*(30\*c^12\*integrate(1/30\*(3\*c^10\*x^11 + 3\*c^8\*x^9 + 4\*c^6\*x^7 + 4\*c^4\*x^5 + 8\*c^2\*x^3 + 8\*x)\*e^(-1/2\*log(c^2\*x^2 + 1) + 1/2\*log(c\*x - 1))/(c^10\*e^(log(c\*x + 1) + log(c\*x - 1) + 1/2\*log(-c\*x + 1)) + sqrt(-c\*x + 1)\*c^10), x) - (3\*c^8\*x^8\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 4\*c^4\*x^4\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + 8\*arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1)))\*sqrt(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1))\*b/c^12

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(a+b\*arccsc(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^{11}}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^11/sqrt(-c^4*x^4 + 1), x)
```

$$3.175 \quad \int \frac{x^7 (a + b \csc^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$$

**Optimal.** Leaf size=268

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{5/2}}{30c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{3/2}}{18c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}}$$

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x) +
(b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
- (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(30*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
- (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)*
(a + b*ArcCsc[c*x]))/(6*c^8) + (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^
2]])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
```

**Rubi [A]** time = 2.0272, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {266, 43, 5247, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{5/2}}{30c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}} + \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{3/2}}{18c^9 x \sqrt{1 - \frac{1}{c^2 x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4], x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x) +
(b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
- (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(30*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
- (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)*
(a + b*ArcCsc[c*x]))/(6*c^8) + (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^
2]])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
```

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 5247

```
Int[((a_) + ArcCsc[(c_)*(x_)])*(b_)]*(u_), x_Symbol] := With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; F
reeQ[{a, b, c}, x]
```

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 6721

Int[(u\_)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 848

Int[((d\_) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_)^(n\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[(d + e\*x)^(m + p)\*(f + g\*x)^n\*(a/d + (c\*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e\*f - d\*g, 0] && EqQ[c\*d^2 + a\*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

### Rule 50

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*(c + d\*x)^n/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 783

Int[((d\_) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[(d + e\*x)^(m + p)\*(f + g\*x)\*(a/d + (c\*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c\*d^2 + a\*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

### Rubi steps



$$\begin{aligned}
\int \frac{x^7 (a + b \csc^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} + \frac{b \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{6c^8 \sqrt{1 - \frac{1}{c^2 x^2}} x^2} dx}{c} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} + \frac{b \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2} dx}{6c^9} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} + \frac{(b\sqrt{1 - c^2 x^2}) \int \frac{(-2 - c^4 x^4)}{x\sqrt{1 - \frac{1}{c^2 x^2}}} dx}{6c^9 \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst}\left(\int \frac{(-2 - c^4 x^4)}{x\sqrt{1 - \frac{1}{c^2 x^2}}} dx\right)}{12c^9 \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst}\left(\int \frac{(-2 - c^4 x^4)}{x\sqrt{1 - \frac{1}{c^2 x^2}}} dx\right)}{12c^9 \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst}\left(\int \frac{(-2 - c^4 x^4)}{x\sqrt{1 - \frac{1}{c^2 x^2}}} dx\right)}{6c^9 \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst}\left(\int \frac{(-2 - c^4 x^4)}{x\sqrt{1 - \frac{1}{c^2 x^2}}} dx\right)}{6c^9 \sqrt{1 - \frac{1}{c^2 x^2}}} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \csc^{-1}(cx))}{6c^8} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} + \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{18c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{30c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{6c^8} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} + \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{18c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{30c^9 \sqrt{1 - \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{6c^8}
\end{aligned}$$

**Mathematica [A]** time = 0.401217, size = 159, normalized size = 0.59

$$\frac{15a\sqrt{1 - c^4 x^4} (c^4 x^4 + 2) + \frac{bcx \sqrt{1 - \frac{1}{c^2 x^2}} \sqrt{1 - c^4 x^4} (3c^4 x^4 + c^2 x^2 + 28)}{c^2 x^2 - 1} + 30b \tan^{-1} \left( \frac{cx \sqrt{1 - \frac{1}{c^2 x^2}}}{\sqrt{1 - c^4 x^4}} \right) + 15b\sqrt{1 - c^4 x^4} (c^4 x^4 + 2) \csc^{-1}(cx)}{90c^8}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(a + b\*ArcCsc[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] -(15\*a\*Sqrt[1 - c^4\*x^4]\*(2 + c^4\*x^4) + (b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[1 - c^4\*x^4]\*(28 + c^2\*x^2 + 3\*c^4\*x^4))/(-1 + c^2\*x^2) + 15\*b\*Sqrt[1 - c^4\*x^4]\*(2 + c^4\*x^4)\*ArcCsc[c\*x] + 30\*b\*ArcTan[(c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/Sqrt[1 - c^4\*x^4]])/(90\*c^8)

---

**Maple [F]** time = 7.663, size = 0, normalized size = 0.

$$\int x^7 (a + b \operatorname{arccsc}(cx)) \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)`

[Out] `int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{6} a \left( \frac{(-c^4 x^4 + 1)^{\frac{3}{2}}}{c^8} - \frac{3 \sqrt{-c^4 x^4 + 1}}{c^8} \right) + \frac{\left( c^8 x^8 \arctan(1, \sqrt{cx+1} \sqrt{cx-1}) + \sqrt{c^2 x^2 + 1} \sqrt{cx+1} \sqrt{-cx+1} c^8 \int \frac{(c^6 x^7 + c^4 x^5 + 2c^2 x^3 + 2cx + 1)c^8}{6 \sqrt{c^2 x^2 + 1}} dx \right)}{6 \sqrt{c^2 x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*(c^8*x^8*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 6*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8*integrate(1/6*(c^6*x^7 + c^4*x^5 + 2*c^2*x^3 + 2*x)*e^(-1/2*log(c^2*x^2 + 1) + 1/2*log(c*x - 1))/(c^6*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + sqrt(-c*x + 1)*c^6), x) + c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^7}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^7/sqrt(-c^4*x^4 + 1), x)
```

$$3.176 \quad \int \frac{x^3(a+b \csc^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$$

**Optimal.** Leaf size=126

$$-\frac{\sqrt{1-c^4x^4}(a+b \csc^{-1}(cx))}{2c^4} - \frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}} + \frac{bx \tan^{-1}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{c^2x^2-1}}\right)}{2c^3\sqrt{c^2x^2}}$$

[Out]  $-(b*x*\text{Sqrt}[1 - c^4*x^4])/(2*c^3*\text{Sqrt}[c^2*x^2]*\text{Sqrt}[-1 + c^2*x^2]) - (\text{Sqrt}[1 - c^4*x^4]*(a + b*\text{ArcCsc}[c*x]))/(2*c^4) + (b*x*\text{ArcTan}[\text{Sqrt}[1 - c^4*x^4]/\text{Sqrt}[-1 + c^2*x^2]])/(2*c^3*\text{Sqrt}[c^2*x^2])$

**Rubi [A]** time = 0.212074, antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {261, 5247, 12, 1572, 1252, 848, 50, 63, 208}

$$-\frac{\sqrt{1-c^4x^4}(a+b \csc^{-1}(cx))}{2c^4} - \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{b\sqrt{1-c^2x^2} \tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(a + b*\text{ArcCsc}[c*x]))/\text{Sqrt}[1 - c^4*x^4], x]$

[Out]  $-(b*\text{Sqrt}[1 - c^2*x^2]*\text{Sqrt}[1 + c^2*x^2])/(2*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)]*x) - (\text{Sqrt}[1 - c^4*x^4]*(a + b*\text{ArcCsc}[c*x]))/(2*c^4) + (b*\text{Sqrt}[1 - c^2*x^2]*\text{ArcTanh}[\text{Sqrt}[1 + c^2*x^2]])/(2*c^5*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)$

#### Rule 261

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p+1)}/(b*n*(p+1)), x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{EqQ}[m, n-1] \ \&\& \ \text{NeQ}[p, -1]$

#### Rule 5247

$\text{Int}[(a_ + \text{ArcCsc}[c_*(x_)]*(b_))*(u_), x\_Symbol] \rightarrow \text{With}\{v = \text{IntHide}[u, x], \text{Dist}[a + b*\text{ArcCsc}[c*x], v, x] + \text{Dist}[b/c, \text{Int}[\text{SimplifyIntegrand}[v/(x^2*\text{Sqrt}[1 - 1/(c^2*x^2)]), x], x], x] /; \text{InverseFunctionFreeQ}[v, x] /; \text{FreeQ}\{a, b, c\}, x]$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 1572

$\text{Int}[(x_)^{(m_)}*((d_) + (e_)*(x_)^{(mn_)})^{(q_)}*((a_) + (c_)*(x_)^{(n2_)})^{(p_)}, x\_Symbol] \rightarrow \text{Dist}[(e^{\text{IntPart}[q]}*(d + e*x^{mn})^{\text{FracPart}[q]})/(x^{mn*\text{FracPart}[q]}*(1 + d/(x^{mn}*e))^{\text{FracPart}[q]}), \text{Int}[x^{(m + mn*q)}*(1 + d/(x^{mn}*e))^q*(a + c*x^{n2})^p, x], x] /; \text{FreeQ}\{a, c, d, e, m, mn, p, q\}, x] \ \&\& \ \text{EqQ}[n2, -2*mn] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !\text{IntegerQ}[q] \ \&\& \ \text{PosQ}[n2]$

#### Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

#### Rule 848

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_),
x_Symbol]
:> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

#### Rule 50

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol]
:> Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol]
:> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol]
:> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \csc^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} + \frac{b \int -\frac{\sqrt{1 - c^4 x^4}}{2c^4 \sqrt{1 - \frac{1}{c^2 x^2} x^2}} dx}{c} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} - \frac{b \int \frac{\sqrt{1 - c^4 x^4}}{\sqrt{1 - \frac{1}{c^2 x^2} x^2}} dx}{2c^5} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \int \frac{\sqrt{1 - c^4 x^4}}{x\sqrt{1 - c^2 x^2}} dx}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{\sqrt{1 - c^4 x^2}}{x\sqrt{1 - c^2 x}} dx, x, x^2 \right)}{4c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{\sqrt{1 + c^2 x}}{x} dx, x, x^2 \right)}{4c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{1}{x\sqrt{1 + c^2 x}} dx, x, x^2 \right)}{4c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \text{Subst} \left( \int \frac{1}{-\frac{1}{c^2} + \frac{x^2}{c^2}} dx, x, x^2 \right)}{2c^7 \sqrt{1 - \frac{1}{c^2 x^2} x}} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}} - \frac{\sqrt{1 - c^4 x^4} (a + b \csc^{-1}(cx))}{2c^4} + \frac{b\sqrt{1 - c^2 x^2} \tanh^{-1} \left( \sqrt{1 + c^2 x^2} \right)}{2c^5 \sqrt{1 - \frac{1}{c^2 x^2} x}}
\end{aligned}$$

**Mathematica [A]** time = 0.175771, size = 138, normalized size = 1.1

$$\frac{\sqrt{1 - c^4 x^4} \left( -ac^2 x^2 + a - bcx \sqrt{1 - \frac{1}{c^2 x^2}} \right) + (b - bc^2 x^2) \tan^{-1} \left( \frac{cx \sqrt{1 - \frac{1}{c^2 x^2}}}{\sqrt{1 - c^4 x^4}} \right) - b(c^2 x^2 - 1) \sqrt{1 - c^4 x^4} \csc^{-1}(cx)}{2c^4 (c^2 x^2 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsc[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] ((a - b\*c\*Sqrt[1 - 1/(c^2\*x^2)]\*x - a\*c^2\*x^2)\*Sqrt[1 - c^4\*x^4] - b\*(-1 + c^2\*x^2)\*Sqrt[1 - c^4\*x^4]\*ArcCsc[c\*x] + (b - b\*c^2\*x^2)\*ArcTan[(c\*Sqrt[1 - 1/(c^2\*x^2)]\*x)/Sqrt[1 - c^4\*x^4]])/(2\*c^4\*(-1 + c^2\*x^2))

**Maple [F]** time = 3.027, size = 0, normalized size = 0.

$$\int x^3 (a + \text{barccsc}(cx)) \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)`

[Out] `int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\left( c^4 \int \frac{(c^2 x^3 + x) e^{\left( -\frac{1}{2} \log(c^2 x^2 + 1) + \frac{1}{2} \log(cx - 1) \right)}}{(cx + 1)(cx - 1) \sqrt{-cx + 1} c^2 + \sqrt{-cx + 1} c^2} dx - \sqrt{c^2 x^2 + 1} \sqrt{cx + 1} \sqrt{-cx + 1} \arctan\left(1, \sqrt{cx + 1} \sqrt{cx - 1}\right) \right) b}{2 c^4} - \frac{\sqrt{-c^4 x^4 + 1} a}{2 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `1/2*(2*c^4*integrate(1/2*(c^2*x^3 + x)*e^(-1/2*log(c^2*x^2 + 1) + 1/2*log(c*x - 1))/(c^2*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + sqrt(-c*x + 1)*c^2), x) - sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arccsc(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)`

$$3.177 \quad \int \frac{a+b \csc^{-1}(cx)}{x\sqrt{1-c^4x^4}} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

**Rubi [A]** time = 0.0890698, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{1-c^4x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{1-c^4x^4}} dx = \int \frac{a+b \csc^{-1}(cx)}{x\sqrt{1-c^4x^4}} dx$$

**Mathematica [A]** time = 0.398408, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x\sqrt{1-c^4x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

**Maple [A]** time = 6.214, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x} \frac{1}{\sqrt{-c^4x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x/(-c^4\*x^4+1)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))/x/(-c^4\*x^4+1)^(1/2), x)



---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a\left(\log\left(\sqrt{-c^4x^4+1}+1\right)-\log\left(\sqrt{-c^4x^4+1}-1\right)\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)}{\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{-cx+1}x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(-c^4\*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/4\*a\*(log(sqrt(-c^4\*x^4 + 1) + 1) - log(sqrt(-c^4\*x^4 + 1) - 1)) + b\*integrate(arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(sqrt(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}(b\text{arccsc}(cx)+a)}{c^4x^5-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(-c^4\*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^4\*x^4 + 1)\*(b\*arccsc(c\*x) + a)/(c^4\*x^5 - x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\text{acsc}(cx)}{x\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x/(-c\*\*4\*x\*\*4+1)\*\*(1/2),x)

[Out] Integral((a + b\*acsc(c\*x))/(x\*sqrt(-(c\*x - 1)\*(c\*x + 1)\*(c\*\*2\*x\*\*2 + 1))), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int\frac{b\text{arccsc}(cx)+a}{\sqrt{-c^4x^4+1}x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x/(-c^4\*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(sqrt(-c^4\*x^4 + 1)\*x), x)

$$3.178 \quad \int \frac{a+b \csc^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}\left(\frac{a+b \csc^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCsc[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

**Rubi [A]** time = 0.100555, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{a+b \csc^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsc[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

[Out] Defer[Int] [(a + b\*ArcCsc[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

Rubi steps

$$\int \frac{a+b \csc^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}} dx = \int \frac{a+b \csc^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}} dx$$

**Mathematica [A]** time = 3.43584, size = 0, normalized size = 0.

$$\int \frac{a+b \csc^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsc[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

[Out] Integrate[(a + b\*ArcCsc[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

**Maple [A]** time = 1.84, size = 0, normalized size = 0.

$$\int \frac{a+b \operatorname{arccsc}(cx)}{x^5} \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsc(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x)

[Out] int((a+b\*arccsc(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8} \left( c^4 \log(\sqrt{-c^4 x^4 + 1} + 1) - c^4 \log(\sqrt{-c^4 x^4 + 1} - 1) + \frac{2\sqrt{-c^4 x^4 + 1}}{x^4} \right) a + b \int \frac{\arctan(1, \sqrt{cx+1}\sqrt{cx-1})}{\sqrt{c^2 x^2 + 1}\sqrt{cx+1}\sqrt{-cx+1} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/8\*(c^4\*log(sqrt(-c^4\*x^4 + 1) + 1) - c^4\*log(sqrt(-c^4\*x^4 + 1) - 1) + 2\*sqrt(-c^4\*x^4 + 1)/x^4)\*a + b\*integrate(arctan2(1, sqrt(c\*x + 1)\*sqrt(c\*x - 1))/(sqrt(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*x^5), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^4 x^4 + 1}(b \operatorname{arccsc}(cx) + a)}{c^4 x^9 - x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^4\*x^4 + 1)\*(b\*arccsc(c\*x) + a)/(c^4\*x^9 - x^5), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsc(c\*x))/x\*\*5/(-c\*\*4\*x\*\*4+1)\*\*(1/2),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{-c^4 x^4 + 1} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsc(c\*x))/x^5/(-c^4\*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsc(c\*x) + a)/(sqrt(-c^4\*x^4 + 1)\*x^5), x)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```

```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1  # File: GradeAntiderivative.mpl
2  # Original version thanks to Albert Rich emailed on 03/21/2017
3
4  #Nasser 03/22/2017  Use Maple leaf count instead since buildin
5  #Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
6  #Nasser 03/24/2017  corrected the check for complex result
7  #Nasser 10/27/2017  check for leafsize and do not call ExpnType()
8  #
9  #Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
10 #
11                          see problem 156, file Apostol_Problems
12
13 GradeAntiderivative := proc(result,optimal)
14 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
15     debug:=false;
16
17     leaf_count_result:=leafcount(result);
18     #do NOT call ExpnType() if leaf size is too large. Recursion problem
19     if leaf_count_result > 500000 then
20         return "B";
21     fi;
22
23     leaf_count_optimal:=leafcount(optimal);
24
25     ExpnType_result:=ExpnType(result);
26     ExpnType_optimal:=ExpnType(optimal);
27
28     if debug then
29         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
30             ExpnType_optimal);
31     fi;
32
33 # If result and optimal are mathematical expressions,
34 # GradeAntiderivative[result,optimal] returns
35 #   "F" if the result fails to integrate an expression that
36 #       is integrable
37 #   "C" if result involves higher level functions than necessary
38 #   "B" if result is more than twice the size of the optimal
39 #       antiderivative
40 #   "A" if result can be considered optimal
41
42 #This check below actually is not needed, since I only
43 #call this grading only for passed integrals. i.e. I check
44 #for "F" before calling this. But no harm of keeping it here.
45 #just in case.
46
47 if not type(result,freeof('int')) then
48     return "F";
49 end if;
50
51 if ExpnType_result<=ExpnType_optimal then
52     if debug then
53         print("ExpnType_result<=ExpnType_optimal");
54     fi;
55     if is_contains_complex(result) then
56         if is_contains_complex(optimal) then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```



```

119   if type(expn,'atomic') then
120       1
121   elif type(expn,'list') then
122       apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124       if type(op(1,expn),'rational') then
125           1
126       else
127           max(2,ExpnType(op(1,expn)))
128       end if
129   elif type(expn,'^^') then
130       if type(op(2,expn),'integer') then
131           ExpnType(op(1,expn))
132       elif type(op(2,expn),'rational') then
133           if type(op(1,expn),'rational') then
134               1
135           else
136               max(2,ExpnType(op(1,expn)))
137           end if
138       else
139           max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140       end if
141   elif type(expn,'+`') or type(expn,'*`') then
142       max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144       max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146       max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148       max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #     Port of original Maple grading function by
3 #     Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #     added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124     else:
125         return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34         #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52                       'sin','cos','tan','cot','sec','csc',
53                       'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54                       'sinh','cosh','tanh','coth','sech','csch',
55                       'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56                       'arctan2','floor','abs'
57                       ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73                      'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74                      'sinh_integral'
75                      'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76                      'polylog','lambert_w','elliptic_f','elliptic_e',
77                      'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91                          hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
          sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```